



University  
of Glasgow | Institute of Neuroscience  
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Centre for Cognitive Neuroimaging (CCNi)



## Introduction to robust estimation of ERP data

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<https://garstats.wordpress.com>



@robustgar

# Take-home messages

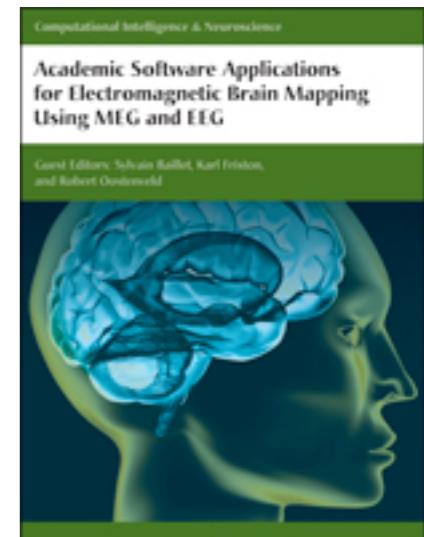
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- *Look at your data*
- *Show your data*
- *A perfect & universal statistical recipe does not exist*
- *Keep exploring: there are many great options, most of them available in free softwares and toolboxes*

## Academic Software Applications for Electromagnetic Brain Mapping Using MEG and EEG

Guest Editors: Sylvain Baillet, Karl Friston, and Robert Oostenveld

- ▶ **EEGLAB, SIFT, NFT, BCILAB, and ERICA: New Tools for Advanced EEG Processing**, Arnaud Delorme, Tim Mullen, Christian Kothe, Zeynep Akalin Acar, Nima Bigdely-Shamlo, Andrey Vankov, and Scott Makeig  
Volume 2011 (2011), Article ID 130714, 12 pages
- ▶ **ELAN: A Software Package for Analysis and Visualization of MEG, EEG, and LFP Signals**, Pierre-Emmanuel Aguera, Karim Jerbi, Anne Caclin, and Olivier Bertrand  
Volume 2011 (2011), Article ID 158970, 11 pages
- ▶ **ElectroMagnetoEncephalography Software: Overview and Integration with Other EEG/MEG Toolboxes**, Peter Peyk, Andrea De Cesarei, and Markus Junghöfer  
Volume 2011 (2011), Article ID 861705, 10 pages
- ▶ **FieldTrip: Open Source Software for Advanced Analysis of MEG, EEG, and Invasive Electrophysiological Data**, Robert Oostenveld, Pascal Fries, Eric Maris, and Jan-Mathijs Schoffelen  
Volume 2011 (2011), Article ID 156869, 9 pages
- ▶ **EEG and MEG Data Analysis in SPM8**, Vladimir Litvak, Jérémie Mattout, Stefan Kiebel, Christophe Phillips, Richard Henson, James Kilner, Gareth Barnes, Robert Oostenveld, Jean Daunizeau, Guillaume Flandin, Will Penny, and Karl Friston  
Volume 2011 (2011), Article ID 852961, 32 pages
- ▶ **EEGIFT: Group Independent Component Analysis for Event-Related EEG Data**, Tom Eichele, Srinivas Rachakonda, Brage Brakedal, Rune Eikeland, and Vince D. Calhoun  
Volume 2011 (2011), Article ID 129365, 9 pages
- ▶ **LIMO EEG: A Toolbox for Hierarchical LInear MOdeling of ElectroEncephaloGraphic Data**, Cyril R. Pernet, Nicolas Chauveau, Carl Gaspar, and Guillaume A. Rousselet  
Volume 2011 (2011), Article ID 831409, 11 pages

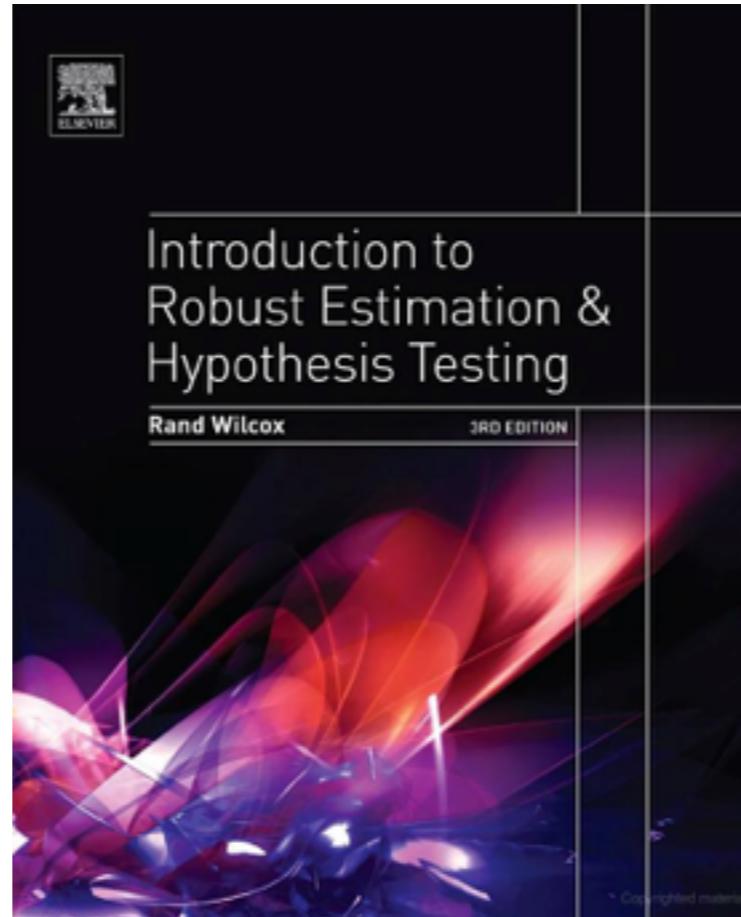


# [http://sccn.ucsd.edu/wiki/EEGLAB Extensions](http://sccn.ucsd.edu/wiki/EEGLAB_Extensions)

## List of data processing extensions

Plug-in name	Version	Short plug-in description	Link	Contact	Comments
rERP	0.4	Estimate overlapping ERPs using multiple regression	<a href="#">Download</a>	M. Burns	User comments
LIMO	1.5	Linear MOdelling of EEG data	<a href="#">Download</a>	C. Pernet	User comments
corrmap	2.02	Cluster ICA components using correlation of scalp maps	<a href="#">Download</a>	S. Debener	User comments
bioelectromag	1.01	Uses Bioelectromagnetism toolbox for ERP peak detection	<a href="#">Download</a>	D. Weber	User comments
VisEd	1.05	Add/Edit dataset events	<a href="#">Download</a>	J. Desjardins	User comments
loreta	1.10	Export and import data to and from LORETA software	<a href="#">Download</a>	A. Delorme	User comments
iirfilt	1.02	Non linear filtering using IIR filter	<a href="#">Download</a>	M. Pozdin	User comments
std_envtopo	2.34	Plot STUDY ICA cluster contribution to ERP	<a href="#">Download</a>	M. Miyakoshi	User comments
std_backproj	0.21	Back-project cluster ICs to channels (beta)	<a href="#">Download</a>	M. Miyakoshi	User comments
dipoleDensity	0.11	Plot STUDY ICA cluster dipole density (beta)	<a href="#">Download</a>	M. Miyakoshi	User comments
std_ErpCalc	0.10	Test and visualize simple effects on ERP (beta)	<a href="#">Download</a>	M. Miyakoshi	User comments
pvaltopo	0.10	Plot topography of percent variance accounted for (beta)	<a href="#">Download</a>	M. Miyakoshi	User comments
trimOutlier	0.15	Trim outlier channels and datapoints interactively (beta)	<a href="#">Download</a>	M. Miyakoshi	User comments
clean_rawdata	0.21	Cleans continuous data using Artifact Subspace Reconstruction	<a href="#">Download</a>	Miyakoshi and Kothe	User comments
Mutual_Info_Clustering	1.00	Group single dataset ICA components by Mutual Information	<a href="#">Download</a>	N. Bigdely	User comments
mass_univ	130502	Mass Univariate ERP Toolbox	<a href="#">Download</a>	D. Groppe	User comments
REGICA	1.00	ICA regression based EOG removal	<a href="#">Download</a>	M. Klados	User comments
MARA	1.1	Multiple Artifact Rejection Algorithm	<a href="#">Download</a>	I. Winkler	User comments
firlift	1.6.1	Routines for designing linear filters	<a href="#">Download</a>	A. Widmann	User comments
PACT	0.14	Computes phase-amplitude coupling for continuous data	<a href="#">Download</a>	M. Miyakoshi	User comments
fMRIb	2.00	Remove fMRI artifacts from EEG	<a href="#">Download</a>	J. Dien & R. Niazy	User comments
SIFT	1.33	Analysis and visualization of multivariate connectivity	<a href="#">Download</a>	T. Mullen	User comments
AAR	131130	ICA-based Automatic Artifact Removal	<a href="#">Download</a>	G. Gomez-Herrero	User comments
Adjust	131130	Automatic Detector - Joint Use of Spatial and Temporal features	<a href="#">Download</a>	Adjust Support	User comments
Cleanline	1.00	Removes sinusoidal artifacts (line noise)	<a href="#">Download</a>	T. Mullen	User comments
Fieldtrip-lite	Daily	Adds source localization and statistics tools to EEGLAB	<a href="#">Download</a>	R. Oostenveld	User comments
EYE-EEG	0.41	Open source MATLAB tool for simultaneous eye tracking & EEG	<a href="#">Download</a>	O. Dimigen	User comments
BERGEN	131130	Remove fMRI artifacts from EEG	<a href="#">Download</a>	M. Moosmann	User comments
CIAC	1.00	Cochlear Implant Artifact Correction	<a href="#">Download</a>	S. Debener	User comments
LR	1.2	Linear Discrimination	<a href="#">Download</a>	P. Sajda	User comments
GEVD	1.00	Generalized Eigenvalue Decomposition (GEVD)	<a href="#">Download</a>	P. Sajda	User comments
CSP	1.1	Common Spatial Patterns	<a href="#">Download</a>	P. Sajda	User comments
Eyesubtract	1.0	Eye Movement Artifact Removal	<a href="#">Download</a>	P. Sajda	User comments
Peakfit	1.0	Single trial EEG peak fitting	<a href="#">Download</a>	P. Sajda	User comments

# Robust statistics: the one book you really need



Wilcox, R. R. (2012)

*Introduction to robust estimation and hypothesis testing* (3rd ed.)

