Appendix D Projection matrix

See figure on next page.

$$
\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{pmatrix}_t =
\begin{pmatrix}
0 & 0 & s_{(3,t)} f_n \left(1-\phi_{(t)}\right) & 0 & 0 & s_{(6,t)} f_c (1-v)\left(1-\phi_{(t)}\right) & s_{(7,t)} f_p \left((1-\psi)\left(1-\phi_{(t)}\right)+ \psi(1-v)\left(1-\phi_{(t)}\right)\right) & 0 \\
s_{(1,t)}(1-m)\left(1-\phi_{(t)}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & s_{(2,t)}\left(1-\phi_{(t)}\right) & s_{(3,t)}\left(1-\phi_{(t)}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & s_{(3,t)} f_n \phi_{(t)} & 0 & 0 & s_{(6,t)} f_c \left(v+\phi_{(t)}-v\phi_{(t)}\right) & s_{(7,t)} \psi f_p \left(v+\phi_{(t)}-v\phi_{(t)}\right)+ (1-\psi)\phi_{(t)} & 0 \\
s_{(1,t)}\phi_{(t)}(1-m) & 0 & 0 & s_{(4,t)}(1-m) & 0 & 0 & 0 & 0 \\
0 & s_{(2,t)}\phi_{(t)} & s_{(3,t)}\phi_{(t)} & 0 & s_{(5,t)} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & s_{(6,t)} & s_{(7,t)} & 0 \\
s_{(1,t)} m & 0 & 0 & s_{(4,t)} m & 0 & 0 & 0 & s_{(8,t)}
\end{pmatrix}
\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{pmatrix}_{t-1}
$$

Figure D.1: Model of brucellosis transmission in the Yellowstone bison population. Elements of the state vector $\mathbf{n}_t$ are defined in Table 2. Definitions of parameters are f_n: number of juveniles recruited per seronegative adult female, f_p: number of juveniles recruited per seropositive adult female, f_c: number of juveniles recruited per seroconverting (infectious) adult female, ψ: proportion of recovered population that is infectious, v: probability of vertical transmission, $\phi_{(t)}$: probability that a susceptible female becomes exposed during $t-1 \to t$, and m: proportion of juveniles surviving to becomes yearling males. The survival probability $s_{(i,t)}$ reflects natural mortality and management removals equation 3. Survival in the absence of removals is p_1 for juveniles, p_2 for yearling and adult females, and p_3 and for yearling and adult males. We estimate $\phi_{(t)}$ at each time step using equation 3 and $s_{(i,t)}$ using equation 4 as described in the text. The dependence of $\phi_{(t)}$ on $\boldsymbol{n}_{(t)}$ means that the model is non-linear.