Online Supplement for "Outsourcing in Place: Should a Retailer Sell its Store-Brand Factory?"

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IIE Transactions

Appendix S1 Proof of Results in Figure 3

In this appendix, we detail the relationship among the wholesale prices under *IP*, $3P^N$ and $3P^S$ in the different capacity regions using four propositions. For each region, we express the relationships among the wholesale prices in the statement of the proposition, followed by its proof.

Proposition A *For K_S in Region 1, where the store-brand factory's capacity constraint is binding under IP, $3P^N$, and $3P^S$, the national brand's equilibrium wholesale prices satisfy $w^{C^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$.*

Proof: Using algebraic manipulations, we can show that the condition $w^{C^*}_{N,IP} > w^{C^*}_{N,3P^N}$ can be written as:

$$\frac{\gamma(\alpha_S - 2K_S) + (\beta_S + \gamma)\alpha_N}{2(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} > 0.5c_N \tag{29}$$

Notice that the left-hand side of the above inequality is equal to the first term in $w^{C^*}_{N,IP}$ for the capacity-constrained case (see Table 1). Therefore, if the above inequality is not satisfied, then $w^{C^*}_{N,IP}$ would not exceed c_N and the national brand would not participate. Thus, we have established that $w^{C^*}_{N,IP} > w^{C^*}_{N,3P^N}$. As shown in the analysis of the $3P^S$ configuration (in Appendix C.3), $w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$. Thus, $w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S} > w^{C^*}_{N,3P^N}$. This completes the proof. ■

Given the above, we consider the relationship between $w^{C^*}_{N,3P^N}$ and $w^{C^*}_{N,IP}$. If the store-brand factory has a binding capacity constraint under $3P^N$ with the third-party charging a mark-up over production cost, then the store-brand factory faces even tighter capacity restrictions in the absence of a wholesale mark-up. To compensate, the retailer has to pass on a high opportunity cost of capacity to customers to drive down demand. Facing little competition, the national brand could charge a high wholesale price. With the introduction of the third-party "middleman" under $3P^N$, the store-brand factory faces less problematic capacity restrictions, which then forces the national brand to price more competitively under *3P*^{N} than under IP.

We now discuss why $w^{C*}_{N,IP} = w^{C*}_{N,3P^S}$, which is a more subtle result. In Region 1, the store-brand factory is operating at capacity under both *3P*^{S} and IP and the store-brand factory provides the same supply to the market whether the factory is operated by the retailer or by a third party. In the IP configuration, the national brand chooses his wholesale price so that the resultant demand for the store-brand product is exactly equal to the factory's capacity. Under *3P*^{S}, the eventual outcome is the same, but the third-party manufacturer performs the capacity-clearing function (to ensure that his capacity constraint is not violated) via his choice of wholesale price in response to the national brand's wholesale price and in view of how the retailer will choose retail prices. In both cases, the national brand uses his first-mover advantage to "push" the store-brand factory to operate at full capacity. This turns out to be preferable for the national brand: although he sells fewer units, he sells each unit at a higher margin, and the differential in the margin more than compensates for the reduction in sales volume.

The same result as in Proposition A applies, with a similar rationale, when K_S is in Region 2, as stated in the next proposition.

Proposition B *For K_S in Region 2, where the store-brand factory is constrained under IP and 3P^S but unconstrained under 3P^N, the national brand's equilibrium wholesale prices satisfy $w^{U^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$.*

Proof: At the leftmost boundary of Region 2, $w^{C^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$ (by Proposition A). At the rightmost boundary, the store-brand factory is unconstrained under both *3P*^{N} and *3P*^{S}. Algebraically we can show that the condition $w^{U^*}_{N,3P^S} > w^{U^*}_{N,3P^N}$ can be written as $\dfrac{\gamma\alpha_S + (\beta_S+\gamma)(2\alpha_N+\gamma c_S)}{4(\beta_N+\gamma)(\beta_S+\gamma)-2\gamma^2} > 0.5c_N$.

