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Appendix A: Details of model development

Single parasitoid models

In the absence of *C. viator* parasitism, the rescaled full parasitoid model has $A_t = N_t$ and $N_{t+1} = P_{t+1}$, so we can eliminate A and P from the model, giving

$$
\begin{aligned}
L_{t+1} &= N_t \exp[r(1 - N_t/K)] & \text{(A.1)}\\
N_{t+1} &= L_{t+1} \exp[-a_d D_{t+1}/(1 + a_d h_d N_t + a_d b_d D_{t+1})] & \text{(A.2)}\\
D_{t+2} &= L_{t+1}\left(1 - \exp[-a_d D_{t+1}/(1 + a_d h_d N_t + a_d b_d D_{t+1})]\right). & \text{(A.3)}
\end{aligned}
$$

A constant but nonzero rate of *C. viator* parasitism leads to the same equations, with mortality due to *C. viator* parasitism absorbed into the intrinsic rate of increase e^r.

Similarly, if parasitism by *Dusona* is absent then $P_{t+1} = L_{t+1}$ and we can eliminate L, giving

$$
\begin{aligned}
A_t &= N_t \exp[-a_c C_{2,t}/(1 + a_c h_c N_t + a_c b_c C_{2,t})] & \text{(A.4)}\\
C_{1,t+1} &= s_c N_t \left(1 - \exp[-a_c C_{2,t}/(1 + a_c h_c N_t + a_c b_c C_{2,t})]\right) & \text{(A.5)}\\
P_{t+1} &= A_t \exp[r(1 - A_t/K)] & \text{(A.6)}\\
N_{t+1} &= P_{t+1} \exp[-a_c C_{1,t+1}/(1 + a_c h_c P_{t+1} + a_c b_c C_{1,t+1})] & \text{(A.7)}\\
C_{2,t+1} &= P_{t+1} \left(1 - \exp[-a_c C_{1,t+1}/(1 + a_c h_c P_{t+1} + a_c b_c C_{1,t+1})]\right), & \text{(A.8)}
\end{aligned}
$$

and the same equations hold after re-scaling if *Dusona* parasitism is constant.

Scaling the maternal effects model

Using Eq. (14) of the text to eliminate E_t from the 3-equation model yields

$$
\begin{aligned}
N_{t+1} &= r N_t \left(-a + b W_t\right) e^{-s N_t(-a+b W_t) + u W_t} & \text{(A.9)}\\
W_{t+1} &= P_{\min} + P_0 e^{-\beta N_t(-a+b W_t)}. & \text{(A.10)}
\end{aligned}
$$

We now make the following substitutions:

$$X_t = \frac{bW_t - a}{bP_0} \tag{A.11}$$

$$r' = rbP_0 e^{au/b} \tag{A.12}$$

$$s' = sbP_0 \tag{A.13}$$

$$u' = uP_0 \tag{A.14}$$

$$x_{\min} = \frac{bP_{\min} - a}{bP_0} \tag{A.15}$$

$$\beta' = \beta bP_0. \tag{A.16}$$

Substituting these into Eqs. (A.9–A.10) and suppressing the primes gives the final form of the model.

Oviposition preference function for food quality model

The data in Fig. 1 of Šmits et al. (2001) presumably represent the number of eggs after some eggs have dropped off the new needles. Thus we need to correct those data to reconstruct the egg laying preference. Let $\hat{e}_m$ be the fraction of eggs observed on mature needles (from the figure) and let μ_e be the fraction of eggs that fall off of new needles. Then the true fraction of eggs laid on mature needles is

$$e_m = \frac{\hat{e}_m}{\hat{e}_m + (1 - \hat{e}_m)/(1 - \mu_e)}. \tag{A.17}$$

We digitized the data from Fig. 1 of Šmits et al. (2001), and corrected the oviposition fraction (using Eq. A.17) for the estimated value of μ_e and for the extreme values observed. After some trial and error we fit the corrected preference data with an equation of the form

