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Appendix A. Model equations indicating the nature of the dynamical system used to investigate the interactions between insects and cotton.

Appendix

Indication of Model Equations – Insect and Cotton Components.

Some of the model equations are presented to indicate the nature of the dynamical system used to investigate the interactions between insects and cotton. The equations are written with notation intended to be explanatory and directly relate to the conceptual model in Figures 3 and 4. The difference equations are solved on a daily time step (Δt) with t representing the current day. T is used to designate a constant parameter; for examples, T_{eggs} is the maturation time for an egg to become a larvae, $T_{Bt\ exposed}$ is the period during which larva are exposed to the pesticide, T_{pupae} is the maturation time for pupa, and similar notation is used for the remaining parameters.

First, the governing equations are defined and then the components that specify the terms are given. Each of the state variables described by the equations corresponds to the boxes shown in the conceptual models (Figures 3 and 4) and represents the number of "individuals" (eggs, larva, pupa, adults) at current time in each of the categories (boxes). For a given state variable or stage, the terms on the right side of the equation are the number of individuals in that stage at the previous time step plus the number of individuals moving (maturing) to that stage minus the number of individuals moving out of the stage because of death or growth. These positive and negative terms represent the inflow and outflow arrows from each box in the conceptual model diagram respectively. In general, these terms are the product of the number of individuals in the contiguous stage at the previous time step and a parameter representing the rate of the flow.

Insect Dynamics in Conventional Cotton

Number of eggs:

$$eggs(t) = eggs(t - \Delta t) + recruit(t - \Delta t) - mature\_egg(t - \Delta t) - death\_eggs(t - \Delta t)$$

Number of Larvae:

$$larvae(t) = larvae(t - \Delta t) + mature\_egg(t - \Delta t) - mature\_larvae(t - \Delta t) - death\_larvae(t - \Delta t)$$

Number of pupae:

$$pupae(t) = pupae(t - \Delta t) + mature\_larvae(t - \Delta t) - mature\_pupae(t - \Delta t) - death\_pupae(t - \Delta t)$$

Number of adults:

$$adults(t) = adults(t - \Delta t) + mature\_pupae(t - \Delta t) + insect\_immigration(t - \Delta t) - adult\_losses(t - \Delta t)$$

Insect Dynamics in *Bt* Cotton

Number of eggs:

$$eggs(t) = eggs(t - \Delta t) + recruit(t - \Delta t) - mature\_egg(t - \Delta t) - death\_eggs(t - \Delta t)$$

Number of larvae in the first category (exposed):

$$Bt\_exposed(t) = Bt\_exposed(t - \Delta t) + mature\_egg(t - \Delta t) - death\_Bt\_exposed(t - \Delta t) - Bt\_survived(t - \Delta t)$$

Number of larvae in the second category (survived the exposure period):

$$Bt\_resistant(t) = Bt\_resistant(t - \Delta t) + Bt\_survived(t - \Delta t) - mature\_larvae(t - \Delta t)$$
$$- death\_Bt\_resistant(t - \Delta t)$$

Number of pupae:

$$pupae(t) = pupae(t - \Delta t) + mature\_larvae(t - \Delta t) - mature\_pupae(t - \Delta t) - death\_pupae(t - \Delta t)$$

Number of adults:

$$adults(t) = adults(t - \Delta t) + mature\_pupae(t - \Delta t) + insect\_immigration(t - \Delta t) -$$
$$- adult\_losses(t - \Delta t)$$

Formulation of terms in the state variables equations

Unless specified, the terms correspond to both the conventional and *Bt* cotton cases.