Using a similar argument to that in the proof of Proposition A, we note that the left hand side of the above inequality is equal to the first term in $w^{U^*}_{N,3P^S}$. Therefore, if the above inequality is not satisfied, then the national brand would not participate. Thus, we have established that $w^{U^*}_{N,3P^S} > w^{U^*}_{N,3P^N}$ in Region 3 (and at the rightmost boundary of Region 2). The facts that in Region 2, $w^{U^*}_{N,3P^N}$ is constant and $w^{C^*}_{N,3P^S}$ is linearly decreasing in K_S complete the proof. ■

Proposition C *For K_S in Region 3, where the store-brand factory is constrained under IP but unconstrained under 3P^N and 3P^S, the national brand's equilibrium wholesale prices in the two outsourcing configurations are such that $w^{U^*}_{N,3P^N} < w^{U^*}_{N,3P^S}$. Furthermore, the wholesale price under IP, $w^{C^*}_{N,IP}$, may be larger, smaller, or in-between $w^{U^*}_{N,3P^N}$ and $w^{U^*}_{N,3P^S}$, depending upon the capacity level of the store-brand factory.*

Proof: See proof of Proposition B above. ■

Recall that the national brand's wholesale price is the same under *IP* and *3P*^{S} in Regions 1 and 2. At the lower boundary of Region 3, the store-brand factory becomes unconstrained under *3P*^{S}. Hence, the national brand's price becomes constant under *3P*^{S} while it continues to fall under *IP* because the store-brand factory is still constrained under *IP* in this region. Thus, the relationship between these two curves is not surprising. What is more interesting is that the price curve under *IP* crosses that of *3P*^{N} in this region. For smaller values of K_S within Region 3, the binding capacity of the store-brand factory under *IP* combined with the national brand's first-mover advantage enables the national brand to charge a high price despite the absence of double marginalization on the store-brand product. On the other hand, as K_S increases, the national brand must be more price-competitive due to greater supply of the store-brand product. Indeed, despite the national brand's first mover advantage under *IP*, when there is near-ample capacity at the store-brand factory, the national brand cannot charge as much as he could have under *3P*^{N}. This occurs because in the latter scenario, the national brand benefits from double marginalization on the store brand product, but such "protection" does not exist when the retailer owns the factory.

Proposition D *For K_S in Region 4, where the store-brand factory is unconstrained under all supply chain configurations, the national brand's equilibrium wholesale prices are such that $w^{U*}_{N,IP} < w^{U*}_{N,3P^N} < w^{U*}_{N,3P^S}$.*

Proof: The proof is straightforward and is omitted.

Appendix S2 Proof of Results in Section 5

Appendix S2.1 Proof of Lemma 1:

Proof of part (a): For K_S in Region 1, we can show that the first derivative of $\Pi^{C*}_{R,3P^N}$ with respect to K_S is strictly greater than zero, and the second derivative is non-negative. For Region 2, 3 and 4, where the factory is unconstrained under $3P^N$, $\Pi^{*}_{R,3P^N}$ does not depend on K_S. Thus, the retailer's profit function is convex increasing in K_S for Region 1, and constant for Regions 2, 3, and 4.

Proof of part (b): The retailer's profit in Regions 1, 2, and 3, under IP, is concave increasing in K_S. For K_S in Region 4, the profit function is constant. To show there a jump discontinuity at K_S^{IP}, we consider $\Pi^{U*}_{R,IP} - \Pi^{C*}_{R,IP}\left(K_S^{IP}\right)$, and show this difference is strictly greater than zero. The retailer's profit function under IP can be found in Table 1. We can show that the difference between the unconstrained profit and the constrained profit at K_S^{IP}, $\Pi_{R,IP}^{U^*} - \Pi_{R,IP}^{C^*}\left(K_S^{IP}\right) > 0$. Thus, we have a jump discontinuity at $K_S^{3P^S}$.

Proof of part (c): For K_S in Regions 1 and 2, the retailer's profit function is convex increasing in K_S.

