A Appendix: Derivation of the moment approximation for a stochastic birth-death-growth process

A.1 Definitions

An individual plant is characterised by a physical location in the plane, given by Cartesian coordinates $x = (x_1, x_2)$, and a measure of size s which could be made in a number of ways (e.g. length, mass). The location x is fixed at birth, and size s can increase over time.

The derivation is based on a 3-dimensional state space, and each plant is at a position in this space given by (x, s). To construct the spatial moments up to order 3, we consider the geometry of three small volumes v, v', v'' in the state space, centred on points $(x, s), (x', s'), (x'', s'')$ respectively (Fig. A.1). The physical space and the point process are assumed to be spatially homogeneous, and we work with the displacement ξ' of x' from x, and the displacement ξ'' of x'' from x. The three volumes do not intersect, and they are small enough for the probability of containing more than one individual to be small relative to the probability of containing a single individual.

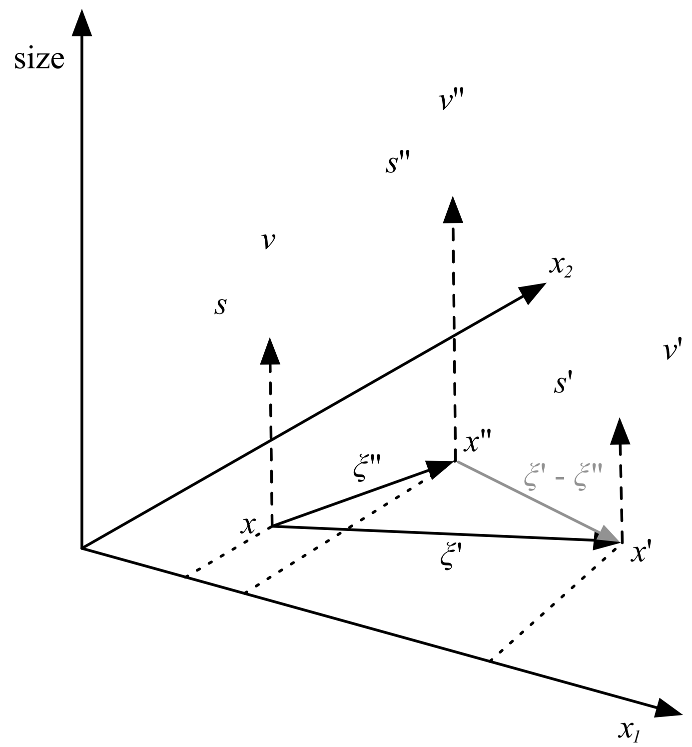

Figure A.1: 3D space describing area (x_1, x_2) and size s; v, v', v'' are small, nonintersecting volumes with relative locations as indicated, which contain one individual at most.

A.1.1 Densities at time t

The first three spatial moments are the densities of single individuals, pairs and triplets, defined in terms of the expected number of individuals in the volumes v, v', v''. In a multispecies community, the species identity of these individuals may be different, and species types are therefore indexed up to third order by i, j, k. The moments become functions of time t in the dynamics below, so they have t as one of their arguments.

The density of individuals of type i with size s at time t is the expected value of the number $n_i(x, s, t)$ of individuals of type i found in a volume v centred on $\{x, s\}$, in the limit as $v \to 0$

$$m_{1,i}(s, t) = \lim_{v \to 0} E\left[\frac{n_i(x, s, t)}{v}\right]; \tag{A.1}$$

this function is the size distribution of type i. The assumption of spatial homogeneity ensures that there is no dependence of densities on location x. The density of pairs of type i, j with sizes s, s', where j is displaced by ξ' from i, is the expected number of pairs of type i and j in the volumes v and v' respectively, in the limit as $v \to 0, v' \to 0$

$$m_{2,ij}(\xi', s, s', t) = \lim_{v \to 0, v' \to 0} E\left[\frac{n_i(x, s, t) n_j(x + \xi', s', t)}{vv'}\right]. \tag{A.2}$$