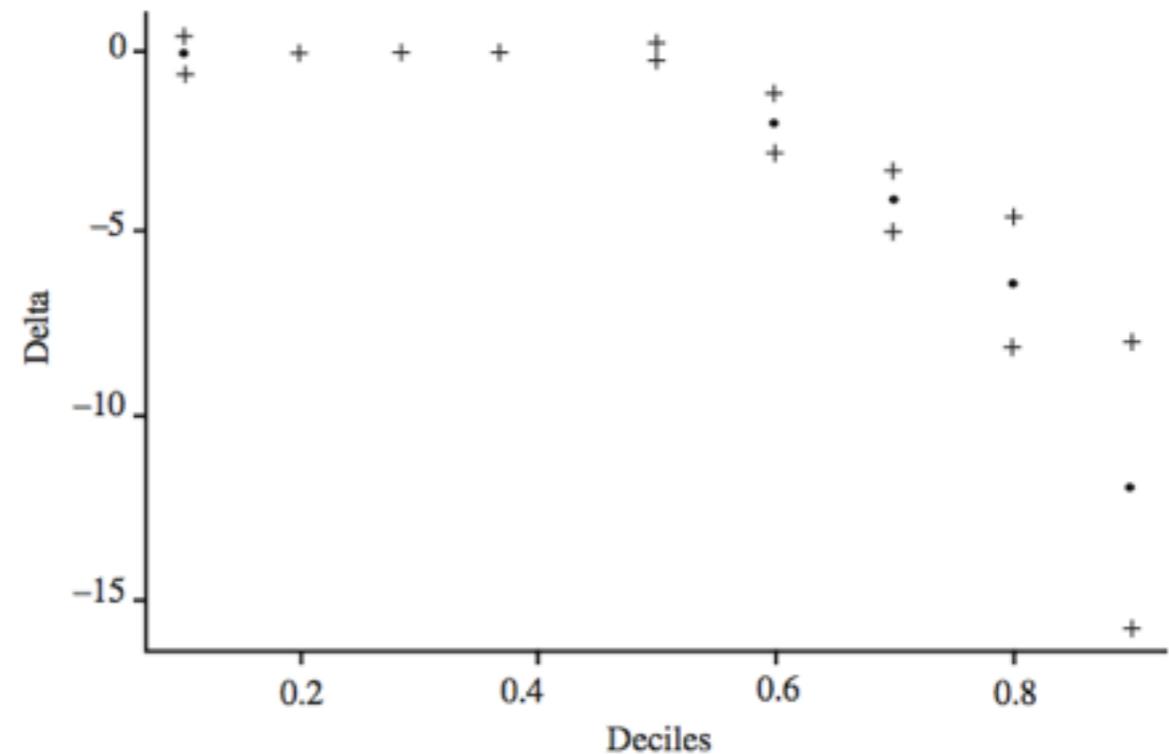
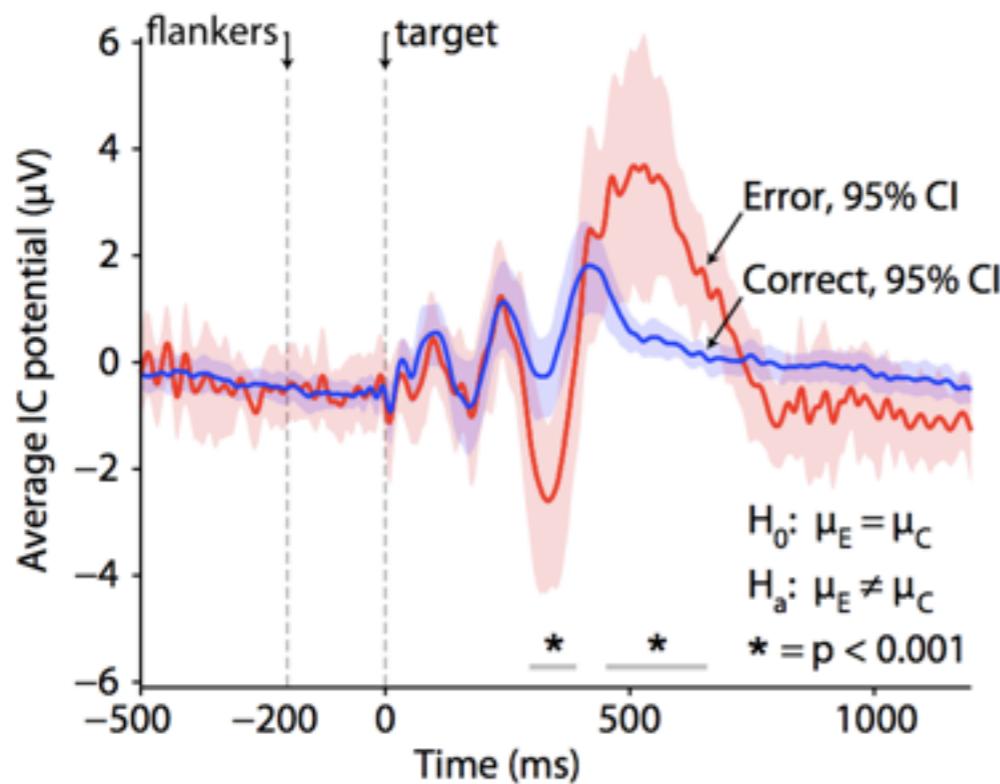
Academic Press

# References: making more informative figures

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Allen, E.A., Erhardt, E.B. & Calhoun, V.D. (2012) Data visualization in the neurosciences: overcoming the curse of dimensionality. *Neuron*, 74, 603-608.

Wilcox, R.R. (2006) Graphical methods for assessing effect size: Some alternatives to Cohen's d. *Journal of Experimental Education*, 74, 353-367.



# ERP references

Rousselet GA and Pernet CR (2011)

**Quantifying the time course of visual object processing using ERPs: it's time to up the game**  
Frontiers in Psychology 2:97. doi: 10.3389/fpsyg.2011.00107

Rousselet GA, Pernet CR, Caldara R and Schyns PG (2011)

**Visual object categorization in the brain: what can we really learn from ERP peaks?**  
Frontiers in Human Neuroscience 5:156. doi: 10.3389/fnhum.2011.00156

# menu

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- [1] Standard ERP figure
- [2] Better figures
- [3] Better stats: robust estimation & hypothesis testing
- [4] Single-subject analyses
- [5] Yet better figures
- [6] Control for multiple comparisons
- [7] ANOVA example
- [8] ANCOVA & linear contrasts example

# dataset

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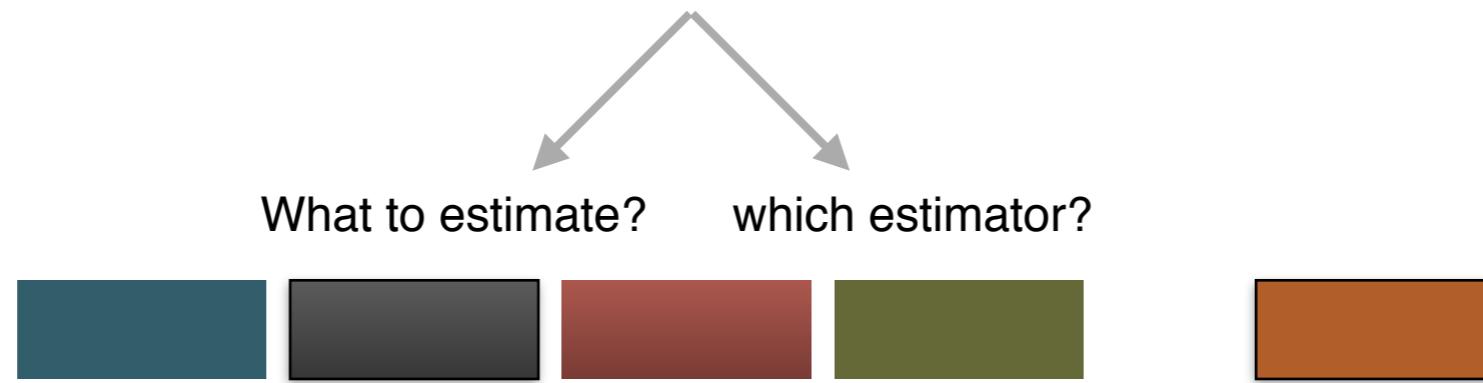
- 1 electrode only:
  - **erps.mat**: 20 subjects, 2 conditions, 500 Hz, -300:600 ms - single-trials
  - **erpm.mat**: averages with size (451x20x2)
  - **S1\_1elec\_epoch\_ancova.set**: 2 subjects, ANCOVA - 500 Hz, single-trials
- All electrodes:
  - **S1\_allelec\_epoch.set**: 2 subjects, 250 Hz, -300:400 ms, single-trials

folders to add to your path:  
eeglab + limo\_eeg  
shift\_function

# Levels of inferences

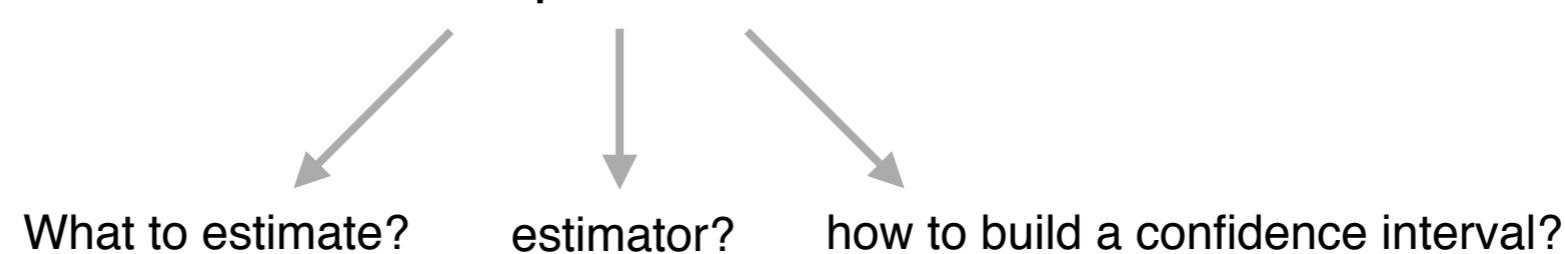
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**Level 1**      Brain 1      Brain 2      Brain 3      Brain 4    ...      Brain n