Appendix S2.2 Proof of Proposition 1

We begin by showing that the retailer's profit under *3P*^{N} exceeds his profits under IP at $K_S = 0$. At $K_S = 0$, the retailer's profit under the IP and *3P*^{N} scenarios are, respectively:

$$\Pi_{R,IP}^{C^*}(K_S = 0) = \frac{\left[\gamma\alpha_S + (\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\,\gamma)c_N\right]^2}{16(\beta_S+\gamma)(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} \tag{30}$$

$$\Pi_{R,3P^N}^{C^*}(K_S = 0) = \frac{(\beta_N+\gamma)^2(\beta_S+\gamma)\left[\gamma\alpha_S + (\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N\right]^2}{4\left[(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)\left(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right)^2\right]} \tag{31}$$

We note that

$$\Pi_{R,3P^N}^{C^*}(K_S = 0) > \frac{(\beta_N+\gamma)^2(\beta_S+\gamma)[\gamma\alpha_S + (\beta_S+\gamma)\alpha_N - (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N]^2}{4\left[(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)(2(\beta_N+\gamma)(\beta_S+\gamma))^2\right]} > \Pi_{R,IP}^{C^*}(K_S = 0)$$

Thus, at $K_S = 0$, the retailer's profit under *3P*^{N} exceeds that under IP. Because $\Pi_{R,3P^N}^*$ is convex increasing in K_S, and $\Pi_{R,IP}^*$ is concave increasing in Region 1, the two functions cross at most twice in Region 1.

Appendix S2.3 Proof of Corollary 1

In Region 2, the retailer's profit under *3P*^{N} is constant, and is increasing under *IP*. Thus, they may cross at most once in Region 2. If the retailer is better off under *3P*^{N} at $K_S^{3P^N}$, then the retailer will be better off for some K_S-interval in Region 2.

Appendix S2.4 The Effect of Substitutability on the Retailer's Profits

We consider an example with $\alpha_N = 100$, $\alpha_S = 50$, $c_N = c_S = 1$, $\beta = 0.15$, two degrees of substitutability: $\gamma = 0.25$ (low) and $\gamma = 0.95$ (high). In Figure S1, we show the retailer's profit as a function of K_S for the two values of γ under both the IP and $3P^N$ configurations. For the high value of γ, we observe, as expected, that the retailer prefers the Nash configuration over a broad range of K_S values. For the low value of γ, we observe that the retailer prefers the Nash configuration over a smaller range of K_S values. Yet, the resulting market share for the store brand product (with $\gamma = 0.25$) is close to 25%, which is consistent with market shares observed in grocery retailing (Geller, 2011). Although the retailer achieves higher profit when γ is high, low substitutability does not preclude the retailer from benefiting from the sale of its store-brand factory. This is because even for low γ values, the effect of competition can dominate the effect of double-marginalization.