Similarly, the density of triplets of type i, j, k with sizes s, s', s'', where j is displaced by ξ' from i and k is displaced by ξ'' from i, is the expected number of triplets of types i, j, k in the volumes v, v', v'' respectively in the limit as $v \to 0, v' \to 0, v'' \to 0$

$$m_{3,ijk}(\xi', \xi'', s, s', s'', t) = \lim_{v \to 0, v' \to 0, v'' \to 0} E\left[\frac{n_i(x, s, t) n_j(x + \xi', s', t) n_k(x + \xi'', s'', t)}{vv'v''}\right]. \tag{A.3}$$

A.1.2 Mark correlation function

For visualisation of the second moment a lower-dimensional mark pair density is introduced. This is a projection of the volumes v, v' on to the horizontal x-plane of Fig. A.1, so the volumes become areas a, a', and expected value of the number of pairs is replaced by the expected value of the product of the sizes and the pairs

$$m_{2,ij}^{(ss')}(\xi, t) = \lim_{a \to 0, a' \to 0} E\left[\frac{[s \times n_i(x, s, t)][s' \times n_j(x + \xi, s', t)]}{aa'}\right]. \tag{A.4}$$

A dimensionless mark correlation function (Stoyan and Penttinen, 2000) is built from this by dividing by the pair density function $m_{2,ij}^{(0)}(\xi, t)$ (which ignores size by removing s and s' from Eqn. A.4) and the product of the mean sizes $\mu_i(t), \mu_j(t)$ of types i and j

$$\hat{m}_{2,ij}(\xi, t) = \frac{1}{\mu_i(t)\mu_j(t)} \frac{m_{2,ij}^{(ss')}(\xi, t)}{m_{2,ij}^{(0)}(\xi, t)}. \tag{A.5}$$

This function shows the spatial structure of the marks after the spatial structure of the locations of individuals has been accounted for.

A.1.3 Indicator functions and probabilities

The moments are transformed to probabilities by multiplying by the volumes in the denominator of each equation. Let $I_{v,i} = 1$ indicate that the volume v is occupied by type i. The probability of there being an individual of type i in v at time t is

$$P(I_{v,i} = 1) = m_{1,i}(s, t)\ v. \tag{A.6}$$

The probability of type i in v and type j in v' at time t is

$$\begin{aligned} P(I_{v,i} = 1 \ \&\ I_{v',j} = 1) &= m_{2,ij}(\xi', s, s', t)\ v\ v', \\ &= P(I_{v',j} = 1 | I_{v,i} = 1)\ P(I_{v,i} = 1), \\ &= P(I_{v',j} = 1 | I_{v,i} = 1)\ m_{1,i}(s, t)\ v, \end{aligned} \tag{A.7}$$

from which the conditional probability of type j in v', given type i in v, is

$$P(I_{v',j} = 1 | I_{v,i} = 1) = \frac{m_{2,ij}(\xi', s, s', t)\ v'}{m_{1,i}(s, t)}. \tag{A.8}$$

The probability of type i in v, type j in v' and type k in v'' at time t is

$$\begin{aligned} P(I_{v,i} = 1 \ \&\ I_{v',j} = 1 \ \&\ I_{v'',k} = 1) &= m_{3,ijk}(\xi', \xi'', s, s', s'', t)\ v\ v'\ v'', \\ &= P(I_{v'',k} = 1 | I_{v,i} = 1 \ \&\ I_{v',j} = 1)\ m_{2,ij}(\xi', s, s', t)\ v\ v', \end{aligned} \tag{A.9}$$

from which the conditional probability of type k in v'', given type i in v and type j in v', is

$$P(I_{v'',k} = 1 | I_{v,i} = 1 \ \&\ I_{v',j} = 1) = \frac{m_{3,ijk}(\xi', \xi'', s, s', s'', t)\ v''}{m_{2,ij}(\xi', s, s', t)}. \tag{A.10}$$

