Joshua V. Ross, David J. Sirl, Philip K. Pollett, and Hugh P. Possingham. 2008. Metapopulation persistence in a dynamic landscape: more habitat or better stewardship? *Ecological Applications* **18:590–598.**

Appendix A: Deterministic fixed points and the transformation. *Ecological Archives* **A018-018-A1.**

Appendix

The deterministic fixed points

The deterministic system which arises in the limit as the number of patches tends to infinity and the proportions of protected and susceptible patches remain constant is

$$\frac{dx}{dt} = r(\rho_u - x) - sx,$$
$$\frac{dy}{dt} = c(y + z)(x - y) - (e + s)y,$$
$$\frac{dz}{dt} = c(y + z)(\rho_p - z) - ez.$$

Setting the three derivatives equal to 0 and solving for $w^* = (x^*, y^*, z^*)$ we get the trivial fixed point $w_0^* = (\frac{r\rho_u}{r+s}, 0, 0)$ and also $w_1^* = (x_1^*, y_1^*, z_1^*)$, where

$$x_1^* = \frac{r\rho_u}{r+s},$$
$$y_1^* = \frac{(e+s)(r+s)(c(\rho_p - \alpha_1) - e) + cre\rho_u}{ec(r+s)},$$
$$z_1^* = \alpha_1,$$

and α_1 is a root of

$$\alpha(z) = sc(r+s)z^2 + ((r+s)(se - 2sc\rho_p) - ce(r + s\rho_p)) z$$
$$+ce\rho_p(r + s\rho_p) + \rho_p(r+s)(sc\rho_p - se - e^2).$$

We can see immediately that in order for this fixed point to be in the appropriate state space $\bar{S} = [0, \rho_u]^2 \times [0, \rho_p]$ it is necessary that $\alpha_1 \in [0, \rho_p]$. In addition we observe numerically that if α has two roots in the interval $[0, \rho_p]$ then using the larger root results in y_1^* being negative. The fixed point w_1^* we seek is therefore that obtained by taking α_1 as the smallest root of $\alpha(z)$ in the interval $[0, \rho_p]$, and if α has no such root then w_0^* is the only fixed point in $\bar{S}$. One can then determine the stability of these fixed points by looking at the eigenvalues of

the Jacobian matrix

$$J(x, y, z) = \begin{pmatrix} -(r+s) & 0 & 0 \\ c(y+z) & c(x-2y-z)-e-s & c(x-y) \\ 0 & c(\rho_p - z) & c(\rho_p - 2z - y) - e \end{pmatrix}.$$

Although some progress can be made in this direction analytically, the formulae so derived are cumbersome and relatively uninformative, so we evaluated the fixed points and determined their stability numerically.

The transformation

Here we describe the transformation used to map the state space S to a set of the form $\{1, 2, \ldots, N\}$, so that numerical calculations could be performed. The state space S is the triangular prismoidal set $S = \{(m, n, p) \in \mathbb{Z}^3 : 0 \leq n \leq m \leq M_u, 0 \leq p \leq M_p\}$ and we wished to transform this to a set of the form $\{1, 2, \ldots, N\}$. It can be easily shown that $N = (M_p + 1)(M_u + 1)(M_u + 2)/2$. One mapping which achieves this is $(m, n, p) \rightarrow m + 1 + n\left(M_u - \frac{n-1}{2}\right) + p(M_u + 1)(M_u + 2)/2$, which has the additional property that the absorbing (extinct) states $(m, 0, 0)$ map to $\{1, 2, \ldots, M_u + 1\}$, which simplified coding.

We also needed to invert this transformation following computations. To do this we firstly noted that the transformation is of the form $y = f_1(n, m, M_u) + p f_2(M_u)$, so that $p = y - 1$ (mod $(M_u + 1)(M_u + 2)/2$), and then $j = y - p(M_u + 1)(M_u + 2)/2$ is sufficient to determine $m, n \in \{0 \leq m \leq n \leq M_u\}$. We did this by checking successive possible values of n, and subsequently found m.