

Appendix A: Mathematical derivations of the theoretical equations for the removal of invasive plants and planting of natives for collective and independent management, and for the penalty/subsidy policy instruments.

All variables used in the mathematical derivations below are either described here or in the main published text.

Collective Optimization Problem

In the collective case, net benefits (benefits minus costs) from native and invasive plants on property 1 and property 2 would be maximized simultaneously, subject to the ecological constraints, where ρ represents the discount rate¹, and μ_{y_i} , μ_{x_i} are the shadow prices (marginal products) of invasive and native plants.

$$\begin{aligned} \max_{H_{1t}, H_{2t}, P_{1t}, P_{2t}} \int_0^{\infty} e^{-\rho t} (B_1(x_{1t}) + B_2(x_{2t}) - D_1(y_{1t}) - D_2(y_{2t}) - C_{H_1}(H_{1t}) - C_{H_2}(H_{2t}) \\ - C_{P_1}(P_{1t}) - C_{P_2}(P_{2t})) \end{aligned} \quad (A.1)$$

s. t.

$$\frac{dx_1}{dt} = G^{x_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{1t}$$

$$\frac{dx_2}{dt} = G^{x_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{2t}$$

$$\frac{dy_1}{dt} = G^{y_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{1t}$$

$$\frac{dy_2}{dt} = G^{y_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{2t}$$

¹ ρ was used for the interest rate in place of r , which is typical in economic modeling, to avoid confusion with r represented as the intrinsic growth rate.

$$\begin{aligned}
H^c = & B_1(x_{1t}) + B_2(x_{2t}) - D_1(y_{1t}) - D_2(y_{2t}) - C_{H_1}(H_{1t}) - C_{H_2}(H_{2t}) - C_{P_1}(P_{1t}) - C_{P_2}(P_{2t}) \\
& + \mu_{x_1}(G^{x_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{1t}) + \mu_{y_1}(G^{y_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{1t}) \\
& + \mu_{x_2}(G^{x_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{2t}) \\
& + \mu_{y_2}(G^{y_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{2t})
\end{aligned} \tag{A.2}$$

The maximum principle is given by:

$$\frac{\partial H^c}{\partial H_{1t}} = -C'_{H_1}(H_{1t}) - \mu_{y_1} = 0 \Rightarrow \mu_{y_1} = -C'_{H_1}(H_{1t}) \tag{A.2a}$$

$$\frac{\partial H^c}{\partial H_{2t}} = -C'_{H_2}(H_{2t}) - \mu_{y_2} = 0 \Rightarrow \mu_{y_2} = -C'_{H_2}(H_{2t}) \tag{A.2b}$$

$$\frac{\partial H^c}{\partial P_{1t}} = -C'_{P_1}(P_{1t}) + \mu_{x_1} = 0 \Rightarrow \mu_{x_1} = C'_{P_1}(P_{1t}) \tag{A.2c}$$

$$\frac{\partial H^c}{\partial P_{2t}} = -C'_{P_2}(P_{2t}) + \mu_{x_2} = 0 \Rightarrow \mu_{x_2} = C'_{P_2}(P_{2t}) \tag{A.2d}$$

$$\frac{d\mu_{x_1}}{dt} = \mu_{x_1}(\rho - G^{x_1}_{x_1}) - B'_1(x_1) - \mu_{x_2}G^{x_2}_{x_1} - \mu_{y_1}G^{y_1}_{x_1} - \mu_{y_2}G^{y_2}_{x_1} \tag{A.2e}$$

$$\frac{d\mu_{x_2}}{dt} = \mu_{x_2}(\rho - G^{x_2}_{x_2}) - B'_2(x_2) - \mu_{x_1}G^{x_1}_{x_2} - \mu_{y_2}G^{y_2}_{x_2} - \mu_{y_1}G^{y_1}_{x_2} \tag{A.2f}$$

$$\frac{d\mu_{y_1}}{dt} = \mu_{y_1}(\rho - G^{y_1}_{y_1}) + D'_1(y_1) - \mu_{x_1}G^{x_1}_{y_1} - \mu_{x_2}G^{x_2}_{y_1} - \mu_{y_2}G^{y_2}_{y_1} \tag{A.2g}$$

$$\frac{d\mu_{y_2}}{dt} = \mu_{y_2}(\rho - G^{y_2}_{y_2}) + D'_2(y_2) - \mu_{x_2}G^{x_2}_{y_2} - \mu_{x_1}G^{x_1}_{y_2} - \mu_{y_1}G^{y_1}_{y_2} \tag{A.2h}$$