A.2 Dynamics of the first moment

A.2.1 Rate terms for growth, birth and death of individuals

The growth rate of a focal individual of type i in volume v has a density-independent component $g_i(s)$, and a density-dependent component as a result of other individuals in its neighbourhood. The latter component sums over all neighbours, allowing for the effect of species j, displacement ξ' and size s', using a dimensionless kernel function $w_{ij}^{(g)}(\xi', s, s')$:

$$\begin{aligned} G_{1,i}(s,t) &= g_i(s) + \sum_j g'_{ij} \iint w_{ij}^{(g)}(\xi', s, s') \boxed{P(I_{v',j} = 1 | I_{v,i} = 1)\ \frac{1}{v'}}\ d\xi'\ ds' \\ &= g_i(s) + \sum_j g'_{ij} \iint w_{ij}^{(g)}(\xi', s, s') \frac{m_{2,ij}(\xi', s, s', t)}{m_{1,i}(s,t)}\ d\xi'\ ds' \ . \end{aligned} \tag{A.11}$$

The boxed term in Eq. A.11 divides the conditional probability by v', to convert the conditional probability into the conditional density of pairs, as in Eq. (A.8).

Similarly, the rate at which a focal individual of type i in volume v dies is

$$D_{1,i}(s,t) = d_i(s) - \sum_j d'_{ij} \iint w_{ij}^{(d)}(\xi', s, s') \frac{m_{2,ij}(\xi', s, s', t)}{m_{1,i}(s,t)}\ d\xi'\ ds' \ . \tag{A.12}$$

Births contribute only to the flux at the left-hand side of the size distribution, and this contribution depends on the neighbourhood of the i individual in v as

$$B_{1,i}(s,t) = b_i(s) + \sum_j b'_{ij} \iint w_{ij}^{(b)}(\xi', s, s') \frac{m_{2,ij}(\xi', s, s', t)}{m_{1,i}(s,t)}\ d\xi'\ ds' \ . \tag{A.13}$$

Birth and death rates, like growth, have a density-independent component $b_i(s), d_i(s)$, and a density-dependent component that sums over all neighbours, using the dimensionless kernel functions $w_{ij}^{(b)}(\xi', s, s')$, $w_{ij}^{(d)}(\xi', s, s')$. Parameters $g'_{ij}, b'_{ij}, d'_{ij}$ weight the effect of neighbours: values < 0 imply neighbours are deleterious, values $= 0$ imply no interactions, values > 0 imply neighbours are beneficial (note the different sign in Eq. A.12). Care needs to be taken to ensure that the rates remain ecologically reasonable; for instance, birth and death rates cannot become negative.

A.2.2 Rate of change of density at size s and time t

The dynamics of $m_{1,i}(s,t)$ are constructed from the change, over a short period of time $[t, t + \delta t]$, in probability that the volume v has an individual of type i. There are two processes involved in this: growth and death. (Birth affects the probability only at the smallest size, and is dealt with separately as a boundary condition.) In the following argument, growth, death and birth events are assumed to take place as independent Poisson processes over time with rates prescribed by Eqns A.11, A.12 and A.13 respectively. This means that the probability of more than one event occurring during a short time interval of length δt is $O(\delta t^2)$.

The probability of finding an individual of type i in v at time $t+\delta t$ is

$$
\begin{aligned}
&m_{1,i}(s,t+\delta t)\ v = \\
&\quad + m_{1,i}(s,t)\ v \ \times\ P(\text{no loss from } v \text{ during } [t,t+\delta t]|I_{v,i}=1 \text{ at } t) \\
&+ (1-m_{1,i}(s,t)\ v)\ \times\ P(\text{growth into } v \text{ during } [t,t+\delta t]|I_{v,i}=0 \text{ at } t).
\end{aligned}
\tag{A.14}
$$

The probabilities in Eq. A.14 break down as follows:

$$
\begin{aligned}
&P(\text{no loss from } v \text{ during } [t,t+\delta t]|I_{v,i}=1 \text{ at } t) \\
&\qquad = P(\text{no growth and no death in } [t,t+\delta t]| \text{ present at time } t) \\
&\qquad = 1 - P(\text{size increment } \delta s \text{ in } [t,t+\delta t]) - P(\text{death in } [t,t+\delta t]) \ + O(\delta t^2) \\
&\qquad = \ 1 - \frac{\delta t}{\delta s} G_{1,i}(s,t) - \delta t D_{1,i}(s,t) + O(\delta t^2)
\end{aligned}
\tag{A.15}
$$