**Level 2**

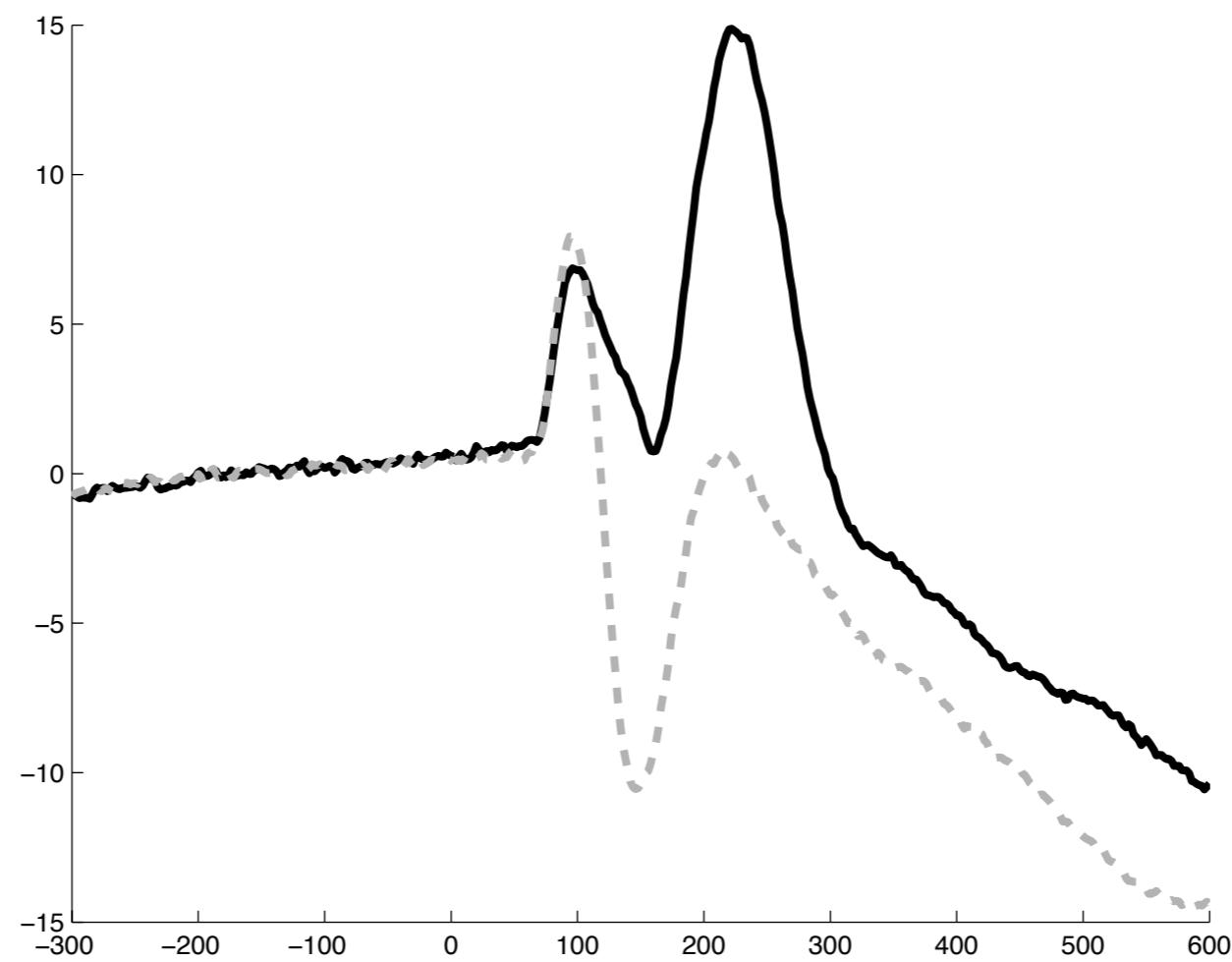
Group inference



let's get started: open Matlab...

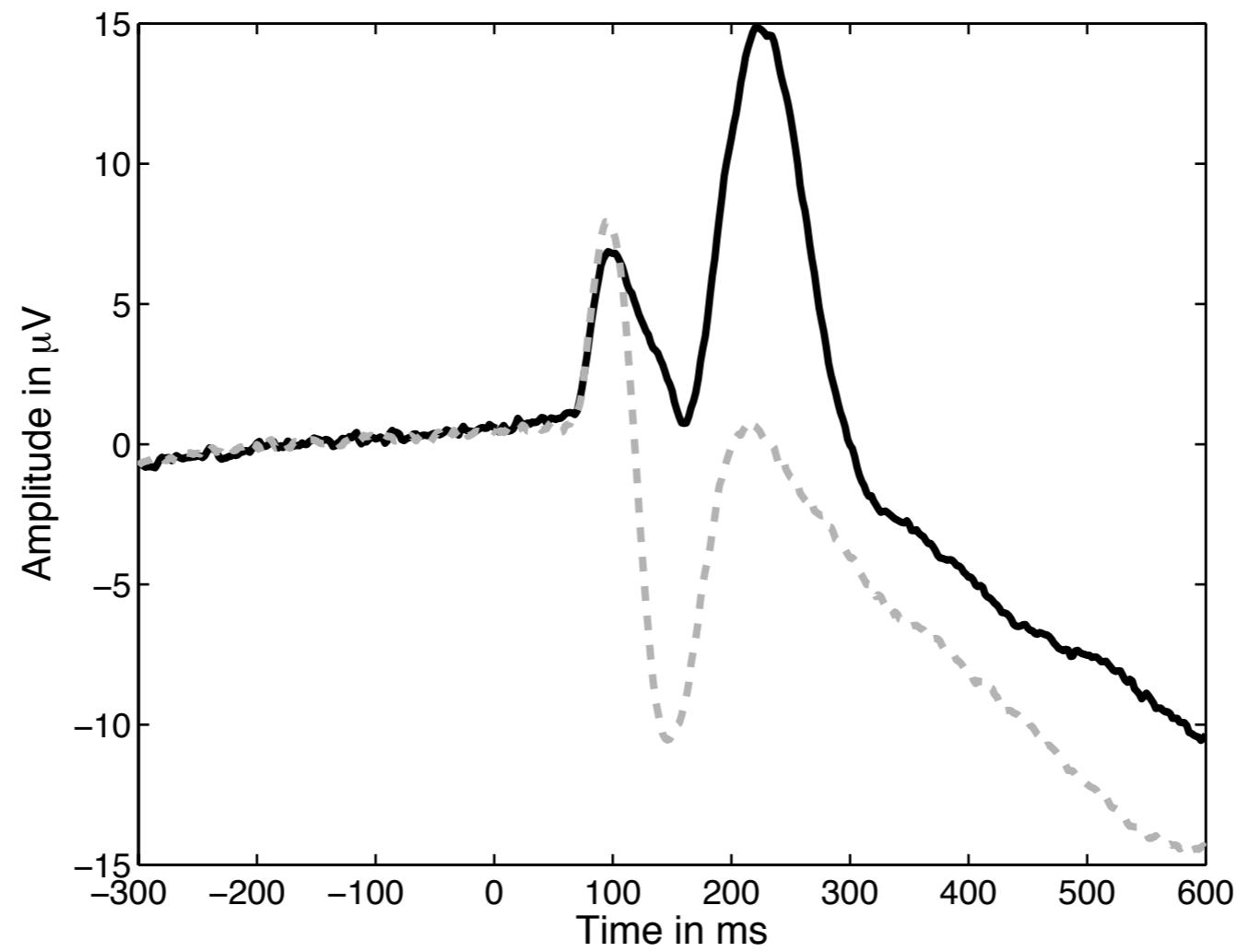
# The standard ERP figure (1)

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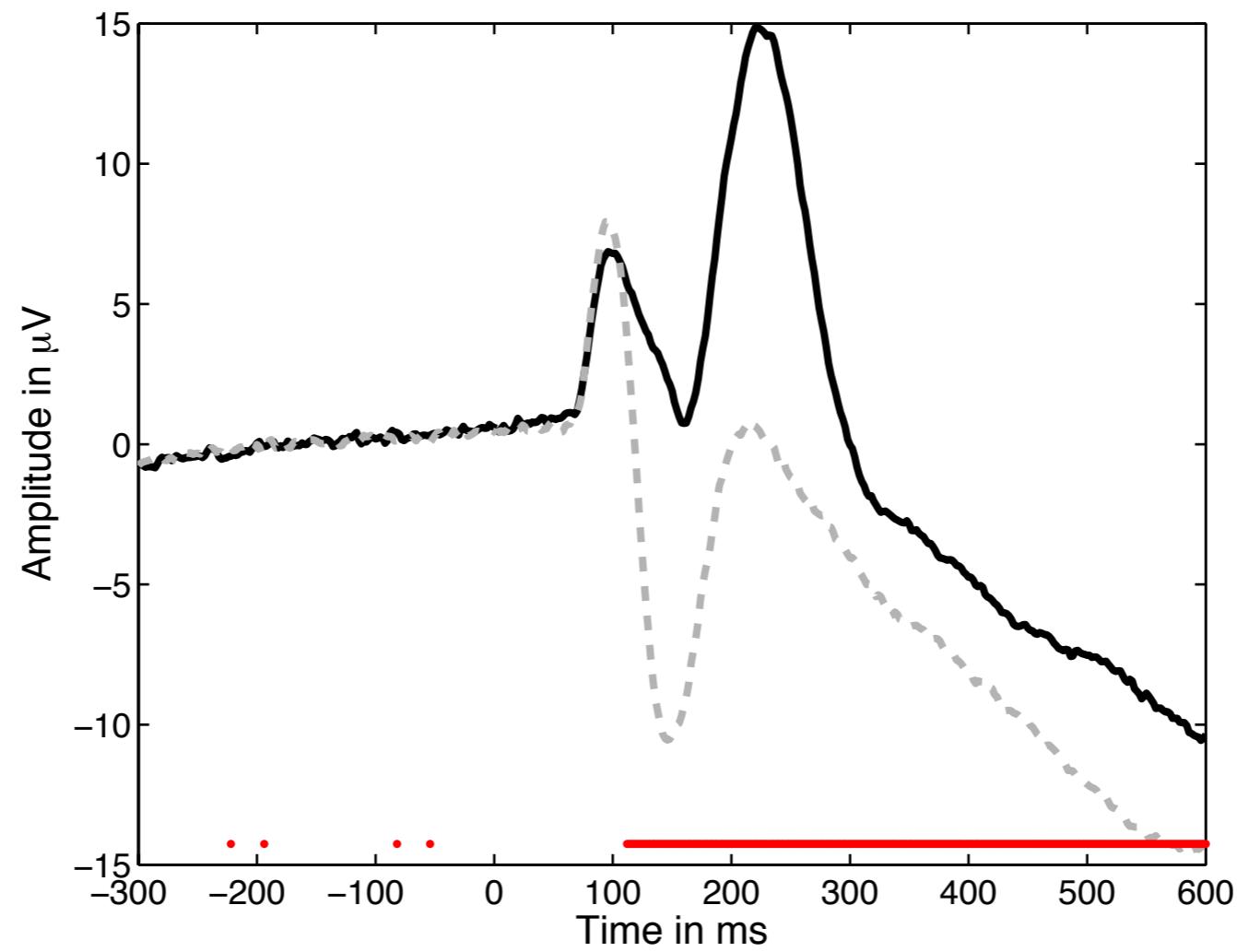
# The standard ERP figure (2)