Plugging (A.2a) – (A.2d) into (A.2e) – (A.2h) and rewriting gives:

$$\frac{d\mu_{x_1}}{dt} = C'_{P_1}(P_{1t})(\rho - G^{x_1}_{x_1}) - B'_1(x_1) - C'_{P_2}(P_{2t})G^{x_2}_{x_1} + C'_{H_1}(H_{1t})G^{y_1}_{x_1} + C'_{H_2}(H_{2t})G^{y_2}_{x_1} \tag{A.2ee}$$

$$\frac{d\mu_{x_2}}{dt} = C'_{P_2}(P_{2t})(\rho - G^{x_2}_{x_2}) - B'_2(x_2) - C'_{P_1}(P_{1t})G^{x_1}_{x_2} + C'_{H_2}(H_{2t})G^{y_2}_{x_2} + C'_{H_1}(H_{1t})G^{y_1}_{x_2} \tag{A.2ff}$$

$$\frac{d\mu_{y_1}}{dt} = -C'_{H_1}(H_{1t})(\rho - G^{y_1}_{y_1}) + D'_1(y_1) - C'_{P_1}(P_{1t})G^{x_1}_{y_1} - C'_{P_2}(P_{2t})G^{x_2}_{y_1} + C'_{H_2}(H_{2t})G^{y_2}_{y_1} \tag{A.2gg}$$

$$\frac{d\mu_{y_2}}{dt} = -C'_{H_2}(H_{2t})(\rho - G_{y_2}^{y_2}) + D'_2(y_2) - C'_{P_2}(P_{2t})G_{y_2}^{x_2} - C'_{P_1}(P_{1t})G_{y_2}^{x_1} + C'_{H_1}(H_{1t})G_{y_2}^{y_1} \quad (A.2hh)$$

We then time differentiated (A.2a) – (A.2d) to give:

$$\frac{\partial H^c}{\partial H_{1t}} = -C''_{H_1}(H_{1t})\dot{H}_1 - \dot{\mu}_{y_1} = 0 \Rightarrow \dot{H}_1 = -\frac{\dot{\mu}_{y_1}}{C''_{H_1}(H_{1t})} \quad (A.2aa)$$

$$\frac{\partial H^c}{\partial H_{2t}} = -C''_{H_2}(H_{2t})\dot{H}_2 - \dot{\mu}_{y_2} = 0 \Rightarrow \dot{H}_2 = -\frac{\dot{\mu}_{y_2}}{C''_{H_2}(H_{2t})} \quad (A.2bb)$$

$$\frac{\partial H^c}{\partial P_{1t}} = -C''_{P_1}(P_{1t})\dot{P}_1 + \dot{\mu}_{x_1} = 0 \Rightarrow \dot{P}_1 = \frac{\dot{\mu}_{x_1}}{C''_{P_1}(P_{1t})} \quad (A.2cc)$$

$$\frac{\partial H^c}{\partial P_{2t}} = -C''_{P_2}(P_{2t})\dot{P}_2 + \dot{\mu}_{x_2} = 0 \Rightarrow \dot{P}_2 = \frac{\dot{\mu}_{x_2}}{C''_{P_2}(P_{2t})}. \quad (A.2dd)$$

Plugging (A.2ee) – (A.2hh) into (A.2aa) – (A.2dd) produced time paths of invader removal and native planting:

$$\frac{dH_1}{dt} = \frac{C'_{H_1}(H_1)}{C''_{H_1}(H_1)}(\rho - G_{y_1}^{y_1}) + \frac{C'_{P_1}(P_1)G_{y_1}^{x_1}}{C''_{H_1}(H_1)} - \frac{D'_1(y_1)}{C''_{H_1}(H_1)} + \frac{C'_{P_2}(P_2)G_{y_1}^{x_2}}{C''_{H_1}(H_1)} - \frac{C'_{H_2}(H_2)G_{y_1}^{y_2}}{C''_{H_1}(H_1)} \quad (A.3)$$