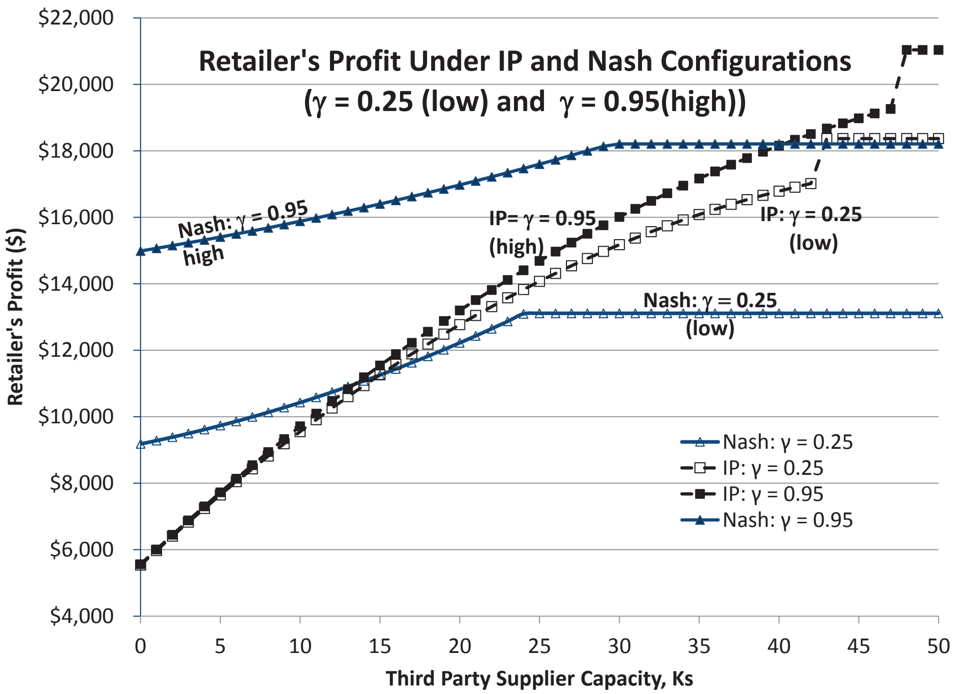

Figure S1: Equilibrium configurations under IP, $3P^N$, and $3P^S$ as a function of K_S.

Appendix S3 Results Regarding National Brand's Profits

Appendix S3.1 Proof of Lemma 2

Recall that the national brand's profits under the IP configuration are given in Table 1. The first and second derivatives of $\Pi^*_{N,IP}$ with respect to K_S are:

$$\frac{\partial \Pi^*_{N,IP}}{\partial K_S} = \begin{cases} \dfrac{\gamma[-(\beta_S+\gamma)\alpha_N + (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N - \gamma(\alpha_S - 2K_S)]}{2(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} < 0 & \text{for } K_S \in [0, K_S^{IP}] \\ 0 & \text{for } K_S \in [K_S^{IP}, \infty) \end{cases}$$

$$\frac{\partial^2 \Pi^*_{N,IP}}{\partial K_S^2} = \begin{cases} \dfrac{\gamma^2}{(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} > 0 & \text{for } K_S \in [0, K_S^{IP}] \\ 0 & \text{for } K_S \in [K_S^{IP}, \infty) \end{cases}$$

Substituting the expression for K_S^{IP} in $\Pi^{*C}_{N,IP}$, we find that all the terms not in the square root cancel, and we are left with

$$\begin{aligned} \Pi^{*C}_{N,IP}(K_S^{IP}) &= \frac{\left[\sqrt{\frac{(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{(\beta_N+\gamma)}}\right]^2}{8(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} = \frac{(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{8(\beta_N+\gamma)} \\ &= \Pi^{*U}_{N,IP}(K_S^{IP}) \end{aligned}$$

The above results imply that the national brand's profit function under the IP configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{IP}]$, and constant for $K_S \geq K_S^{IP}$.

For the *3P*^{S} configuration, we showed in Section 3.2.2 that $\Pi^{*C}_{N,3P^S} = \Pi^{*C}_{N,IP}$ for $K_S \in [0, K_S^{3P^S}]$. Thus, by the preceding analysis, we know the national brand's profit is convex decreasing in K_S for $K_S \in [0, K_S^{3P^S}]$, and constant for $K_S \geq K_S^{3P^S}$. The national brand's profit, given by (25), is:

$$\Pi^{C*}_{N,3P^S}(K_S = K_S^{3P^S}) = \left[\frac{\left[(\beta_S+\gamma)\alpha_N + \gamma\left(\alpha_S - 2K_S^{3P^S}\right) - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N\right]^2}{8(\beta_S+\gamma)(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} \right]$$

$$= \frac{1}{16}\left[\frac{[2(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S + (\beta_S+\gamma)c_S) - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N]^2}{(\beta_S+\gamma)(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)}\right] = \Pi^{U*}_{N,3P^S}\left(K_S^{3P^S}\right)$$

which is a constant. Thus, in summary, the national brand's profit function under the *3P*^{S} configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{3P^S}]$, and constant for $K_S \geq K_S^{3P^S}$.
The national brand supplier's profit functions under the *3P*^{N} configuration are given by (19) and (21) in the unconstrained and constrained cases, respectively. The first and second derivatives of (19) and (21) with respect to K_S are:

$$\frac{\partial \Pi^*_{N,3P^N}}{\partial K_S} = \begin{cases} \dfrac{2\gamma(\beta_N+\gamma)[-(\beta_S+\gamma)\alpha_N + (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N - \gamma(\alpha_S - 2K_S)]}{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]^2} < 0, & K_S \in [0, K_S^{3P^N}] \\ 0, & K_S \in [K_S^{3P^N}, \infty) \end{cases}$$

$$\frac{\partial^2 \Pi^*_{N,3P^N}}{\partial K_S^2} = \begin{cases} \dfrac{4\gamma^2(\beta_N+\gamma)}{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]^2} > 0 & \text{for } K_S \in [0, K_S^{3P^N}] \\ 0 & \text{for } K_S \in [K_S^{3P^N}, \infty) \end{cases}$$

