Ecological Archives **E092-049-A2**

Michael Dowd and Ruth Joy. 2011. Estimating behavioral parameters in animal movement models using a State Augmented Particle Filter. *Ecology* 92:568-575.

Appendix B. A primer on particle filtering.

B : Appendix - Particle Filtering

Numerical solutions for state space models are based on Monte Carlo algorithms that generate samples from probability distributions describing the system state. These samples can be used to reconstruct approximations of summary quantities of interest (e.g. the mean and variance of the state or even its probability distribution). Here, we have used a particle filter based on sequential importance resampling, which is the most common approach for solving the nonlinear and non-Gaussian state space model. The interested reader is referred to Gordon et al. (1993) and Kitagawa (1996), and the book of (Ristic et al. 2004), for further details. The state augmented particle filter provides the basis for parameter estimation using the multiple iterated filter at the end of Section 4. At each iteration, the particle filter is run to produce estimates for the augmented state (comprised of the vertical velocity, z_t, the dummy variable, ζ_t, and the movement parameters, a_1 and a_2). The particle filter is therefore a central element of the approach, and is explained in detail below.

Consider a sample from our 4×1 augmented state vector $\tilde{x}$. Each member of the sample is also a 4×1 vector, and there are n_p of these in the total sample. The sample members, or particles, are each denoted by $\tilde{x}^{(i)}$ where $i = 1, \ldots, n_p$; the complete sample, or ensemble, with its n_p members is designated as $\{\tilde{x}^{(i)}\}$. Suppose this sample is drawn from the (multivariate) probability density function (pdf), $p(\tilde{x})$, as designated by the notation

$$\{\tilde{x}^{(i)}\} \sim p(\tilde{x}), \qquad i = 1, \ldots, n_p.$$

This is a sample based approximation to the pdf of the augmented state. As the sample size $n_p \to \infty$ it provides for an exact representation (we use $n_p = 500$ in this study). A particle filter is a recursive algorithm that produces an ensemble of particles whose marginal density approximates the target pdf, and hence provides a Monte Carlo solution for the state space model. The particle filtering algorithm is recursive, and so we need only consider the time transition of the system from $t-1$ to t to specify the procedure completely.

Suppose we are at time $t-1$ and have a sample from the system state,

$$\{\tilde{x}^{(i)}_{t-1|t-1}\} \sim p(\tilde{x}_{t-1}|y_{1:t-1}), \qquad i = 1, \ldots, n_p. \tag{B.1}$$

The subscripting on $\tilde{x}$ designates that this is a sample at time $t-1$ using information up to and including time $t-1$. It is a draw from the pdf of $\tilde{x}_{t-1}$ conditional on having observations up to and including time $t-1$, or $y_{1:t-1}$. The target distribution of the particle filter is the pdf (or sample thereof) at time t as represented by

$$\{\tilde{x}^{(i)}_{t|t}\} \sim p(\tilde{x}_{t}|y_{1:t}), \qquad i = 1, \ldots, n_p. \tag{B.2}$$

Monte Carlo methods can be used to carry out the transition from (B.1) to (B.2) for sequential data assimilation, and provide a means to generate the new sample at time t from the current sample at time $t-1$.

The algorithm for the particle filter used to generate the target sample $\{\tilde{x}^{(i)}_{t|t}\}$ from the current sample, $\{\tilde{x}^{(i)}_{t-1|t-1}\}$ is the following:

1. *Forecasting/Prediction Step.* The forecasting step predicts the augmented state forward to the next observation time. This produces a sample from the forecast pdf, $p(\tilde{x}_t|y_{1:t-1})$, designated as $\{\tilde{x}^{(i)}_{t|t-1}\}$. This is done by treating each member of the sample at time $t-1$, $\{\tilde{x}^{(i)}_{t-1|t-1}\}$ as an initial condition for prediction via the state evolution equation (4), i.e.

$$\tilde{x}^{(i)}_{t|t-1} = g(\tilde{x}^{(i)}_{t-1|t-1}) + \tilde{n}^{(i)}_{t}, \qquad i = 1, \ldots, n_p \tag{B.3}$$

where $\tilde{n}_t^{(i)}$ represents an independent realization of the augmented system noise and is a draw from the normal mixture model (8) for the vertical velocity state, and from the disturbance terms ν_t in (7) associated with the movement parameters.

2. *Resampling/Filtering Step.* The filtering step involves drawing a sample with replacement from the forecast ensemble with a probability proportional to a set of weights computed from the likelihood. To do this, we calculate a weight for each sample member

$$w_t^{(i)} = p(y_t|\tilde{x}_{t|t-1}^{(i)}).$$

Here, $p(y_t|\tilde{x}_{t|t-1}^{(i)})$ is the likelihood of the observations y_t conditional on knowledge of the predicted state for the ith sample member from step 1. Note that only the first element of $\tilde{x}$ is observed and so this is a univariate likelihood. It hence reflects the observation error e_t in (2), which is $\sim N(0, \sigma_o^2)$ and shown in Figure 3 i. A standard weighted resampling (with replacement) of the sample $\{\tilde{x}_{t|t-1}^{(i)}\}$ is undertaken using the corresponding weights $\{w_t^{(i)}\}$. After resampling, this yields a sample $\{x_{t|t}^{(i)}\}$ from the target probability distribution $p(\tilde{x}_t|y_{1:t})$.

Therefore given a starting value for the state at $t = 0$, this sequential importance resampling algorithm can be run forward in time to sequentially generate the required samples for estimation of the augmented state for the duration of the analysis period.

Additional References

Gordon, NJ, Salmond DJ, Smith AFM. 1993. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F* 140(2):107-113.

Kitagawa G. 1996. Monte Carlo filter and smoother for non-Gaussian nonlinear state space models. *Journal of Computational and Graphical Statistics* 5: 1-25.