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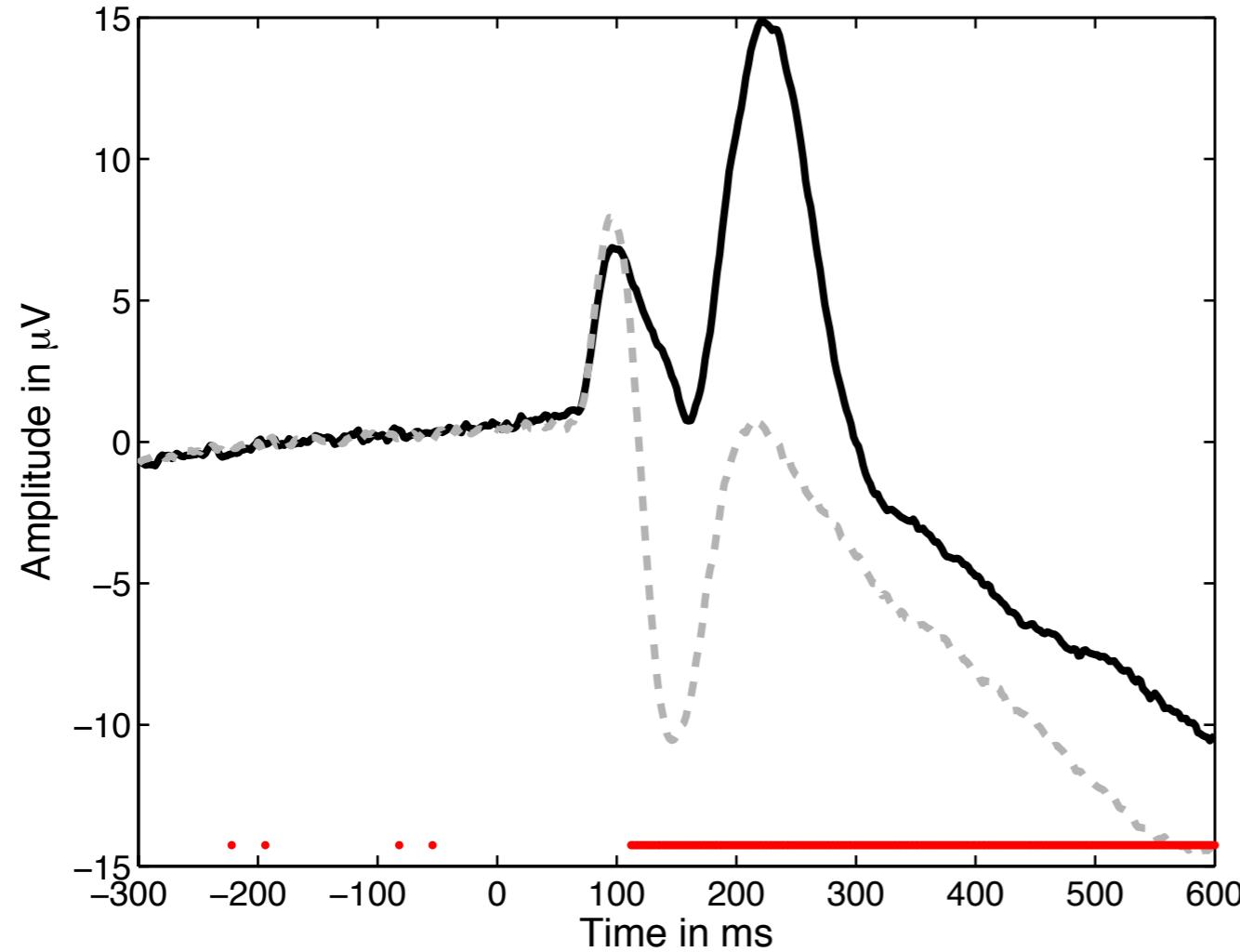
# The standard ERP figure (3)

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erp\_workshop\_1\_standard\_figure.m

# Why the standard figure is not good enough



Interpretations should be limited to what was measured: group differences in means

Significant effect?

- interesting?
- how many subjects?
- effect size?

Non-significant effect?

- not there?
- lack of power?
- how many subjects?
- effect size?

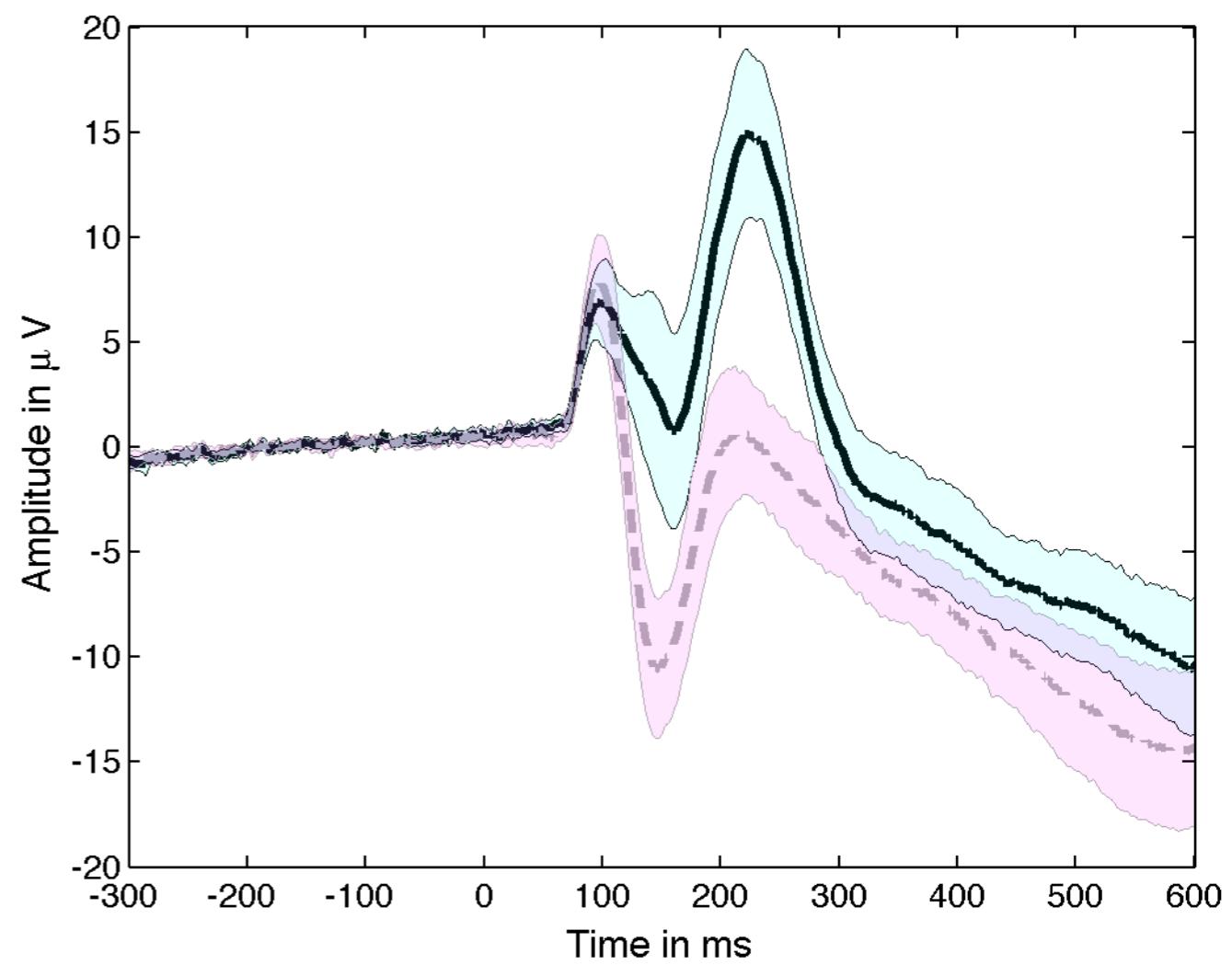
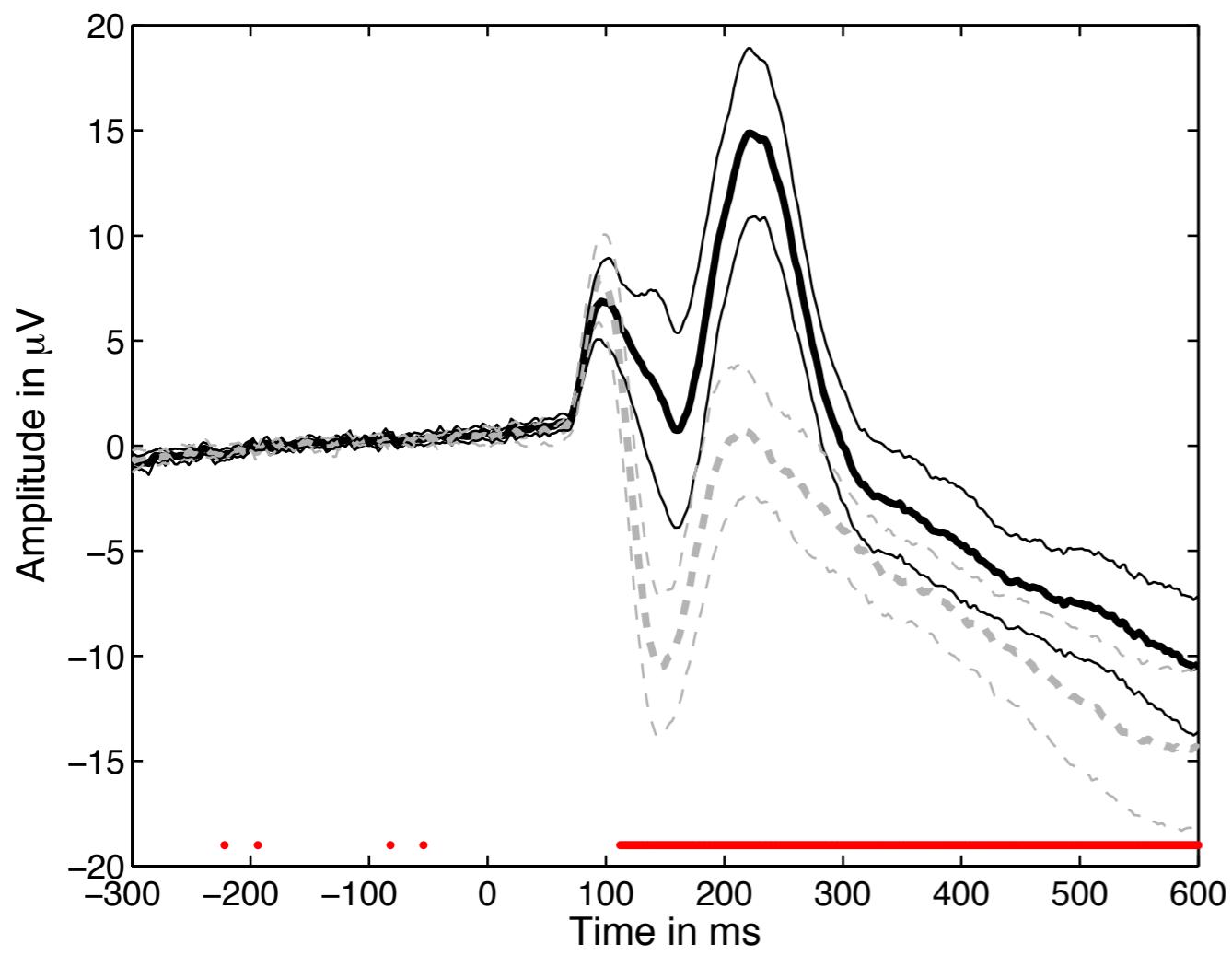
# Confidence intervals