Although it is algebraically messy, it is straightforward to show that $\Pi^{C*}_{N,3P^N}(K_S^{3P^N}) = \Pi^{U*}_{N,3P^N}(K_S^{3P^N})$. Hence, the national brand's profit function under the *3P*^{N} configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{3P^N}]$, and constant for $K_S \geq K_S^{3P^N}$.

Appendix S3.2 Proof of Lemma 3

We now compare the national brand's profits under different configurations, region by region.

We first prove that *for K_S in Region 1 (i.e., $K_S \in [0, K_S^{3P^N}]$)*, $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$. In Section 3.2.3, we showed that $\Pi^*_{N,IP} = \Pi^*_{N,3P^S}$ in Region 1, Thus, we only need to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$ in Region 1, which we establish next.The national brand's profits under the Nash and Stackelberg settings in Region 1 are given by (21) and (27), respectively. Dividing (27) by (21), we have

$$\frac{\Pi^*_{N,3PS}}{\Pi^*_{N,3PN}} = \frac{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]^2}{4(\beta_N+\gamma)(\beta_S+\gamma)\left[(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]} \geq 1.$$

Thus, $\Pi^*_{N,3P^N} < \Pi^*_{N,3PS} = \Pi^*_{N,IP}$ for all K_S in Region 1.

We now prove that *for K_S in Region 2 (i.e., $K_S \in [K_S^{3P^N}, K_S^{3P^S}]$)*, $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$. In Lemma 4, we established that $\Pi^*_{N,3PN}$ is constant in Region 2. In Section 3.2.3 we showed that $\Pi^*_{N,3PS} = \Pi^*_{N,IP}$ and in Lemma 4, we showed that these functions are decreasing in Region 2. Thus, to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$ in Region 2, we only need to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,IP}$ at $K_S^{3P^S}$, the rightmost boundary of Region 2, which we establish next.

$$
\begin{aligned}
\Pi^*_{N,3P^N}(K_S^{3P^S}) &= \frac{(\beta_N+\gamma)\cdot C_1}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} = \frac{(\beta_N+\gamma)(\beta_S+\gamma)\cdot C_1}{2(\beta_S+\gamma)\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \\
&< \frac{C_1}{4(\beta_S+\gamma)\left[\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma\right]} < \frac{C_1}{16(\beta_S+\gamma)\left[\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma\right]} \\
&= \Pi^*_{N,IP}(K_S^{3P^S})
\end{aligned}
$$

where $C_1 = \left[\gamma\alpha_S + (\beta_S+\gamma)\left[2\alpha_N+\gamma c_S\right] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2$.

We now prove: *For K_S in Region 3 (i.e., $K_S \in [K_S^{3P^S}, K_S^{IP}]$)*, $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$ *and* $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$. We have already shown that at $K_S^{3P^S}$ (the leftmost boundary of Region 3), the national brand's profit under IP equals his profit under *3P*^{S}, which exceeds his profit under *3P*^{N}. Because the national brand's profits under both *3P*^{N} and *3P*^{S} are constant in Region 3, it follows that for K_S in Region 3 (i.e., $K_S \in [K_S^{3P^S}, K_S^{IP}]$), $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$. Finally, because the national brand's profit under IP is decreasing in Region 3, and the profit under *3P*^{S} is constant, it must be the case that $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$ in Region 3.