$$
\begin{aligned}
&P(\text{growth into } v \text{ during } [t,t+\delta t]|I_{v,i}=0) \\
&\qquad = P(\text{at size } s-\delta s \text{ and location } x \text{ at time } t) \\
&\qquad \times P(\text{growth from } s-\delta s \text{ to } s \text{ during } [t,t+\delta t]) \\
&\qquad = \ m_{1,i}(s-\delta s,t)\ v \ \times \frac{\delta t}{\delta s} G_{1,i}(s-\delta s,t) + O(\delta t^2),
\end{aligned}
\tag{A.16}
$$

where $G_{1,i}$ and $D_{1,i}$ are as in (A.11) and (A.12). The terms δs and δt specify the size increment and time period respectively, cancelling the dimensions of the growth and death rates, making the expressions dimensionless. Substituting (A.15) and (A.16) into (A.14), subtracting $m_1(s,t)$ from each side, dividing by δt, and simplifying by removing the terms still multiplied by v (which are negligible compared to the other terms because v is small), gives

$$
\begin{aligned}
\frac{m_{1,i}(s,t+\delta t) - m_{1,i}(s,t)}{\delta t} &= - \left[\frac{m_{1,i}(s,t)G_{1,i}(s,t) - m_{1,i}(s-\delta s,t)G_{1,i}(s-\delta s,t)}{\delta s}\right] \\
&\quad - m_{1,i}(s,t)D_{1,i}(s,t)\ .
\end{aligned}
\tag{A.17}
$$

In the limit $\delta s \to 0, \delta t \to 0$, Eq. (A.17) becomes

$$
\frac{\partial m_{1,i}(s,t)}{\partial t} = -\frac{\partial\left[m_{1,i}(s,t)G_{1,i}(s,t)\right]}{\partial s} - m_{1,i}(s,t)D_{1,i}(s,t)\ , \tag{A.18}
$$

which is a size-based version of the von Foerster equation. Thus the rate of change of the density of individuals at size s over time is a function of the growth rate of the individual with respect to its own size, and the death rate, both of which are coupled to the second moment, which describes the arrangement of the individuals in the plane.

Individuals are born at the smallest size $s_{0,i}$ of type i. At this boundary on body size, the term $m_{1,i}(s_0-\delta s,t)G_{1,i}(s_0-\delta s,t)$ in Eq. (A.17) is replaced by the total rate at which births are taking place

$$
\int m_{1,i}(s,t)\ B_{1,i}(s,t)\ ds\ , \tag{A.19}
$$

where $B_{1,i}$ is as given in Eq. (A.13).

A.3 Dynamics of the second moment

The dynamics of the second moment are more intricate than those of the first moment because they have to deal with events leading to gain and loss of individuals both in volume v and in v' of the pair.

A.3.1 Rate terms for growth, birth and death of pairs

The contribution that growth of a type i individual in volume v makes to the rate of change in density of the i,j pair, where j is in v' and displaced by ξ' from v, has to allow for neighbours of the pair of type k in volume v'' displaced by ξ'' from v. This requires the conditional dependence (A.10), shown as the boxed term below:

$$\begin{aligned} G_{2,ij}(\xi',s,s',t) &= g_i(s) \\ &\quad + \sum_k g'_{ik} \iint w_{ik}^{(g)}(\xi'',s,s'') \boxed{P(I_{v'',k}=1 | I_{v,i}=1 \;\&\; I_{v',j}=1)\ \frac{1}{v''}}\ d\xi''\ ds'' \ +\ g'_{ij}\ w_{ij}^{(g)}(\xi',s,s') \\ &= g_i(s) + \sum_k g'_{ik} \iint w_{ik}^{(g)}(\xi'',s,s'') \frac{m_{3,ijk}(\xi',\xi'',s,s',s'',t)}{m_{2,ij}(\xi',s,s',t)}\ d\xi''\ ds'' \ +\ g'_{ij}\ w_{ij}^{(g)}(\xi',s,s')\ , \end{aligned} \tag{A.20}$$