$$\frac{dH_2}{dt} = \frac{C'_{H_2}(H_2)}{C''_{H_2}(H_2)}(\rho - G_{y_2}^{y_2}) + \frac{C'_{P_2}(P_2)G_{y_2}^{x_2}}{C''_{H_2}(H_2)} - \frac{D'_2(y_2)}{C''_{H_2}(H_2)} + \frac{C'_{P_1}(P_1)G_{y_2}^{x_1}}{C''_{H_2}(H_2)} - \frac{C'_{H_1}(H_1)G_{y_2}^{y_1}}{C''_{H_2}(H_2)} \quad (A.4)$$

$$\frac{dP_1}{dt} = \frac{C'_{P_1}(P_1)}{C''_{P_1}(P_1)}(\rho - G_{x_1}^{x_1}) + \frac{C'_{H_1}(H_1)G_{x_1}^{y_1}}{C''_{P_1}(P_1)} - \frac{B'_1(x_1)}{C''_{P_1}(P_1)} - \frac{C'_{P_2}(P_2)G_{x_1}^{x_2}}{C''_{P_1}(P_1)} + \frac{C'_{H_2}(H_2)G_{x_1}^{y_2}}{C''_{P_1}(P_1)} \quad (A.5)$$

$$\frac{dP_2}{dt} = \frac{C'_{P_2}(P_2)}{C''_{P_2}(P_2)}(\rho - G_{x_2}^{x_2}) + \frac{C'_{H_2}(H_2)G_{x_2}^{y_2}}{C''_{P_2}(P_2)} - \frac{B'_2(x_2)}{C''_{P_2}(P_2)} - \frac{C'_{P_1}(P_1)G_{x_2}^{x_1}}{C''_{P_2}(P_2)} + \frac{C'_{H_1}(H_1)G_{x_2}^{y_1}}{C''_{P_2}(P_2)}. \quad (A.6)$$

We can rewrite (A.3) through (A.6) in equilibrium. For simplicity, we have only rewritten equations (A.3) and (A.5). We assumed an interior solution such that eradication has not occurred.

$$0 = \frac{dH_1}{dt} = C'_{H_1}(H_1) = \frac{-C'_{P_1}(P_1)G_{y_1}^{x_1} + D'_1(y_1) - C'_{P_2}(P_2)G_{y_1}^{x_2} + C'_{H_2}(H_2)G_{y_1}^{y_2}}{(\rho - G_{y_1}^{y_1})} \quad (A.7)$$

$$0 = \frac{dP_1}{dt} = C'_{P_1}(P_1) = \frac{-C'_{H_1}(H_1)G^{y_1}_{x_1} + B'_1(x_1) + C'_{P_2}(P_2)G^{x_2}_{x_1} - C'_{H_2}(H_2)G^{y_2}_{x_1}}{(\rho - G^{x_1}_{x_1})}. \quad (\text{A. 8})$$

Using equations (A. 7) and (A. 8), we calculated the fundamental equation of resource economics, which defines the optimal collective steady state values for invasive and native plants on property 1;

$$\rho = G^{y_1}_{y_1} + \frac{D'_1(y_1) - C'_{P_1}(P_1)G^{x_1}_{y_1} - C'_{P_2}(P_2)G^{x_2}_{y_1} + C'_{H_2}(H_2)G^{y_2}_{y_1}}{C'_{H_1}(H_1)} \quad (\text{A. 9})$$

$$\rho = G^{x_1}_{x_1} + \frac{B'_1(x_1) - C'_{H_1}(H_1)G^{y_1}_{x_1} + C'_{P_2}(P_2)G^{x_2}_{x_1} - C'_{H_2}(H_2)G^{y_2}_{x_1}}{C'_{P_1}(P_1)}. \quad (\text{A. 10})$$

Steady state invasive plant abundance adjusts according to equation (A.9), which is driven by the tradeoffs from removal. More specifically, in equilibrium removal will occur until the discount rate (rate of return on investments elsewhere in the economy) equals the marginal reduction in net growth rate plus a stock term. The stock term measures the avoided marginal damages from the invasive plants (numerator) relative to the marginal cost of removal (denominator). The numerator in the stock term reflects the combination of reduced future damages from leaving fewer invasive plants on the property because fewer invasive plants leave more room for natives and less spread to other properties. The reduction in future damages is then discounted by the cost of removal. A larger stock term implies larger avoided damages, which means the lower $G^{x_1}_{x_1}$ has to be to balance the equation for any value of ρ , and invasive plant abundance will fall. This resource equation is different relative to other resource equations because the return from removal is the avoided loss due to invasive plants.