We now show that *for K_S in Region 4 (i.e., $K_S \in [K_S^{IP}, \infty]$)*, $\Pi^*_{N,IP} \leq \Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$. We first prove that $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$. Because the national brand's profit under IP was shown above to be less or equal to his profit under *3P*^{S} (a constant) at the left boundary of Region 3, and his profit under IP is decreasing in Region 3 while his profit under *3P*^{S} is constant, it must be the case that in Region 4, it remains less than his profit under *3P*^{S}. We next show that $\Pi^*_{N,IP} \leq \Pi^*_{N,3P^N}$. We can write $\Pi^*_{N,IP}$ as:

$$
\begin{aligned}
\Pi^*_{N,IP} &= \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)[2\alpha_N+2\gamma c_S] - [2(\beta_N+\gamma)(\beta_S+\gamma)]\,c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)\right]^2} \\
&< \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)[2\alpha_N+2\gamma c_S] - [2(\beta_N+\gamma)(\beta_S+\gamma)]\,c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \equiv \Pi^{UpperBound}_{N,IP} \qquad (32)
\end{aligned}
$$

where $\Pi^{UpperBound}_{N,IP}$ is an upper bound on $\Pi^*_{N,IP}$.

Recall that ${\Pi_{N,3P^N}}^* = \dfrac{(\beta_N+\gamma)\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N+\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2}$. Dividing $\Pi_{N,3P^N}{}^*$by (32), we obtain

$$\frac{\Pi_{N,3P^N}{}^*}{\Pi_{N,IP}^{UpperBound}} = \frac{\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N + \gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right] c_N\right]^2}{\left[(\beta_S+\gamma)[2\alpha_N + 2\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)\right] c_N\right]^2} \geq 1$$

The inequality follows from the fact that the difference between the numerator and denominator is non-negative, i.e., $\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N + \gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right] c_N\right] -$

$$[(\beta_S+\gamma)[2\alpha_N + 2\gamma c_S] - [2(\beta_N+\gamma)(\beta_S+\gamma)]\, c_N] = \gamma\,[\alpha_S - (\beta_S+\gamma)c_S + \gamma c_N] = \gamma D_S(c_N, c_S) \geq 0 \quad (33)$$

Thus, we have shown that $\Pi^*_{N,IP} < \Pi_{N,IP}^{UpperBound} \leq \Pi^*_{N,3P^N}$ in Region 4.

Appendix S4 Proof of Lemma 4

It is straightforward to compare the demand expressions given in Tables 1, 2, and the demand expressions in Appendix C at $K_S = 0$ and at the various K_S thresholds to establish the claims in Lemma 4. The expressions for $D^{C^*}_{N,IP}$ and $D^{U^*}_{N,IP}$ can be found in Table 1. Table 2 contains the expressions for $D^{C^*}_{N,3P^N}$ and $D^{U^*}_{N,3P^N}$. Lastly, the expressions for $D^{C^*}_{N,3P^S}$ and $D^{U^*}_{N,3P^S}$ are shown in Appendix C.

For Region 1, we must show that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} \leq D^{C^*}_{N,3P^N}$. Region 1's leftmost boundary is $K_S = 0$. Recall from Tables 1 and 3 that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S}$, whereby we only need to establish the inequality between $D^{C^*}_{N,3P^S}$ and $D^{C^*}_{N,3P^N}$. Both demands are linearly decreasing in K_S (cf. Tables 1 and 3). Hence, we need only show that their values at the boundaries of Region 1 have the stated relationship. Substituting $K_S = 0$ into the expression for $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S}$, through simple algebraic manipulations we find that

$$\begin{aligned} D^{C^*}_{N,IP}(0) &= \frac{(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N + \gamma\alpha_S}{4(\beta_S+\gamma)} \\ &< \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta - N\gamma + \beta_S\gamma)c_N + \gamma(\alpha_S - 2K_S)\right]}{2[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]} = D^{C^*}_{N,3P^N}(0) \end{aligned}$$

Substituting $K_S^{3P^N}$ into the expression for $D^{C^*}_{N,IP}$, we obtain

$$\begin{aligned} D^{C^*}_{N,IP}(K_S^{3P^N}) &= \frac{(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)\left[2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]c_N\right]}{4(\beta_S+\gamma)[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]} \\ &\leq \frac{(\beta_N+\gamma)\left[2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]c_N\right]}{2[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]} \\ &= D^{C^*}_{N,3P^N}(K_S^{3P^N}) \end{aligned}$$

Thus, we have shown that $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} \le D^{C*}_{N,3P^N}$ in Region 1.