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- *limo\_ttest*
- *limo\_trimci* / *limo\_yuend\_ttest* / *limo\_yuen\_ttest*
- *limo\_pbc*
- *limo\_bootttest1*
- *limo\_yuen\_ttest\_boot*
- *limo\_central\_tendency\_and\_ci*
- *limo\_robust\_ci*

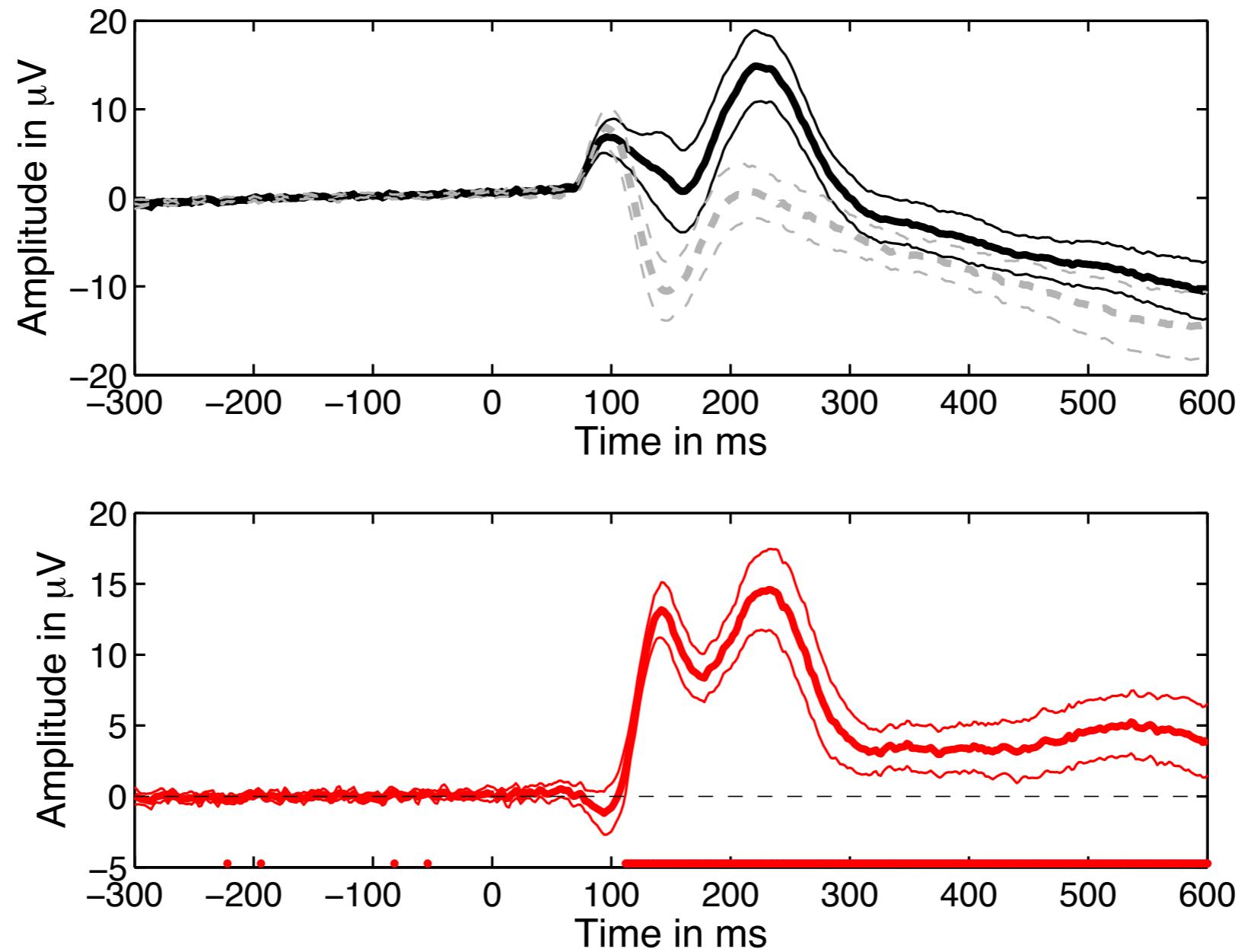
# Add confidence intervals

---

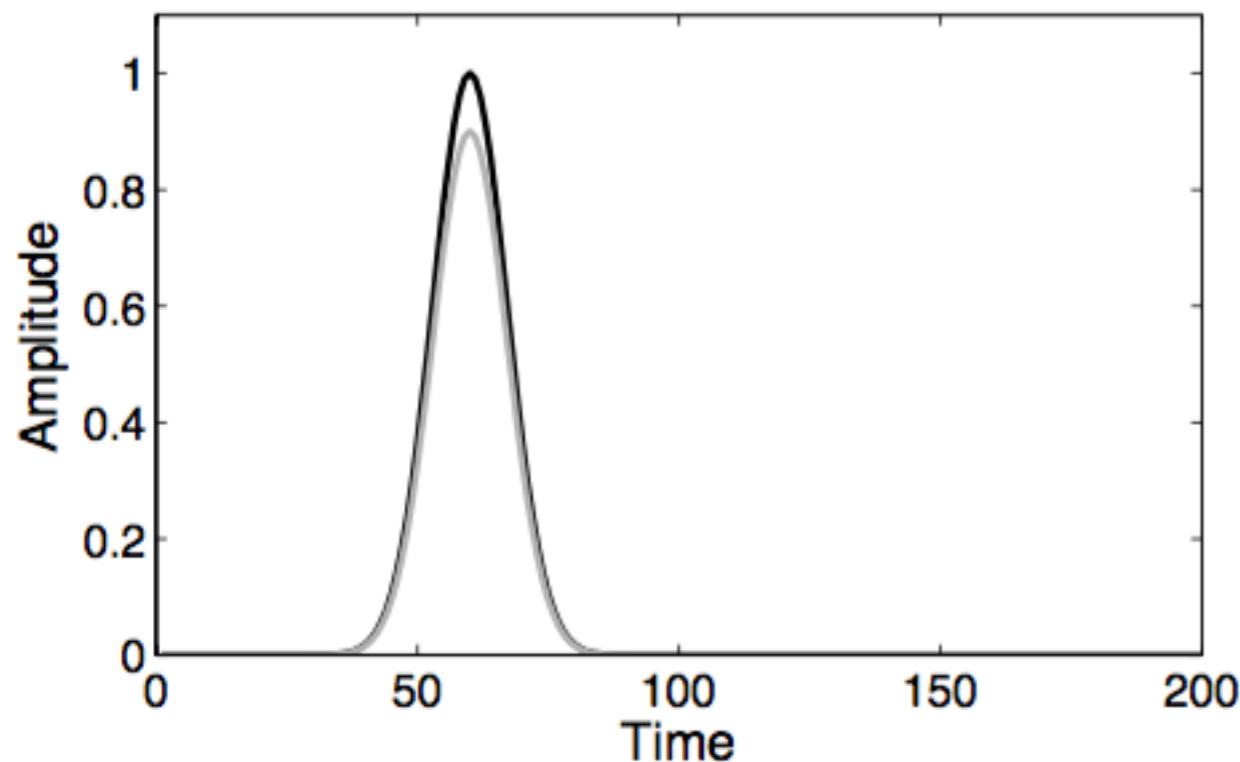


# Add plot of the difference

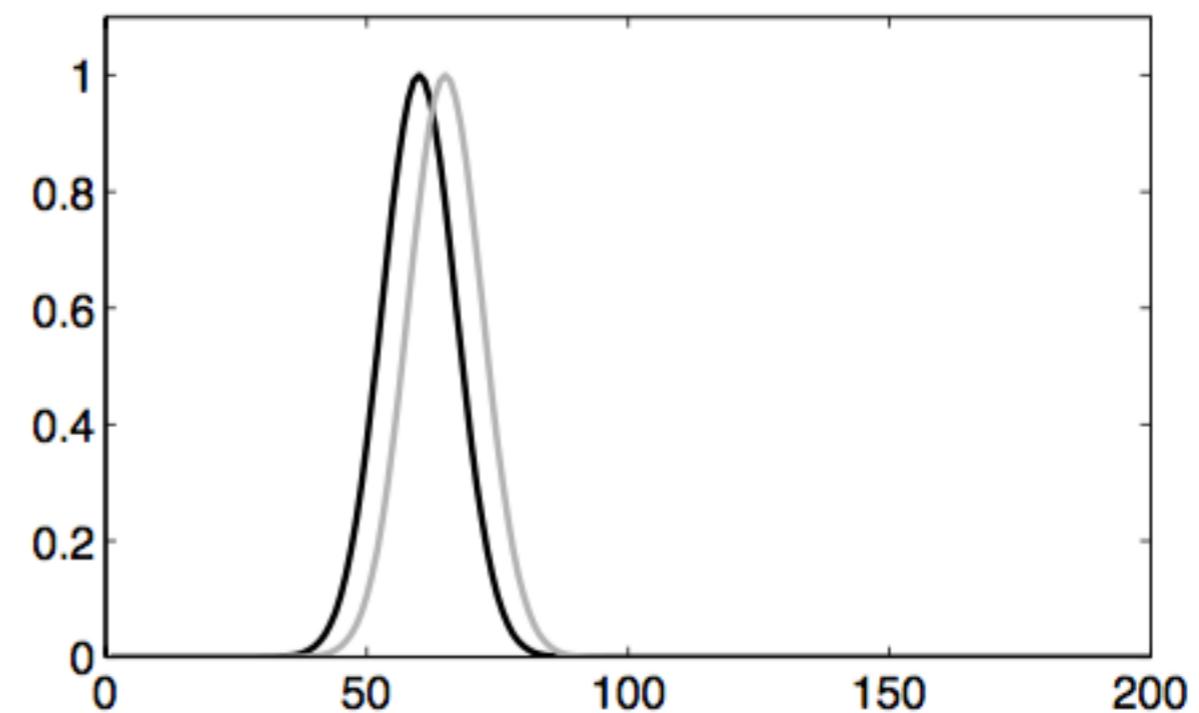
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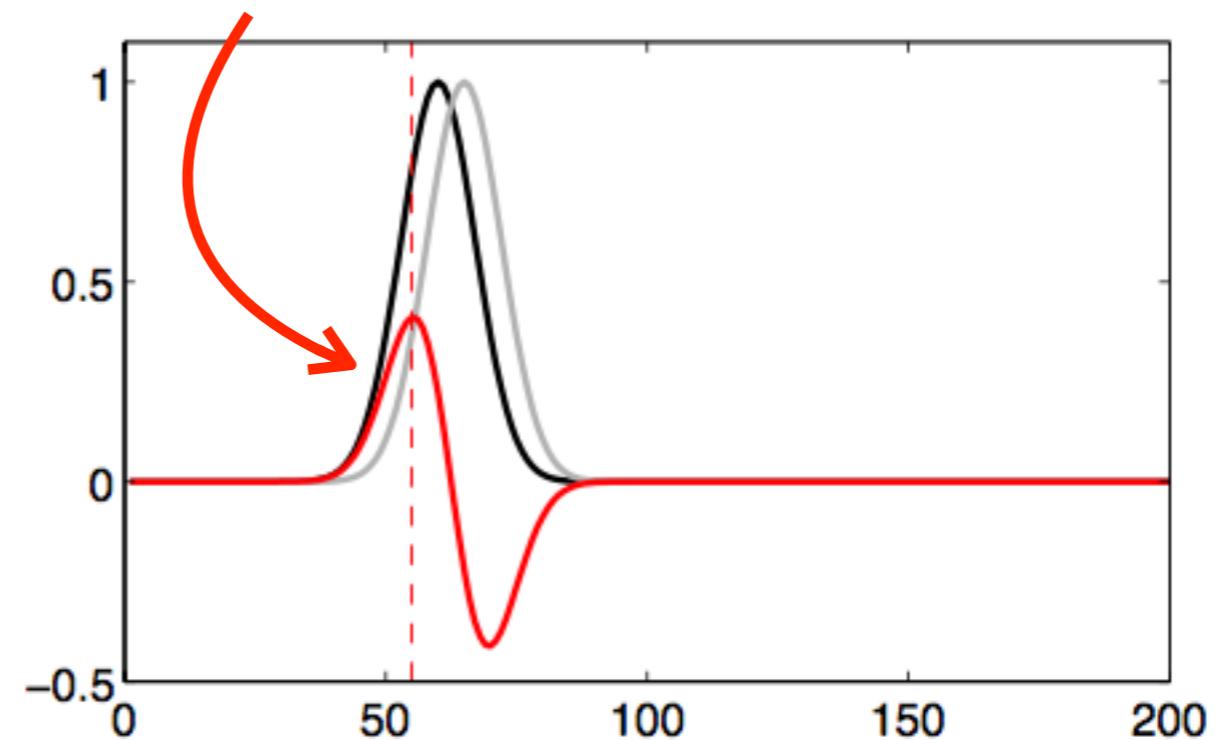
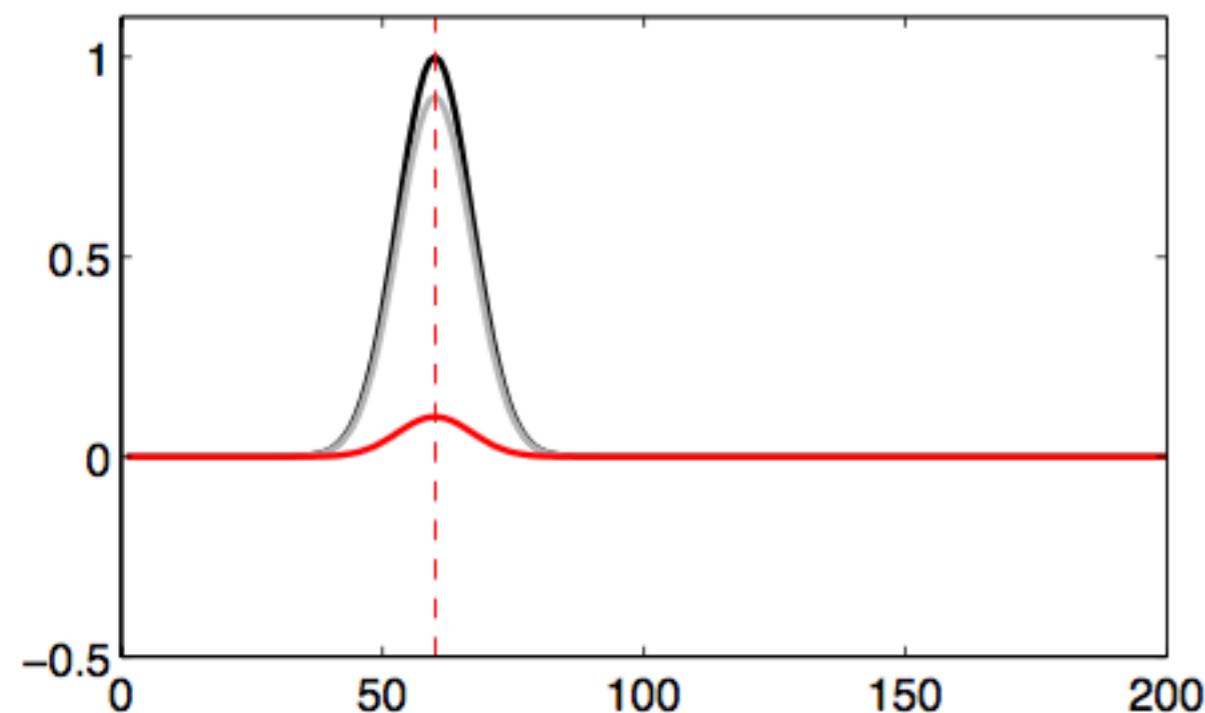
**Amplitude difference**