From equation (A. 10), optimal steady state native plant abundance has been reached when the opportunity cost of managing the resource (the discount rate) equals the marginal increase in net native growth rate plus a stock effect. The numerator in the stock term reflects the

combination of future benefits of native plants because more natives can disperse to other properties and prevent invasive plant spread/establishment. These benefits are then discounted by the cost of planting natives (denominator). An increase in the stock effect reflects higher benefits from native plants, which leads to an increase in planting efforts and larger native plant abundance.

Independent Optimization Problem

In the independent case, individuals maximize their own net benefits, which produce two optimization problems if we are looking at two landowners (i.e., properties). The net benefits are maximized subject to the ecological constraints for invasive plants and native plants in each individual property, where ρ represents the discount rate and μ_{y_i} , μ_{x_i} are the shadow prices (marginal products) of invasive and native plants.

$$\max_{H_{1t}, P_{1t}} \int_0^{\infty} e^{-\rho t} (B_1(x_{1t}) - D_1(y_{1t}) - C_{H_1}(H_{1t}) - C_{P_1}(P_{1t})) dt \quad (A.11)$$

s. t.

$$\frac{dx_1}{dt} = G^{x_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{1t}$$

$$\frac{dy_1}{dt} = G^{y_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{1t}$$

$$H^c = B_1(x_{1t}) - D_1(y_{1t}) - C_{H_1}(H_{1t}) - C_{P_1}(P_{1t}) + \mu_{x_1}(G^{x_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{1t}) + \mu_{y_1}(G^{y_1}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{1t}) \quad (A.12)$$

The maximum principle is given by:

$$\frac{\partial H^c}{\partial H_{1t}} = -C'_{H_1}(H_{1t}) - \mu_{y_1} = 0 \Rightarrow \mu_{y_1} = -C'_{H_1}(H_{1t}) \quad (A.13a)$$

$$\frac{\partial H^c}{\partial P_{1t}} = -C'_{P_1}(P_{1t}) + \mu_{x_1} = 0 \Rightarrow \mu_{x_1} = C'_{P_1}(P_{1t}) \quad (A.13b)$$

$$\frac{d\mu_{x_1}}{dt} = \mu_{x_1}(\rho - G_{x_1}^{x_1}) - B'_1(x_1) - \mu_{y_1} G_{x_1}^{y_1} \quad (A.13c)$$

$$\frac{d\mu_{y_1}}{dt} = \mu_{y_1}(\rho - G_{y_1}^{y_1}) + D'_1(y_1) - \mu_{x_1} G_{y_1}^{x_1}. \quad (A.13d)$$

Plugging (A.13a) – (A.13b) into (13c) – (13d) and rewriting gives:

$$\frac{d\mu_{x_1}}{dt} = C'_{P_1}(P_{1t})(\rho - G_{x_1}^{x_1}) - B'_1(x_1) + C'_{H_1}(H_{1t})G_{x_1}^{y_1} \quad (A.13cc)$$

$$\frac{d\mu_{y_1}}{dt} = -C'_{H_1}(H_{1t})(\rho - G_{y_1}^{y_1}) + D'_1(y_1) - C'_{P_1}(P_{1t})G_{y_1}^{x_1} \quad (A.13dd)$$

We then time differentiated (A.13a) – (A.13b) to give:

$$\frac{\partial H^c}{\partial H_{1t}} = -C''_{H_1}(H_{1t})\dot{H}_1 - \dot{\mu}_{y_1} = 0 \Rightarrow \dot{H}_1 = -\frac{\dot{\mu}_{y_1}}{C''_{H_1}(H_{1t})} \quad (A.13aa)$$

$$\frac{\partial H^c}{\partial P_{1t}} = -C''_{P_1}(P_{1t})\dot{P}_1 + \dot{\mu}_{x_1} = 0 \Rightarrow \dot{P}_1 = \frac{\dot{\mu}_{x_1}}{C''_{P_1}(P_{1t})} \quad (A.13bb)$$

