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**Cang Hui and Melodie A. McGeoch. 2008. Does the self-similar species distribution model lead to unrealistic predictions? *Ecology* 89:2946–2952.**

## Appendix B: Notation used for Deriving Species Richness Fractions in a Community with a Scale-Heritage Assumption.

We first explain the notation used in figure B1. Let

$$u_j = \left(\frac{j}{8}\right)^{-\ln a / \ln 2} = a^{3 - \ln j / \ln 2},$$

where  $j = 1, 2, \dots, 8$ . Here the maximum value 8 of  $j$  is controlled by the size of the smallest cell (rectangular landscape unit). For describing the situation in figure B1, the smallest quadrate is 1/8 of the original  $A_0$ . We have four special cases:  $u_8 = 1$ ,  $u_4 = a$ ,  $u_2 = a^2$  and  $u_1 = a^3$ . The species richness fractions can be expressed by a polynomial term,

$$\sum_{j=1}^8 \text{sign} \cdot \theta_j u_j,$$

where  $\theta_j$  is the absolute value of the parameter of the term  $u_j$  and  $\text{sign}$  is a function that equals +1 if the parameter of  $u_j$  is positive and is -1 if the parameter is negative. Therefore, a sequence,  $\theta_8 \theta_7 \theta_6 \theta_5 \theta_4 \theta_3 \theta_2 \theta_1$ , can be used to describe the above polynomial term. Note that we

also underline the numbers which have a negative sign. For example, in figure B1 we have

$L(o, o, o) = 10\underline{2}00020$ , indicating that  $L(o, o, o) = 1 - 2 \cdot a^{3 - \ln 6 / \ln 2} + 2a^2$ . The algorithmic

calculation can readily be obtained from the combination of these sequences by cross-sequence serial application of the appropriate algorithm. For example, we have  $F(o, x, o, o) = \underline{1}\underline{1}\underline{1}\underline{1}\underline{1}\underline{1}\underline{1}$ , and therefore,

$$\begin{aligned} F(o, o, o, o) &= L(o, o, o) - 2F(o, x, o, o) \\ &= 10\underline{2}00020 - \underline{2}\underline{2}\underline{2}\underline{2}\underline{2}\underline{2}\underline{2} \\ &= \underline{1}\underline{2}0\underline{2}\underline{2}\underline{2}02 \end{aligned}$$

After the bisection of the left side quadrate (figure 1B), seven non-overlapping species richness fractions are generated (figure 1B), i.e. those which are interlinked in figure 2.

Without Harte's probability rule [PR] constraint, there are two independent fractions:  $L(o, x, x)$  and  $L(o, x, o)$ . When considering Harte's PR ( $L(o, x, n) = a(1 - a)$ ), it is only necessary to know one fraction  $L(o, x, x)$ . The value is calculated according to the power-law SAR such that  $L(o, x, x) = 10^{-1000000}$ , following which all other fractions may be calculated (figure 2). After further bisection (figure 1C), 15 non-overlapping fractions are obtained with four independent fractions:  $F(o, x, x, x)$ ,  $F(o, x, o, x)$ ,  $F(o, x, x, o)$  and  $F(o, x, o, o)$ . Because of the marginal constraint of Harte's PR, it is only necessary to know three of them. First, the value of  $F(o, x, x, x)$  is calculated according to the power-law SAR (see figure B1), and then according to intersection calculus for non-exclusive events (joint events) in set theory (Enderton 1977)  $A \cap B = \overline{\overline{A} \cup \overline{B}}$  we have  $F(o, x, n, x)$  and  $F(o, x, x, n)$ , from which we have  $F(o, x, o, x)$  and  $F(o, x, x, o)$ . Then, using the same probability principle we obtain  $F(o, x, o, o)$  (see figure B1 for all the values).

#### LITERATURE CITED

- Harte, J., A. Kinzig, and J. Green. 1999. Self-similarity in the distribution and abundance of species. *Science* 284:334-346
- Maddux, R. D. 2004. Self-similarity and the species-area relationship. *American Naturalist* 163:616-626.

### Figure legend for Figure B1

**Figure B1:** The fraction of species that occur in particular subsets generated by the bisection of the original cell, the left cell and the top left cell (see figure 1). Each situation is described using the notation of Maddux's (2004), i.e.  $H(o,x)$ ,  $L(o,x,x)$  and  $F(o,x,x,x)$  (e.g. numeral I; see also Appendix A). The two rows (an example indicated with the numerals (II) and (III)) below each case of the above notation are derived following the non-heritage assumption (II) and the scale-heritage assumption (III). Solid arrow heads indicate the process of bisection; open arrow heads indicate the combinations of cases required to obey Harte et al.'s (1999) probability rule.

Figure B1 for Appendix B