**Latency difference**

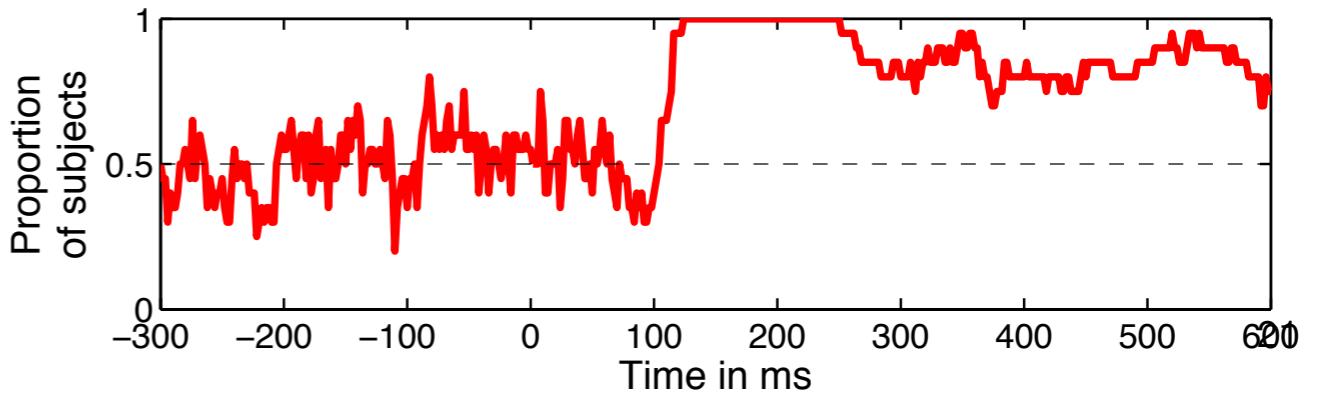
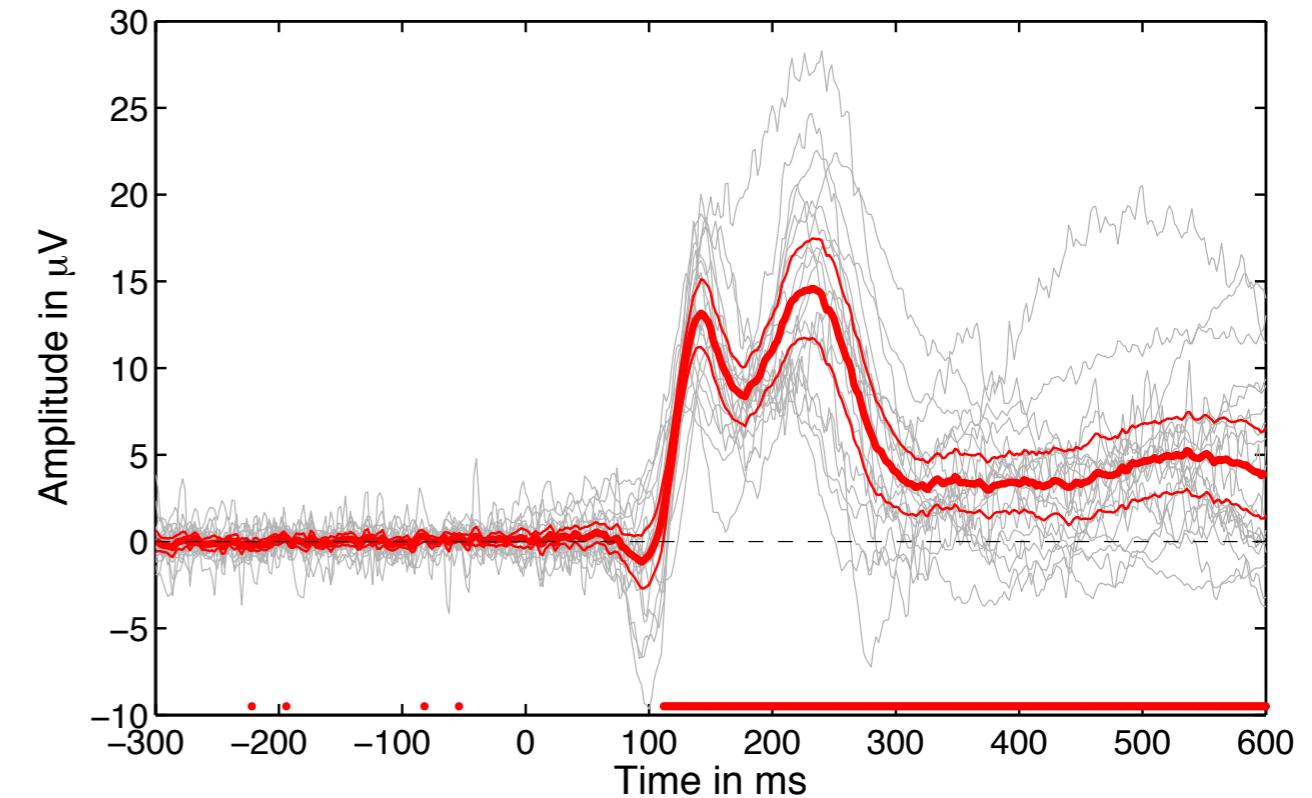
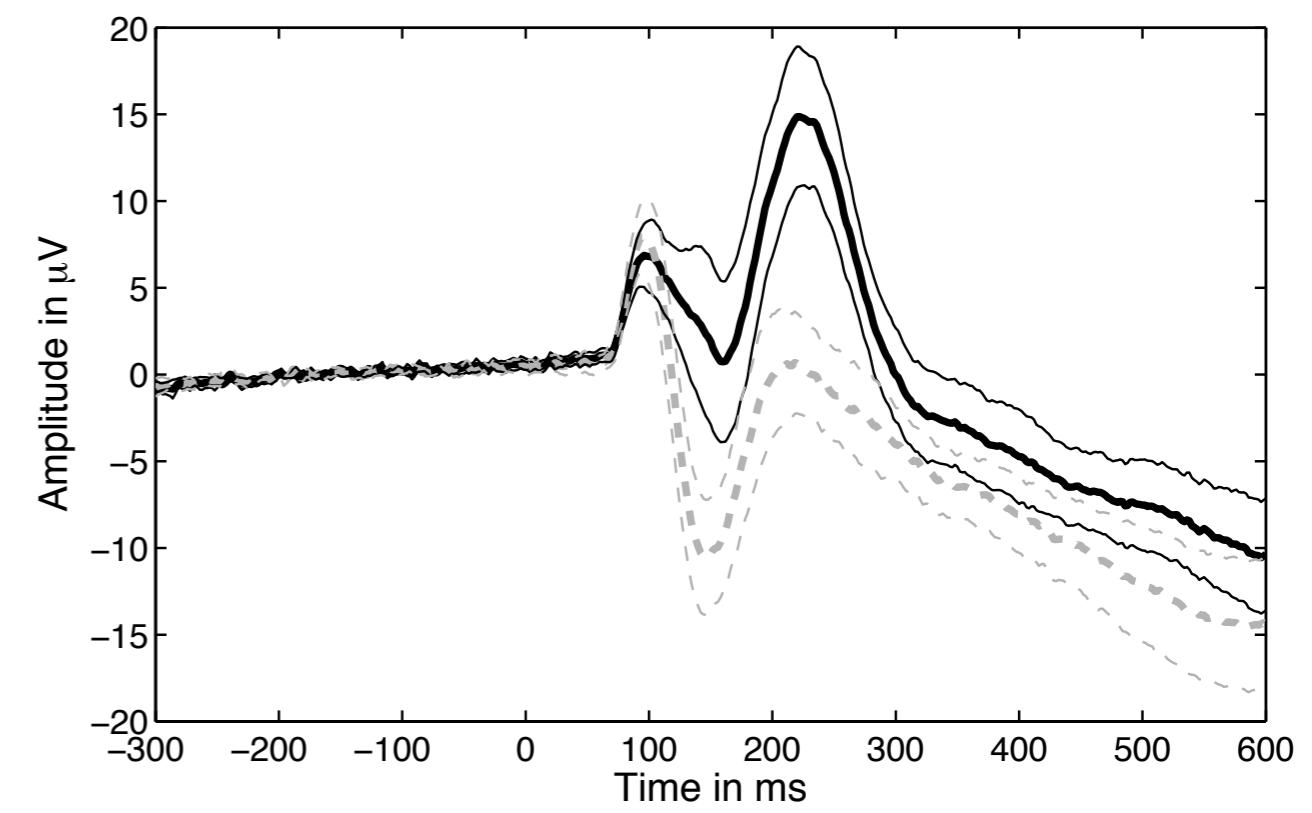