where the parameters are as defined in Section A.2.1. The final term in (A.20) is needed to allow for the effect of type j in v' on growth of i in v. The contribution that death of an individual of type i in volume v makes to the rate of change in density of the i,j pair is obtained in a similar way as

$$D_{2,ij}(\xi',s,s',t) = \ d_i(s) - \sum_k d'_{ik} \iint w_{ik}^{(d)}(\xi'',s,s'') \frac{m_{3,ijk}(\xi',\xi'',s,s',s'',t)}{m_{2,ij}(\xi',s,s',t)}\ d\xi''\ ds'' \ -\ d'_{ij}\ w_{ij}^{(d)}(\xi',s,s'). \tag{A.21}$$

Births contribute only to the flux at the left-hand side of the size distribution, but this contribution depends on the neighbourhood of the pair as

$$B_{2,ij}(\xi',s,s',t) = \ b_i(s) + \sum_k b'_{ik} \iint w_{ik}^{(b)}(\xi'',s,s'') \frac{m_{3,ijk}(\xi',\xi'',s,s',s'',t)}{m_{2,ij}(\xi',s,s',t)}\ d\xi''\ ds'' \ +\ b'_{ij}\ w_{ij}^{(b)}(\xi',s,s')\ . \tag{A.22}$$

Parameters for the birth and death rates are as defined in Section A.2.1.

A.3.2 Rate of change in pair density

The dynamics of $m_{2,ij}(\xi',s,s',t)$ are constructed from the change over a short period of time $[t,t+\delta t]$ in the probability that both the volume v contains an individual of type i and also the volume v' contains an individual of type j. Growth and death both contribute to this change, but birth in general does not, because it generates an input only at the smallest size $s_{0,i}$ (this is dealt with separately as a boundary condition). The assumption that growth, death and birth events are independent Poisson processes over time remains in place, now with rates prescribed by Eqns A.20, A.21 and A.22 respectively. This means that the probability of more than one event occurring during a short time interval of length δt is $O(\delta t^2)$.

Taking the components of the flux apart, using the rules of conditional probability, the probability at $t+\delta t$ is:

$$\begin{aligned} m_{2,ij}(\xi',s,s',t+\delta t)\ v\ v' \quad &= \\ +\ m_{2,ij}(\xi',s,s',t)\ v\ v' \quad &\times P(\text{no loss from } v \text{ or } v' \text{ during } [t,t+\delta t] | I_{v,i}=1 \;\&\; I_{v',j}=1 \text{ at } t) \\ +\ m_{1,i}(s,t)\ v \quad &\times P(\text{growth by } j \text{ into } v' \text{ during } [t,t+\delta t] | I_{v,i}=1 \;\&\; I_{v',j}=0 \text{ at } t) \\ +\ m_{1,j}(s',t)\ v' \quad &\times P(\text{growth by } i \text{ into } v \text{ during } [t,t+\delta t] | I_{v,i}=0 \;\&\; I_{v',j}=1 \text{ at } t)\ . \end{aligned} \tag{A.23}$$

The second and third unconditional probabilities have been simplified here by taking just the lowest order terms in v and v', i.e.

$$\begin{aligned} P(I_{v,i}=1 \;\&\; I_{v',j}=0 \text{ at } t) \ &= \ m_{1,i}(s,t)\ v - m_{2,ij}(\xi',s,s',t)\ v\ v' \ \approx\ m_{1,i}(s,t)\ v \\ P(I_{v,i}=0 \;\&\; I_{v',j}=1 \text{ at } t) \ &= \ m_{1,j}(s',t)\ v' - m_{2,ij}(\xi',s,s',t)\ v\ v' \ \approx\ m_{1,j}(s',t)\ v' \end{aligned}$$

because the higher order terms in v and v' are negligible for small volumes.