Plugging (A.13cc) and (A.13dd) into (A.13aa) and (A.13bb) and which solving the optimization problem from property 1 produces the following time paths for removal and planting:

$$\frac{dH_1}{dt} = \frac{C'_{H_1}(H_1)}{C''_{H_1}(H_1)}(\rho - G_{y_1}^{y_1}) + \frac{C'_{P_1}(P_1)G_{y_1}^{x_1}}{C''_{H_1}(H_1)} - \frac{D'_1(y_1)}{C''_{H_1}(H_1)} \quad (A.14)$$

$$\frac{dP_1}{dt} = \frac{C'_{P_1}(P_1)}{C''_{P_1}(P_1)}(\rho - G_{x_1}^{x_1}) + \frac{C'_{H_1}(H_1)G_{x_1}^{y_1}}{C''_{P_1}(P_1)} - \frac{B'_1(x_1)}{C''_{P_1}(P_1)}. \quad (A.15)$$

In equilibrium, we can rewrite (A.14) and (A.15) as,

$$0 = \frac{dH_1}{dt} \Rightarrow C'_{H_1}(H_1) = \frac{D'_1(y_1) - C'_{P_1}(P_1)G_{y_1}^{x_1}}{(\rho - G_{y_1}^{y_1})} \quad (A.16)$$

$$0 = \frac{dP_1}{dt} \Rightarrow C'_{P_1}(P_1) = \frac{B'_1(x_1) - C'_{H_1}(H_1)G_{x_1}^{y_1}}{(\rho - G_{x_1}^{x_1})}. \quad (A.17)$$

Using equations (A.16) and (A.17) produces the fundamental equation of resource economics, which defines the optimal independent steady state values for invasive and native plants on property 1,

$$\rho = G_{y_1}^{y_1} + \frac{D_1'(y_1) - C_{P_1}'(P_1)G_{y_1}^{x_1}}{C_{H_1}'(H_1)} \quad (A.18)$$

$$\rho = G_{x_1}^{x_1} + \frac{B_1'(x_1) - C_{H_1}'(H_1)G_{x_1}^{y_1}}{C_{P_1}'(P_1)} \quad (A.19)$$

The steady state independent invasive plant abundance is determined from equation (A.18), where the discount rate equals the avoided net growth rate of invasives plus a stock effect. The stock effect measures the avoided marginal damages of invasive plants normalized by removal costs. The avoided damages are smaller than the collective case, so we would expect invasive plant abundance to fall, but less than the collective case.

From equation (A.19), the optimal native plant abundance has been reached when the discount rate equals the growth rate of natives plus a stock term. The stock term captures the marginal value of native population growth relative to planting costs. The benefits are smaller than the collective case so we would expect planting and native abundance to be lower than the collective case.

The above process for property 1 is repeated for property 2,

$$\max_{H_{2t}, P_{2t}} \int_0^\infty e^{-\rho t} (B_2(x_{2t}) - D_2(y_{2t}) - C_{H_2}(H_{2t}) - C_{p_2}(P_{2t})) dt \quad (A.20)$$

s. t.

$$\frac{dx_2}{dt} = G^{x_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) + P_{2t}$$

$$\frac{dy_2}{dt} = G^{y_2}(x_{1t}, x_{2t}, y_{1t}, y_{2t}) - H_{2t}$$

Solving the optimization problem from property 2 produces the following time paths for removal and planting:

$$\frac{dH_2}{dt} = \frac{C'_{H_2}(H_2)}{C''_{H_2}(H_2)}(\rho - G_{y_2}^{y_2}) + \frac{C'_{P_2}(P_2)G_{y_2}^{x_2}}{C''_{H_2}(H_2)} - \frac{D'_2(y_2)}{C''_{H_2}(H_2)} \quad (A. 21)$$

$$\frac{dP_2}{dt} = \frac{C'_{P_2}(P_2)}{C''_{P_2}(P_2)}(\rho - G_{x_2}^{x_2}) + \frac{C'_{H_2}(H_2)G_{x_2}^{y_2}}{C''_{P_2}(P_2)} - \frac{B'_2(x_2)}{C''_{P_2}(P_2)}. \quad (A. 22)$$