maximum difference before the peaks!



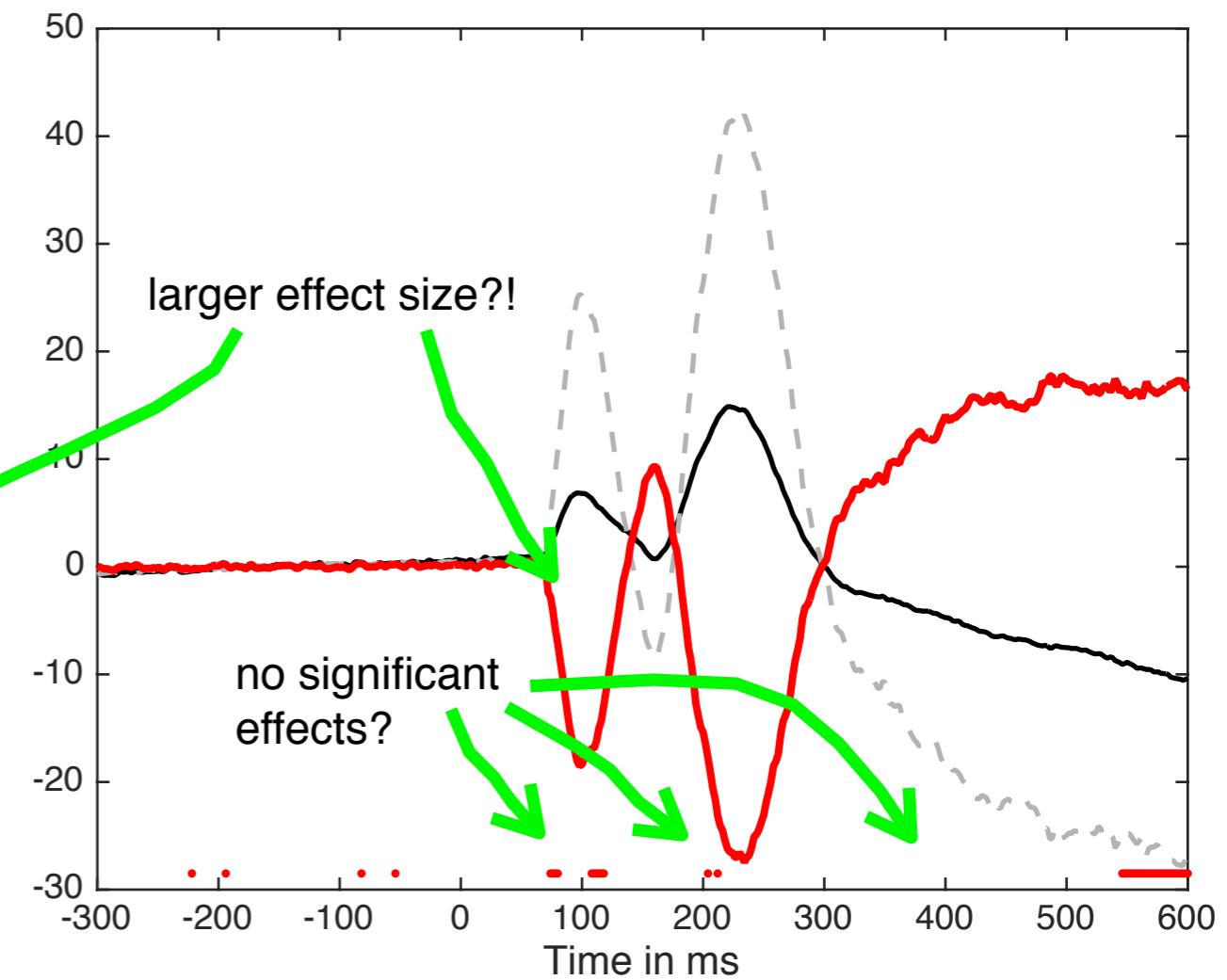
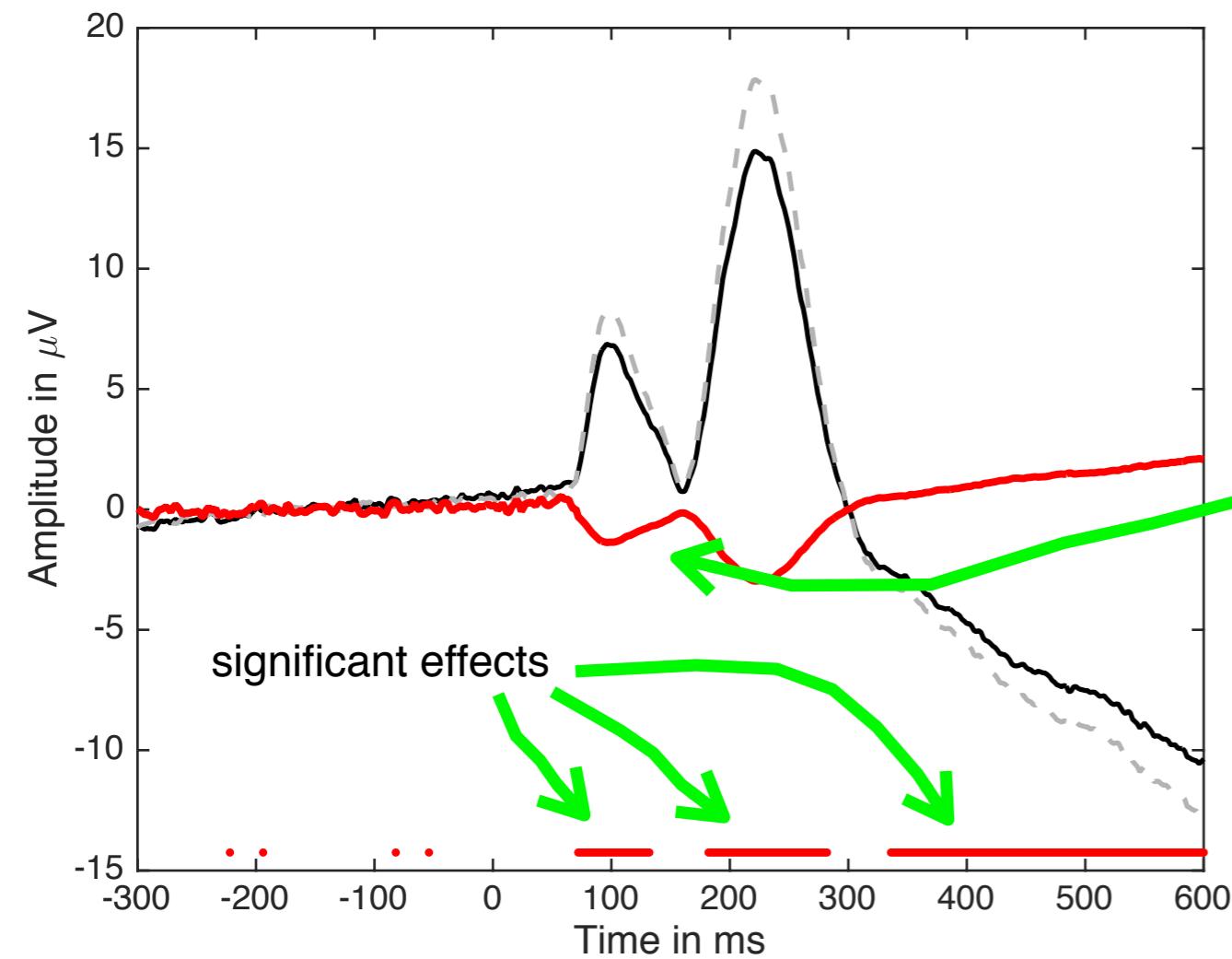
# How many subjects show an effect?