In Region 2, we require $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} < D^{U*}_{N,3P^N}$. At Region 2's leftmost boundary ($K_S = K_S^{3P^N}$), we have $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} < D^{C*}_{N,3P^N} = D^{U*}_{N,3P^N}$. The national brand's demand under the $3P^N$, $D^{U*}_{N,3P^N}$, is constant in K_S in Region 2, while the national brand's demand under $3P^S$ and IP continues to decrease in K_S. This implies that $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} < D^{U*}_{N,3P^N}$ in Region 2.

In Region 3, we require that $D^{C*}_{N,IP} \le D^{U*}_{N,3P^S} \le D^{U*}_{N,3P^N}$. For $K_S \ge K_S^{3P^S}$, a direct comparison of $D^{U*}_{N,3P^N}$ and $D^{U*}_{N,3P^S}$ (see Tables 2 and 3 for the expressions, both of which are constants) reveals that $D^{U*}_{N,3P^S} \le D^{U*}_{N,3P^N}$ because the two expressions share the same numerator, but the numerator for $D^{U*}_{N,3P^S}$ is larger. We now turn to a comparison of $D^{C*}_{N,IP}$ and $D^{U*}_{N,3P^S}$. At the leftmost boundary in Region 3 ($K_S = K_S^{3P^S}$), we have

$$D^{C*}_{N,IP}(K_S^{3P^S}) = \underbrace{\sqrt{\frac{2[(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}}}_{\le 1} \cdot \underbrace{\frac{2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]c_N}{8(\beta_S+\gamma)}}_{=D^{U*}_{N,3P^S}}$$

Thus, $D^{C*}_{N,IP} \le D^{U*}_{N,3P^S}$ at the leftmost boundary in Region 3. Combining this result with the facts that $D^{C*}_{N,IP}$ is decreasing in K_S in Region 3 while $D^{U*}_{N,3P^S}$ is constant, we have that $D^{C*}_{N,IP} \le D^{U*}_{N,3P^S} \le D^{U*}_{N,3P^N}$ in Region 3.

Finally, in Region 4, for $K_S \ge K_S^{IP}$, we have $D^{C*}_{N,IP} = \dfrac{\alpha_N - (\beta_N+\gamma)c_N + \gamma c_s}{4}$. Multiplying both the numerator and denominator by $2(\beta_S+\gamma)$, the expression becomes $\frac{[2(\beta_S+\gamma)\alpha_N + 2(\beta_S+\gamma)\gamma c_S - 2(\beta_N+\gamma)(\beta_S+\gamma)c_N]}{8(\beta_S+\gamma)}$, which is less than or equal to $D^{U*}_{N,3P^S}$ (cf. Table 3). Together with the fact that $D^{U*}_{N,3P^S} \le D^{U*}_{N,3P^N}$ for $K_S \ge K_S^{3P^S}$, we have that $D^{C*}_{N,IP} \le D^{U*}_{N,3P^S} \le D^{U*}_{N,3P^N}$ in Region 4.

Appendix S5 Third-Party Supplier Profits

In this appendix, we show that the third-party supplier prefers (profit-wise) that the national-brand supplier is the Stackelberg leader for K_S in Regions 1, 3, and 4. We conjecture that the result also holds in Region 2, although we have not been able to prove it because of the complicated form of $K_S^{3P^S}$. We have observed through numerous simulations that Region 2 covers a very narrow range of capacity levels (e.g., $K_S^{3P^S}$ is only a few percent larger than $K_S^{3P^N}$) and we have been unable to generate a counterexample to our conjecture.

The store-brand supplier's profits under the third party production configurations ($3P^N, 3P^S$) for both the unconstrained and constrained equilibrium settings are given below:

$$\Pi^{U*}_{S,3P^N} = \frac{(\beta_S+\gamma)\left[\gamma\alpha_N + (\beta_N+\gamma)(2\alpha_S+\gamma c_N) - \left(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)c_S\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2}$$

