

Digital Appendix F. On using the exponential to quantify fluctuations.

In general, a theoretical curve predicting $dN(t)$ in a bin of width dt is transformed to become a function of diameter b in the following way:

$$\frac{dN}{db} = \frac{dN}{dt} \frac{dt}{db} \quad (\text{F.1})$$

where $\frac{db}{dt}$ is rate of increase in dbh.

Thus if $t = \kappa b$, $\frac{db}{dt} = \frac{1}{\kappa}$ and if $\frac{dN}{dt} = e^{-\delta t}$ then

$$\frac{dN}{db} = e^{-\kappa\delta b} \kappa \quad (\text{F.2})$$

– simple scaling.

Suppose however

$$(i) \quad t = \kappa b^2 \quad \frac{db}{dt} = \frac{1}{2\kappa b}$$

The size profile becomes an exponential in κb^2 and

$$\frac{dN}{db} = e^{-\kappa\delta b^2} 2\kappa b \quad (\text{F.3})$$

which is a skewed gaussian with a maximum at $b = \frac{1}{\sqrt{2\kappa\delta}}$. This case, which corresponds to

the cross sectional area growing at a constant rate, is very different from an exponential.

$$(ii) \quad \text{Suppose } t = \kappa \ln \frac{b}{b(0)}$$

$$\text{when } \frac{db}{dt} = \frac{b}{\kappa}$$

$$\text{then } \frac{dN}{db} = e^{-\delta \kappa \ln \frac{b}{b(0)}} \frac{\kappa}{b}$$

$$= \frac{\kappa}{b} \left(\frac{b(0)}{b} \right)^{\delta \kappa} \quad (\text{F.4})$$

which is an inverse power law. This is reasonable enough, but diameter increasing exponentially with age is pathological (see Ogden 1981, Worbes et al. 2003).

There are several cases in the data of Kelly et al. (2001) where the size profiles exhibit a maximum which is right shifted from zero – in at least two of these the growth rate data are sufficient to rule out the form (i) (see Fig. 2 in Kelly et al. 2001). In all probability, the growth rates are dominated by a term independent of diameter and in such cases the size profiles will have much the same shape as the age profiles.

An exponential is thus a sensible single parameter choice for the shape taken as underlying both the size and age profiles, for the purpose of revealing recruitment fluctuations. We have made this argument with trees in mind, but a comparable argument might be made for any organism with a sufficient correlation between age and some measurable aspect of size.

Literature cited

- Kelly, C. K., H. Banyard Smith, Y. M. Buckley, R. Carter, M. Franco, W. Johnson, T. Jones, B. May, R. Perez Ishiwara, A. Perez-Jimenez, A. Solis Magallanes, H. Steers, and C. Waterman. 2001. Investigations in commonness and rarity: a comparative analysis of co-occurring, congeneric Mexican trees. *Ecology Letters* **4**:618-627.
- Ogden, J. 1981. Dendrochronological studies and the determination of tree ages in the Australian tropics. *Journal of Biogeography* **8**:405-420.
- Worbes, M., R. Staschel, A. Roloff, and W. J. Junk. 2003. Tree ring analysis reveals age structure, dynamics and wood production of a natural forest stand in Cameroon. *Forest Ecology and Management* **173**:105-123.