The next step takes apart the conditional probabilities in (A.23). The probability in the first term, conditional on the presence of the pair $I_{v,i}=1, I_{v',j}=1$ at time t, breaks down as

$$\begin{aligned}
&P(\text{no loss from } v \text{ or } v' \text{ during } [t,t+\delta t] | I_{v,i}=1 \ \& \ I_{v',j}=1 \text{ at } t)\\
&\quad = +1\\
&\qquad - P(\text{size increment } \delta s \text{ by } i \text{ in } v \text{ during } [t,t+\delta t])\\
&\qquad - P(\text{size increment } \delta s \text{ by } j \text{ in } v' \text{ during } [t,t+\delta t])\\
&\qquad - P(\text{death of } i \text{ in } v \text{ during } [t,t+\delta t])\\
&\qquad - P(\text{death of } j \text{ in } v' \text{ during } [t,t+\delta t]) + O(\delta t^2)\\
&\quad = 1 - \frac{\delta t}{\delta s} G_{2,ij}(\xi', s, s', t) - \frac{\delta t}{\delta s} G_{2,ji}(-\xi', s', s, t)\\
&\qquad - \delta t\ D_{2,ij}(\xi', s, s', t) - \delta t\ D_{2,ji}(-\xi', s', s, t) + O(\delta t^2).
\end{aligned} \tag{A.24}$$

The probability in the second term, conditional on $I_{v,i}=1, I_{v',j}=0$ at time t, breaks down as

$$\begin{aligned}
&P(\text{growth by } j \text{ into } v' \text{ during } [t,t+\delta t] | I_{v,i}=1 \ \& \ I_{v',j}=0 \text{ at } t)\\
&\quad = P(\text{type } j \text{ at size } s'-\delta s \text{ at } t \mid \text{type } i \text{ at size } s \text{ in } v \text{ at } t)\\
&\quad \times P(\text{size increment } \delta s \text{ by } j \text{ into } v' \text{ during } [t,t+\delta t])\\
&\quad = \frac{m_{2,ji}(-\xi', s'-\delta s, s, t)\ v'}{m_{1,i}(s,t)} \times \frac{\delta t}{\delta s} G_{2,ji}(-\xi', s'-\delta s, s, t) + O(\delta t^2),
\end{aligned} \tag{A.25}$$

with the assumption that the volume occupied by j at size $s'-\delta s$ is v'. The probability in the third term, conditional on $I_{v,i}=0, I_{v',j}=1$ at time t, breaks down as

$$\begin{aligned}
&P(\text{growth by } i \text{ into } v \text{ during } [t,t+\delta t] | I_{v,i}=0 \ \& \ I_{v',j}=1 \text{ at } t)\\
&\quad = P(\text{type } i \text{ at size } s-\delta s \text{ at } t \mid \text{type } j \text{ at size } s' \text{ in } v' \text{ at } t)\\
&\quad \times P(\text{size increment } \delta s \text{ by } i \text{ into } v \text{ during } [t,t+\delta t])\\
&\quad = \frac{m_{2,ij}(\xi', s-\delta s, s', t)\ v}{m_{1,j}(s',t)} \times \frac{\delta t}{\delta s} G_{2,ij}(\xi', s-\delta s, s', t) + O(\delta t^2),
\end{aligned} \tag{A.26}$$

with the assumption that the volume occupied by i at size $s-\delta s$ is v.

Lastly, the conditional probabilities (A.24), (A.25), (A.26) are substituted into (A.23). In doing this, the term $m_{2,ij}(\xi', s, s', t)$ is subtracted from both sides, and the equation divided by δt, leaving

$$\begin{aligned}
&\frac{1}{\delta t}\ [m_{2,ij}(\xi', s, s', t+\delta t) - m_{2,ij}(\xi', s, s', t)]\\
&= -\ \frac{1}{\delta s}\ [m_{2,ij}(\xi', s, s', t) G_{2,ij}(\xi', s, s', t) \ - \ m_{2,ij}(\xi', s-\delta s, s', t) G_{2,ij}(\xi', s-\delta s, s', t)]\\
&\quad -\ \frac{1}{\delta s}\ [m_{2,ji}(-\xi', s', s, t) G_{2,ji}(-\xi', s', s, t) \ - \ m_{2,ji}(-\xi', s'-\delta s, s, t) G_{2,ji}(-\xi', s'-\delta s, s, t)]\\
&\quad - m_{2,ij}(\xi', s, s', t) D_{2,ij}(\xi', s, s', t) \ - \ m_{2,ji}(-\xi', s', s, t) D_{2,ji}(-\xi', s', s, t)\ ;
\end{aligned} \tag{A.27}$$