Number of eggs deposited by adults:

$$recruit(t) = \begin{cases} 0.5 \cdot eggs\_daily \cdot adults(t) & \text{if } 0 < t \le 200 \\ 0 & 200 < t \le 365 \end{cases}$$

Mortality rate of eggs:

$$eggs\_mortality(t) = \begin{cases} egg\_pest\_mort & \left.\begin{array}{l} \textit{when a spray occurs and for a period of 3 days} \\ \textit{(spray residual effect period)} \end{array}\right. \\ egg\_after\_spray\_mort & \textit{during 3 days after the spray residual effect period} \\ egg\_mort & \textit{any other period} \end{cases}$$

Number of eggs maturing to larva stage:

$$mature\_egg(t) = recruit(t - T_{eggs}) . \prod_{i=0}^{T_{eggs}-1}\left(1 - eggs\_mortality(t - T_{eggs} - i)\right)$$

Number of eggs lost due to death:

$$death\_eggs(t) = eggs\_mortality(t) . eggs(t - \Delta t)$$

Mortality rate of larvae in conventional cotton:

$$larvae\_mortality(t) = \begin{cases} larvae\_pest\_mort & \left.\begin{array}{l} \textit{when a spray occurs and for a} \\ \textit{period of 3 days (spray residual effect period)} \end{array}\right. \\ larvae\_after\_spray\_mort & \textit{during 3 days after the spray residual effect period} \\ larvae\_mort & \textit{during any other period} \end{cases}$$

Number of larvae lost due to death in the conventional cotton model:

$$death\_larvae(t) = larvae\_mortality(t - \Delta t) . larvae(t - \Delta t)$$

Number of larvae maturing to pupae stage in conventional cotton:

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$$mature\_larvae(t) = mature\_eggs(t - T_{larvae}) \cdot \prod_{i=0}^{T_{larvae}-1}(1 - larvae\_mortality(t - T_{larvae} - i))$$

Mortality rate of *Bt* exposed larvae in *Bt* cotton:

$$Bt\_exposed\_mortality(t) = \begin{cases} larvae\_pest\_mort & \begin{array}{l}\text{when a spray occurs and for a period of 3 days}\\ \text{(spray residual effect period)}\end{array} \\ larvae\_after\_spray\_mort & \text{during 3 days after the spray residual effect period} \\ Bt\_exposed\_mort & \text{during any other period} \end{cases}$$

Mortality rate of *Bt* resistant larvae in *Bt* cotton:

$$Bt\_resistant\_mortality(t) = \begin{cases} larvae\_pest\_mort & \begin{array}{l}\text{when a spray occurs and for a period of 3 days}\\ \text{(spray residual effect period)}\end{array} \\ larvae\_after\_spray\_mort & \text{during 3 days after the spray residual effect period} \\ Bt\_larvae\_mort & \text{during any other period} \end{cases}$$

Number of *Bt* exposed larvae surviving the exposed period in *Bt* cotton:

$$Bt\_survived(t) = mature\_eggs(t - T_{Bt\_exposed}) \cdot \prod_{i=0}^{T_{Bt\_exposed}-1}(1 - Bt\_exposed\_mortality(t - T_{Bt\_exposed} - i))$$

Number of *Bt* resistant larvae maturing to pupa stage in *Bt* cotton:

$$mature\_larvae(t) = Bt\_resistant(t - T_{Bt\_resistant}) \cdot \prod_{i=0}^{T_{Bt\_resistant}-1}(1 - Bt\_resistant\_mortality(t - T_{Bt\_resistant} - i))$$

Number of pupae maturing to adult stage:

$$mature\_pupae(t) = mature\_larvae(t - T_{pupae}) \cdot (1 - pupae\_mortality)^{T_{pupae}}$$

Number of pupae lost due to death:

$$death\_pupae(t) = pupae\_mortality \cdot pupae(t - \Delta t)$$

Number of adults immigrating into the 1 ha plot:

$$insect\_immigration(t) = \frac{\exp\left(-\frac{1}{2}\left(\frac{t - \mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}} \cdot total\_insect\_immig$$

Number of adults lost due to death:

$$adult\_losses(t) = (adult\_mortality + spray\_mortality + bat\_consumption + emigration) \cdot adults(t - \Delta t)$$