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Haridas Chirakkal and Leah R. Gerber. 2010. Short- and long-term population response to changes in vital rates: implications for demographic population viability analysis. *Ecological Applications* 20:783–788.

Appendix A. Calculation of short-term elasticity.

We follow methods outlined in Haridas and Tuljapurkar (2007) to derive the short-term elasticities. Let the current random projection matrix in year t be $\mathbf{A}(t)$ and let this be perturbed to $\mathbf{A}(t)+\boldsymbol{\varepsilon}\mathbf{C}(t)$, where ε is small. Let population growth rate in year t be $\lambda(t)$ and let $\mathbf{y}(t)$ denote age-structure vector. Then

$$\lambda(t) = |\,\mathbf{A}(t)\mathbf{y}(t-1)\,|, \tag{A.1}$$

where $|\,\mathbf{x}\,|$, denotes the sum of elements of vector $\mathbf{x}$.
Let age-vector $\mathbf{y}(t)$ change to $\overline{\mathbf{y}}(t) = \mathbf{y}(t) + \boldsymbol{\varepsilon}\mathbf{w}(t)$, as a result of perturbing projection matrix. Then $\overline{\mathbf{y}}(t)$ iterates as

$$\overline{\mathbf{y}}(t) = \frac{(\mathbf{A}(t) + \boldsymbol{\varepsilon}\mathbf{C}(t))\overline{\mathbf{y}}(t-1)}{\overline{\boldsymbol{\lambda}}(t)}\ , \tag{A.2}$$

where

$$\overline{\lambda}(t) = |\,(\mathbf{A}(t) + \boldsymbol{\varepsilon}\mathbf{C}(t))\overline{\mathbf{y}}(t-1)\,|$$

$$= \lambda(t) + \varepsilon\mu(t) + O(\varepsilon^2)\,, \tag{A.3}$$

where

$$\mu(t) = |\,\mathbf{C}(t)\mathbf{y}(t-1) + \mathbf{A}(t)\mathbf{w}(t-1)\,|$$

and $O(\varepsilon^2)$ denotes terms of order ε^2 and more.

The elasticity in year t is computed as the proportional change

$$e(t) = \lim_{\varepsilon \to 0}(\log\overline{\lambda}(t) - \log\lambda(t))$$

$$= \mu(t)/\lambda(t)$$

$$= \frac{|\mathbf{C}(t)\mathbf{y}(t-1)|}{\lambda(t)} + \frac{|\mathbf{A}(t)\mathbf{w}(t-1)|}{\lambda(t)}. \quad \text{(A.4)}$$

First component in the above equation is the short-term (single year) effect due to perturbing the projection matrix (i.e., increasing a vital rate or its mean/variance) keeping age-structure fixed at the observed value $\mathbf{y}(t-1)$ at the beginning of the year. The second component is the effect due to changes in age-structure accumulated over years. If a single vital rate is perturbed, say $A_{ij}(t)$, then first component is simply

$$e_1(t) = \frac{C_{ij}(t)y_j(t-1)}{\lambda(t)}. \quad \text{(A.5)}$$

In this paper we consider two perturbations: firstly change mean without changing variance in which case $C_{ij}(t) = \mu_{ij}$, the mean of the vital rate $A_{ij}(t)$. Secondly change variance without changing mean in which case, $C_{ij}(t) = A_{ij}(t) - \mu_{ij}$. Note that $e_1(t)$ is random and changes with year but its long-run average over time converges and this limit gives the short-term stochastic elasticity discussed in the paper. Denoting this average by $\overline{E}_1$, we have

$$\overline{E}_1 = \lim_{T \to \infty} \left(\frac{1}{T}\right) \sum_{t=1}^{T} e_1(t). \quad \text{(A.6)}$$

Literature Cited

Haridas, C.V., and S. Tuljapurkar, S. 2007. Time, transients and elasticity. Ecology Letters 10:1143–1153.