the symmetry $m_{2,ij}(\xi', s, s', t) = m_{2,ji}(-\xi', s', s, t)$ has been exploited in writing this equation. Taking the limit of Eq. (A.27) as $\delta s \to 0, \delta t \to 0$, gives the dynamics of the second spatial moment

$$\begin{aligned}
&\frac{\partial m_{2,ij}(\xi', s, s', t)}{\partial t}\\
&\quad = -\ \frac{\partial\, [m_{2,ij}(\xi', s, s', t) G_{2,ij}(\xi', s, s', t)]}{\partial s} - \frac{\partial\, [m_{2,ij}(\xi', s, s', t) G_{2,ji}(-\xi', s', s, t)]}{\partial s'}\\
&\qquad - \ m_{2,ij}(\xi', s, s', t) D_{2,ij}(\xi', s, s', t) \ - \ m_{2,ij}(\xi', s, s', t) D_{2,ji}(-\xi', s', s, t)\ ,
\end{aligned} \tag{A.28}$$

where the symmetry $m_{2,ij}(\xi', s, s', t) = m_{2,ji}(-\xi', s', s, t)$ has again been used. This equation has a structure similar to that of the first-moment dynamics (A.18), with a growth term and a death term, but it now deals with the changes in both individuals in the pair.

A.3.3 Boundary conditions for pair density

Let $m_i^{(b)}(-\xi')$ be a dispersal kernel function describing the displacement $-\xi'$ at which a newborn individual of type i comes to rest, relative to its parent (also of type i); the function is normalised to sum to unity over all displacements. Consider the boundary on body size comprising an ij pair, in which type i has size $s_{0,i}$ (the offspring), and type j has size s' and is displaced by ξ' from i. At this boundary, the term $m_{2,ij}(\xi', s_0 - \delta s, s', t)G_{2,ij}(\xi', s_0 - \delta s, s', t)$ in Eq. (A.27) is replaced by the total rate at which births of type i create this pair. There are five ways in which this happens, most easily understood in terms of geometries in Fig. A.2. The corresponding rate terms for each are

$$\begin{aligned}
&\iint b_i(s'')\ m_i^{(b)}(-\xi'')\ m_{2,ij}(\xi'-\xi'', s'', s', t)\ ds''d\xi'' \\
&+\sum_k b'_{ik} \iint m_i^{(b)}(-\xi'') \iint w_{ik}^{(b)}(\xi''', s'', s''')\ m_{3,ijk}(\xi'-\xi'', \xi''', s'', s', s''', t)\ ds'''d\xi'''ds''d\xi'' \\
&+b'_{ij} \iint m_i^{(b)}(-\xi'')\ w_{ij}^{(b)}(\xi'-\xi'', s'', s')\ m_{2,ij}(\xi'-\xi'', s'', s', t)\ ds''d\xi'' \\
&+m_i^{(b)}(-\xi')\ b_i(s')\ m_{1,i}(s', t) \\
&+m_i^{(b)}(-\xi')\ \sum_k b'_{ik} \iint w_{ik}^{(b)}(\xi''', s', s''')\ m_{2,ik}(\xi''', s', s''', t)\ ds'''d\xi'''. \qquad \text{(A.29)}
\end{aligned}$$

The first term comes from the intrinsic birth rate of a parent in v'' (Fig. A.2a). The second term comes from the modification of this birth rate due to neighbours of that parent (Fig. A.2b). The third term adds in the effect of j to the modification of the birth rate of the parent in v'', as j is itself a neighbour of the parent (Fig. A.2c). The fourth and fifth terms come from the contributions made when the parent is located in v', with the intrinsic birth rate (Fig. A.2d), and the modification by neighbours (Fig. A.2e). There is a corresponding set of terms for the boundary $m_{2,ij}(\xi', s, s_{0,j}, t)$.

