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Sebastian J. Schreiber, Reinhard Bürger, and Daniel I. Bolnick. 2011. The community effects of phenotypic and genetic variation within a predator population. ***Ecology*** **92:1582–1593.**

Appendix E: Quantifying ecological trade-offs

In our model, various kinds of trade offs result from the assumptions that a given predator phenotype, x, differs in its attack rates, $a_1(x)$ and $a_2(x)$, on the two preys and that the distribution of phenotypes is Gaussian with a fixed variance. Because the attack rates are Gaussian functions of the phenotypic value, a phenotype with a high attack rate on prey 1 will generally have a low attack rate on prey 2, and vice versa. More precisely, if a_1 is the attack rate of a phenotype on prey 1, a simple calculation shows that its attack rate on prey 2 is

$$a_2 = a_1 \exp\left[\frac{-2\theta^2 + 2\theta\tau\sqrt{-2\ln(a_1/\alpha)}}{\tau^2}\right], \tag{E.1}$$

where the notation of Appendix C is used. It is straightforward to show that the trade-off curve is concave (convex) if θ is small (large), whereas it is 'wave like' for intermediate θ (see Figure E1).

In addition, a trade off between the components $\bar{a}_1(\bar{x})$ and $\bar{a}_2(\bar{x})$ of mean fitness is induced. This trade-off depends not only on θ, α, and τ, but in particular on the phenotypic variance σ^2. It is given by

$$\bar{a}_2 = \bar{a}_1 \exp\left[\frac{-2\theta^2 + 2\theta\sqrt{-2(\sigma^2+\tau^2)\ln\dfrac{\bar{a}_1\sqrt{\sigma^2+\tau^2}}{\alpha\tau}}}{\sigma^2+\tau^2}\right]. \tag{E.2}$$

Of course, in the absence of phenotypic variation ($\sigma^2 = 0$), (E.2) reduces to (E.1). It is not difficult to show that $\bar{a}_2$ is a concave function of $\bar{a}_1$ whenever $\theta \le \sqrt{\sigma^2+\tau^2}$. Otherwise, it may be convex, concave, or more complicated (see Figure E1).

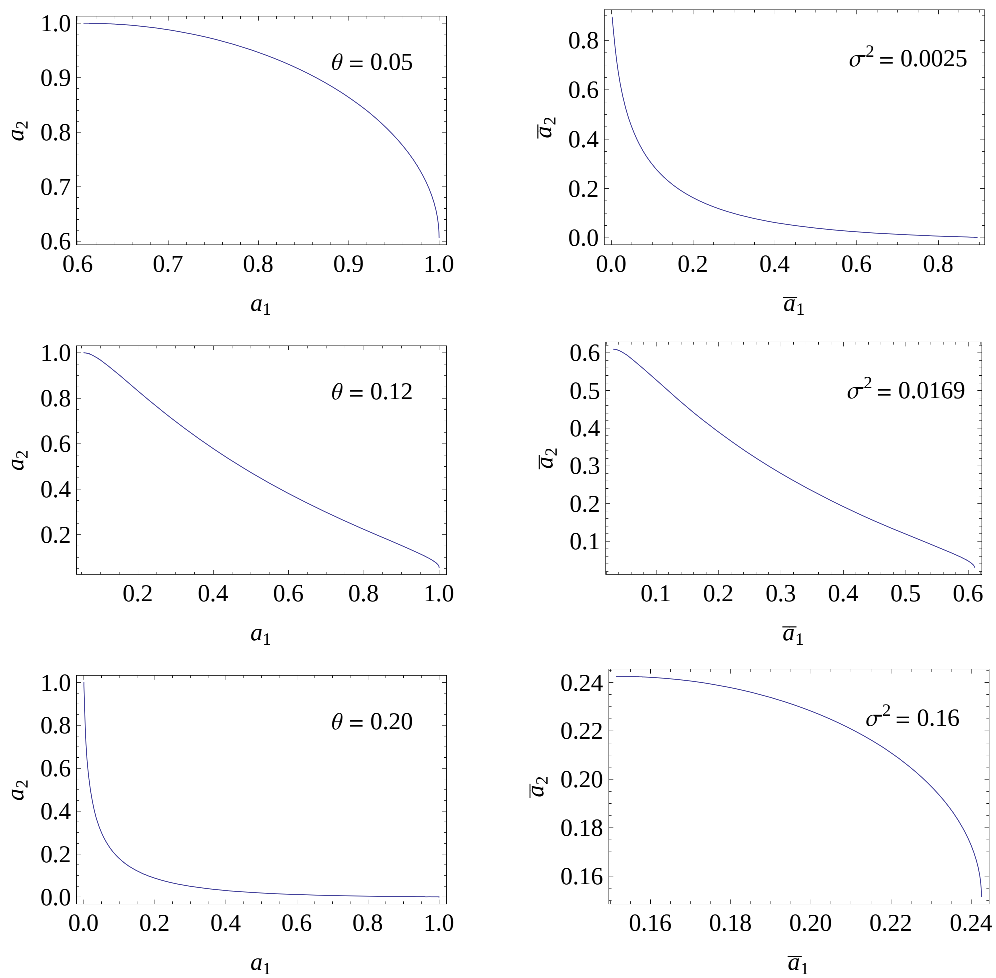

Fig. E1.— Phenotypic and fitness trade offs. (a) The three graphs in the left panel display the trade-off curves resulting from (E.1) for the three indicated values of θ, $\alpha = 1$ (which solely determines the scale on both axes), and $\tau = 0.1$. The trade-off functions are shown on their range of admissible values, $[\exp(-2\theta^2/\tau^2)\alpha, \alpha]$. (b) The three graphs in the right panel display the trade-off curves resulting from (E.2) for the three indicated values of σ^2, and $\alpha = 1$ (which solely determines the scale on both axes), $\theta = 0.2$, $\tau = 0.1$. The trade-off functions are shown on their range of admissible values, $\left[\exp\left(\frac{-2\theta^2}{\sigma^2+\tau^2}\right)\frac{\alpha\tau}{\sqrt{\sigma^2+\tau^2}}, \frac{\alpha\tau}{\sqrt{\sigma^2+\tau^2}}\right]$.