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Brett A. Melbourne and Peter Chesson. 2006. The scale transition: scaling up population dynamics with field data. Ecology 87:1478–1488.

Appendix A: Exact and approximate stream-scale models of periphyton dynamics.

We begin with the exact model of Eq. 3 in the main text:

$$\frac{d\overline{R}}{dt} = \overline{g(R)} - \overline{f(R)C}\,. \tag{A.1}$$

From the definition of covariance:

$$\overline{f(R)C} = \overline{f(R)}\;\overline{C} + \mathrm{Cov}(f(R), C) \tag{A.2}$$

(see, for example, Rice 1995 or Chesson et al. 2005). By definition, the scale transition in periphyton growth rate T_g is the difference between the full spatial model and the mean-field model:

$$T_g = \overline{g(R)} - g(\overline{R}) \tag{A.3}$$

Similarly, the scale transition in the functional response T_f is

$$T_f = \overline{f(R)} - f(\overline{R}). \tag{A.4}$$

By substitution into Eq. A.1:

$$\frac{d\overline{R}}{dt} = g(\overline{R}) - f(\overline{R})\overline{C} + T_g - T_f\overline{C} - \mathrm{Cov}(f(R), C), \tag{A.5}$$

which is Eq. 4 in the main text, where Cov(f(R),C) = T_{fC}.

To find an approximate stream-scale model, we use the delta method to approximate $\overline{g(R)}$ and $\overline{f(R)}$, by approximating $g(R)$ and $f(R)$ as second order Taylor polynomials (see, for example, Rice 1995, appendix in Chesson 1998, or Chesson et al. 2005):

$$\overline{g(R)} \approx g(\overline{R}) + \frac{1}{2} g''(\overline{R})\mathrm{Var}(R) \tag{A.6}$$

$$\overline{f(R)} \approx f(\overline{R}) + \frac{1}{2} f''(\overline{R})\mathrm{Var}(R) \tag{A.7}$$

To approximate the covariance term, recall that

$$\mathrm{Cov}(f(R), C) = \frac{1}{n}\sum^{n}\left(f(R) - \overline{f(R)}\right)\left(C - \overline{C}\right). \tag{A.8}$$

Substituting the approximations

$$f(R) \approx f(\overline{R}) + f'(\overline{R})(R - \overline{R}), \tag{A.9}$$

$$\overline{f(R)} \approx f(\overline{R}) \tag{A.10}$$

into Eq. A.8 we obtain

$$\mathrm{Cov}(f(R), C) \approx f'(\overline{R})\frac{1}{n}\sum^{n}(R - \overline{R})(C - \overline{C}) = f'(\overline{R})\mathrm{Cov}(R, C), \tag{A.11}$$

where $f'(R)$ is the first derivative of $f(R)$. The approximation in Eq. A.11 has the same order of accuracy as the approximations in Eq. A.6 and A.7 (Chesson et al. 2005). By substitution of Eq. A.6, A.7 and A.11 into Eq. A.2 and A.1, we obtain:

$$\frac{d\overline{R}}{dt} \approx g(\overline{R}) - f(\overline{R})\overline{C} + \tfrac{1}{2}g''(\overline{R})\mathrm{Var}(R) - \tfrac{1}{2}f''(\overline{R})\mathrm{Var}(R)\overline{C} - f'(\overline{R})\mathrm{Cov}(R, C), \tag{A.12}$$

which is Eq. 5 in the main text.

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