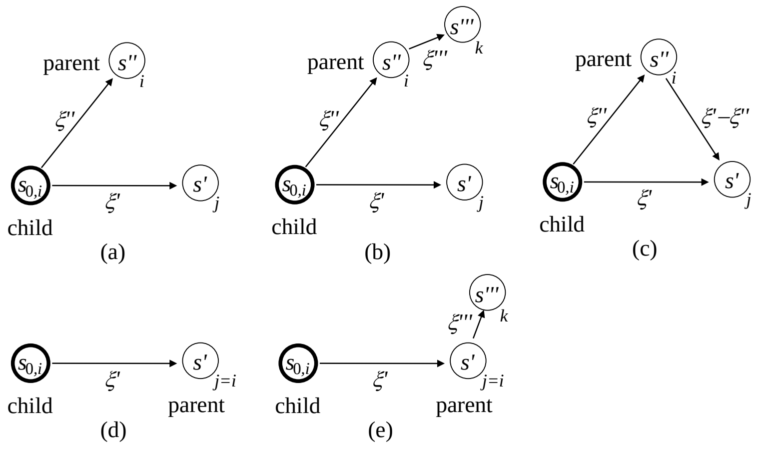

Figure A.2: Geometry of birth events contributing to the boundary term $m_{2,ij}(\xi', s_{0,i}, s', t)$ as described by expressions A.29: (a) intrinsic births by parent; (b) modification to parent's birth rate due to its neighbours; (c) modification to birth rate of parent due to pre-existing component of the pair; (d) intrinsic births from a parent in the pair; (e) modification to birth rate of parent in the pair due its neighbours.

A.4 Effect of δs on the dynamics

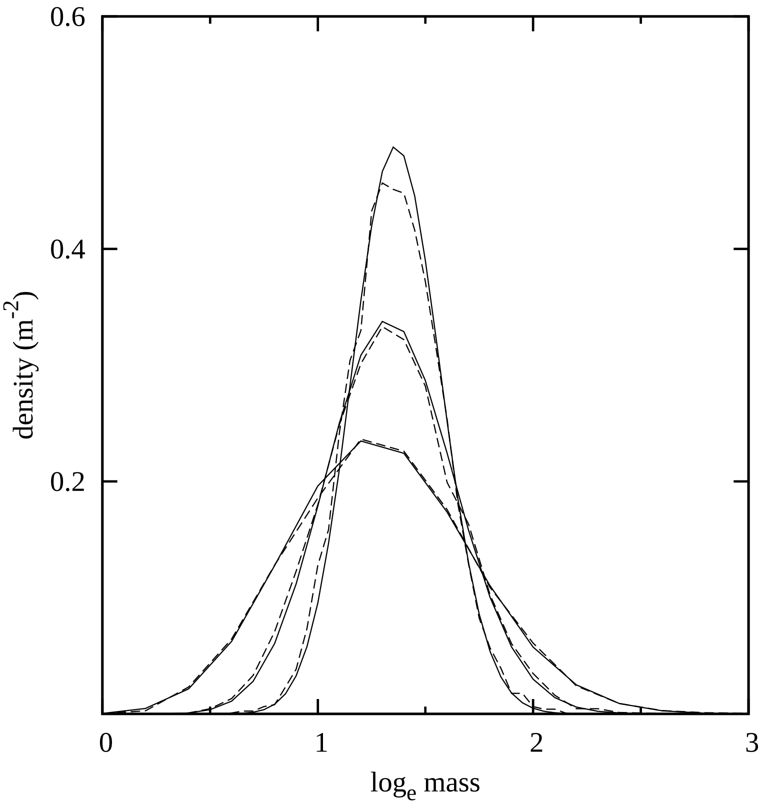

Figure A.3: Effect of three values of δs on the size distribution after 25 years, starting from the same size distribution at year 0. The values of δs, in order of increasing variance, are: 0.05, 0.1, 0.2. Dashed lines: average of 10 realizations of the stochastic process; continuous lines: moment dynamics. Parameter values set as in Figure 4a to exclude any interactions among trees.

References

Stoyan, D., and A. Penttinen. 2000. Recent Applications of Point Process Methods in Forestry Statistics. Statistical Science 15:61–78.