$$1 = \left(\frac{A_{(m)t}}{A_t}\right)^\alpha + (1 - F_t)^\alpha \tag{A.18}$$

$$\frac{A_{(m)t}}{A_t} = \left[(1 - F_t)^\alpha\right]^{1/\alpha}, \tag{A.19}$$

which appears to provide a satisfactory fit (Fig. A.1). The parameter estimates are in Table A.1.
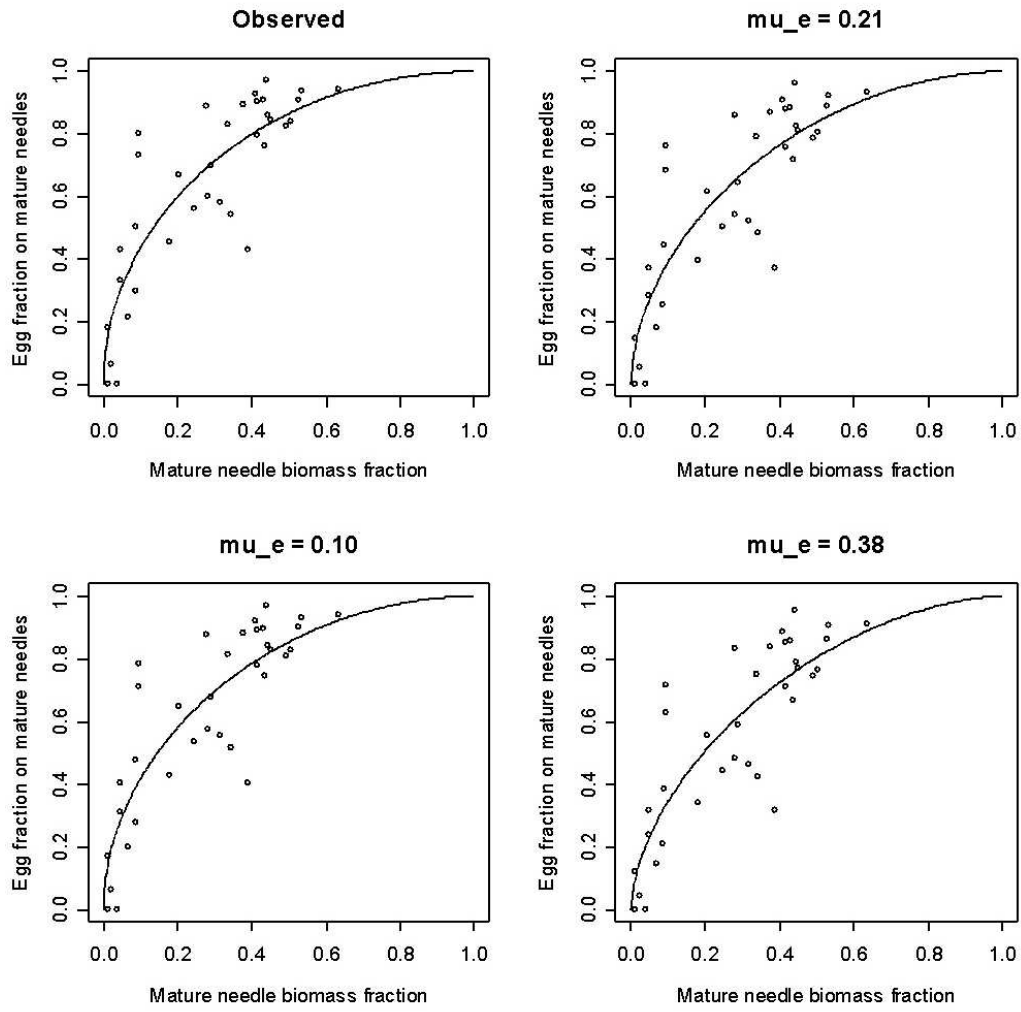
Figure A.1: Fits of Eq. (A.18) under different egg drop assumptions. The circles are data; the lines are the fitted curves.

Table A.1: Estimated value of α from Eq. (A.18), along with its standard error and the standard error of the residuals.

Data	$\hat{\alpha}$	SE(α)	residual SE
Observed	1.998	0.114	0.151
$\mu_e = 0.21$	1.843	0.102	0.154
$\mu_e = 0.10$	1.927	0.109	0.152
$\mu_e = 0.38$	1.695	0.091	0.156

Literature Cited

Šmits, A., S. Larsson, and R. Hopkins. 2001. Reduced realised fecundity in the pine looper *Bupalus piniarius* caused by host plant defoliation. Ecological Entomology **26**:417–424.