$$
\begin{aligned}
\Pi^{C*}_{S,3P^N} &= \frac{\left[\gamma\alpha_N + (\beta_N+\gamma)\left[2\alpha_S + \gamma c_N - 4K_S\right] - \left(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right) c_S\right] K_S}{2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2} \\
\Pi^{U*}_{S,3P^S} &= \frac{\begin{array}{l}\left[2\gamma(\beta_S+\gamma)\alpha_N + \left(4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right)\alpha_S\right. \\ \qquad \left. + \left(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right)\gamma c_N - (\beta_S+\gamma)(4(\beta_N+\gamma)(\beta_S+\gamma) - 3\gamma^2)c_S\right]^2\end{array}}{2(\beta_S+\gamma)\left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right]^2} \\
\Pi^{C*}_{S,3P^S} &= \frac{\begin{array}{l}\left[\gamma(\beta_S+\gamma)\alpha_N + (2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)(\alpha_S - 2K_S)\right. \\ \qquad \left. +((\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)(\gamma c_N - 2(\beta_S+\gamma)c_S)\right] K_S\end{array}}{2(\beta_S+\gamma)((\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)}
\end{aligned}
$$

To show that the third-party supplier always prefers the $3P^S$ to the $3P^N$ configuration in Regions 1, 3, and 4, we compare the applicable profits in each K_S region.

In Region 1, the store-brand factory has a binding capacity constraint under both *3P*^{S} and *3P*^{N}. We can write $\Pi^{C*}_{S,3P^S} - \Pi^{C*}_{S,3P^N}$ as:

$$
\Pi^{C*}_{S,3P^S} - \Pi^{C*}_{S,3P^N} = \frac{\gamma^3 K_S \cdot \left[(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S - 2K_S) - \left((\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right) c_N\right]}{2(\beta_S+\gamma)\left[(\beta_N+\gamma)(\beta_S+\gamma) - \gamma\right]\left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma\right]} \geq 0
$$

This follows from (29), where we show the bracketed term in the numerator is positive.

We next compare the third-party supplier's profits in Regions 3 and 4, where the capacity constraint is not binding under either *3P*^{S} or *3P*^{N}. The difference $\Pi^{U*}_{S,3P^S} - \Pi^{U*}_{S,3P^N}$ can be written as:

$$
\Pi^{U*}_{S,3P^S} - \Pi^{U*}_{S,3P^N} = \frac{\begin{array}{c}\gamma^3 \cdot \left[(\gamma^2 + \beta_S\gamma)c_S - (\gamma^2 + 2\beta_S\gamma + 2\beta_S\beta_N + 2\gamma\beta_N)c_N + (2\gamma + 2\beta_S)\alpha_N + \gamma\alpha_S\right] \cdot \\ \left[\begin{array}{c}\left(8(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right)\left[2(\beta_S+\gamma)\gamma\alpha_N + (4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)\alpha_S + \right. \\ \left. (2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)\gamma c_N - (\beta_S+\gamma)(4(\beta_N+\gamma)(\beta_S+\gamma) - 3\gamma^2)c_S\right] \\ -4\gamma^2((\beta_S+\gamma)\gamma\alpha_N + (\beta_N+\gamma)(\beta_S+\gamma)\alpha_S - (\beta_S+\gamma)((\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)c_S)\end{array}\right]\end{array}}{32(\beta_S+\gamma)\left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma\right]^2\left[4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma\right]^2}
$$

which we can show is decreasing in c_s. In Appendix D.1, we showed that (28) is an upper bound on c_s. Substituting this upper bound into the above expression, we obtain a lower bound on the profit difference:

$$
\Pi^{U*}_{S,3P^S} - \Pi^{U*}_{S,3P^N} \geq \frac{\gamma^6\left[\gamma\alpha_S + (\beta_N+\gamma)\alpha_N - \left((\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right) c_N\right]^2}{2(\beta_S+\gamma)\left[4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma\right]^2\left[4(\beta_N+\gamma)(\beta_S+\gamma) - 3\gamma\right]^2} \geq 0
$$

Thus, the third-party supplier always prefers that the national brand is the Stackelberg leader for K_S in Regions 1, 3, and 4.

References

Geller, M. (2011). Store-brand food seen eating up market share. `www.reuters.com/article/2011/03/24/us-food-privatelabel-idUSTRE72N4U320110324`. [Online; accessed 27-July-2015].