Economic Policy Instruments: penalty/subsidy

This section introduces policy instruments such that independent management's removal of invaders and planting of natives is equal to the collective scenario, where t_1 is the penalty on property 1's invasive plants and s_1 is a subsidy on property 1's natives.

$$\max \int_0^\infty e^{-\rho t} (B_1(x_{1t}) - D_1(y_{1t}) - C_{H_1}(H_{1t}) - C_{P_1}(P_{1t}) + t_1 y_{1t} + s_1 x_{1t}) \quad (A. 23)$$

s. t.

$$\frac{dx_1}{dt} = G^{x_1}(x_1, x_2, y_1, y_2) + P_1$$

$$\frac{dy_1}{dt} = G^{y_1}(x_1, x_2, y_1, y_2) - H_1$$

Solving the optimization problem from property 1 produces new time paths for removal and planting:

$$\frac{dH_1}{dt} = \frac{C'_{H_1}(H_1)}{C''_{H_1}(H_1)}(\rho - G_{y_1}^{y_1}) + \frac{C'_{P_1}(P_1)G_{y_1}^{x_1}}{C''_{H_1}(H_1)} - \frac{D'_1(y_1)}{C''_{H_1}(H_1)} + \frac{t_1}{C''_{H_1}(H_1)} \quad (A. 24)$$

$$\frac{dP_1}{dt} = \frac{C'_{P_1}(P_1)}{C''_{P_1}(P_1)}(\rho - G_{x_1}^{x_1}) + \frac{C'_{H_1}(H_1)G_{x_1}^{y_1}}{C''_{P_1}(P_1)} - \frac{B'_1(x_1)}{C''_{P_1}(P_1)} - \frac{s_1}{C''_{P_1}(P_1)}. \quad (A. 25)$$

Comparing equations (A. 24) and (A. 25) to equations (A. 3) and (A. 5), the penalty and subsidy necessary to achieve the collective solution are:

$$t_1 = C'_{P_2}(P_2)G_{y_1}^{x_2} - C'_{H_2}(H_2)G_{y_1}^{y_2}$$

$$s_1 = C'_{P_2}(P_2)G^{x_2}_{x_1} - C'_{H_2}(H_2)G^{y_2}_{x_1}.$$

This process is repeated for property 2:

$$\max \int_0^\infty e^{-\rho t} (B_2(x_2) - D_2(y_2) - C_{H_2}(H_2) - C_{P_2}(P_2) + t_2 y_2 + s_2 x_2) \\ s. t.$$

$$\frac{dx_2}{dt} = G^{x_2}(x_1, x_2, y_1, y_2) + P_2$$

$$\frac{dy_2}{dt} = G^{y_2}(x_1, x_2, y_1, y_2) - H_2$$

Solving the optimization problem from property 2 produces new time paths for removal and planting:

$$\frac{dH_2}{dt} = \frac{C'_{H_2}(H_2)}{C''_{H_2}(H_2)}(\rho - G^{y_2}) + \frac{C'_{P_2}(P_2)G^{x_2}_{y_2}}{C''_{H_2}(H_2)} - \frac{D'_2(y_2)}{C''_{H_2}(H_2)} + \frac{t_2}{C''_{H_2}(H_2)} \quad (A. 26)$$

$$\frac{dP_2}{dt} = \frac{C'_{P_2}(P_2)}{C''_{P_2}(P_2)}(\rho - G^{x_2}) + \frac{C'_{H_2}(H_2)G^{y_2}_{x_2}}{C''_{P_2}(P_2)} - \frac{B'_2(x_2)}{C''_{P_2}(P_2)} - \frac{s_2}{C''_{P_2}(P_2)}. \quad (A. 27)$$

Comparing equations (A. 26) and (A. 27) to equations (A. 4) and (A. 6), the penalty and subsidy necessary to achieve the collective solution are:

$$t_2 = C'_{P_1}(P_1)G^{x_1}_{y_2} - C'_{H_1}(H_1)G^{y_1}_{y_2}$$

$$s_2 = C'_{P_1}(P_1)G^{x_1}_{x_2} - C'_{H_1}(H_1)G^{y_1}_{x_2}$$