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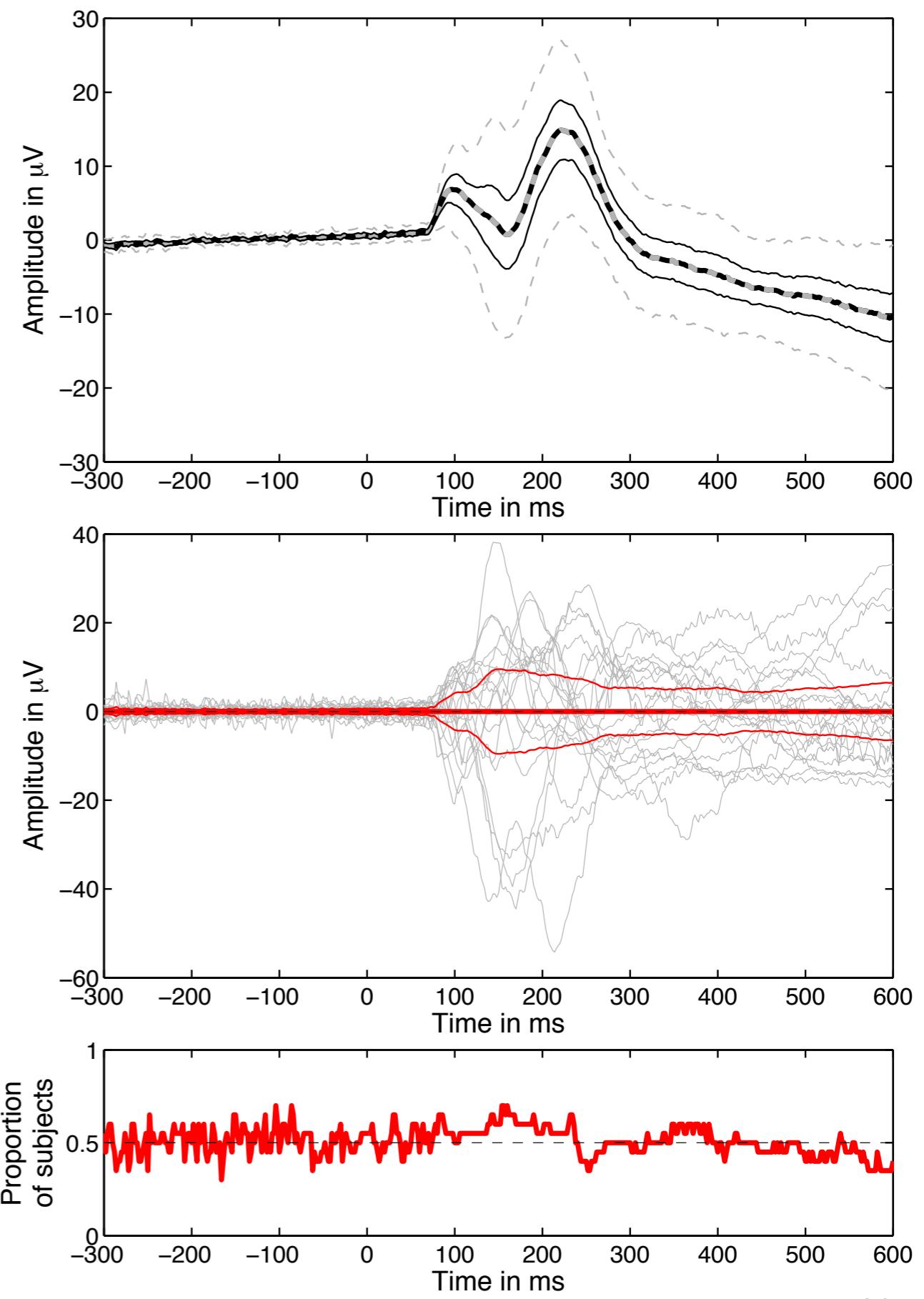


erp\_workshop\_2\_better\_figure.m

# No effect? Case study 1



# No effect? Case study 2

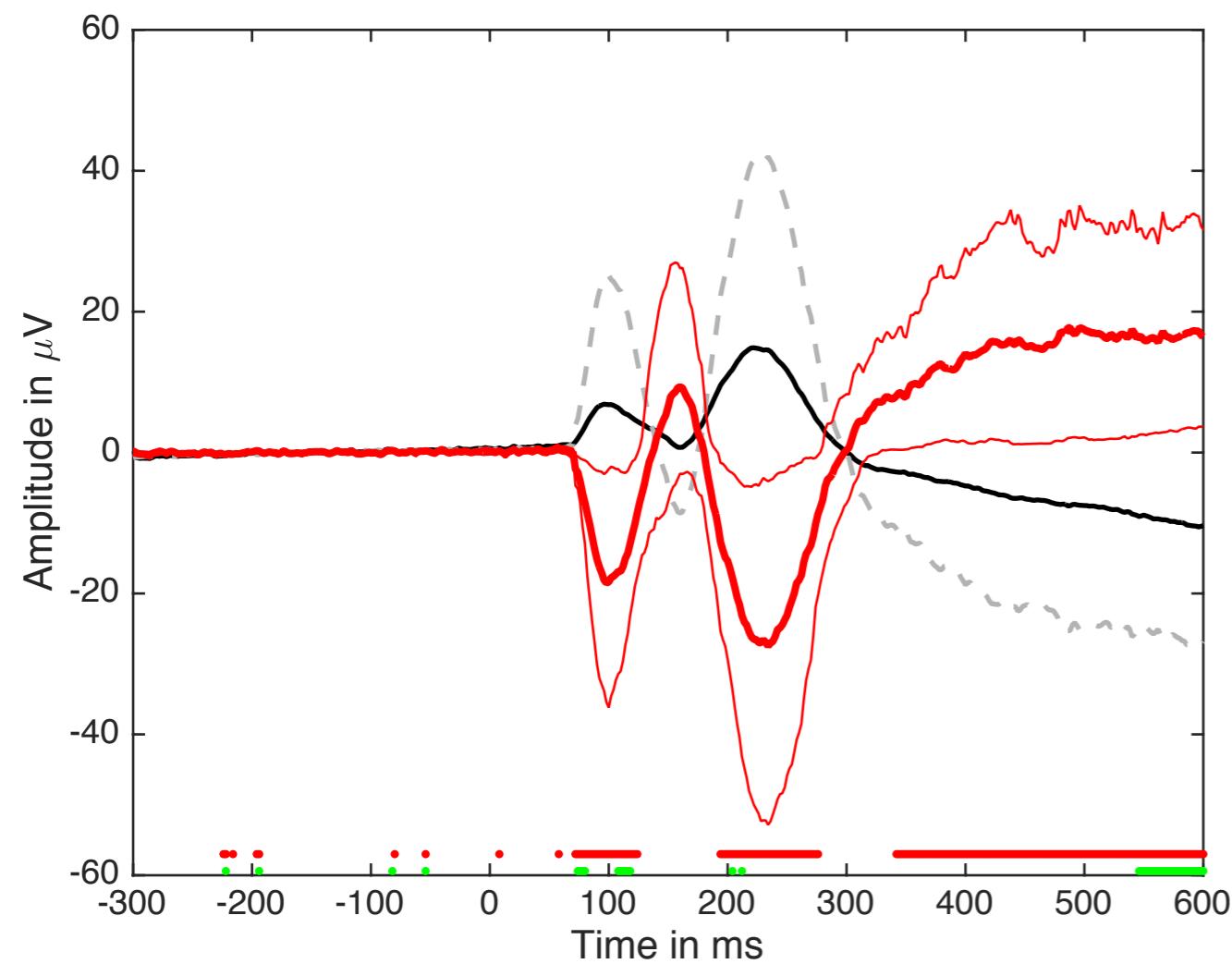


erp\_workshop\_3\_case\_studies.m

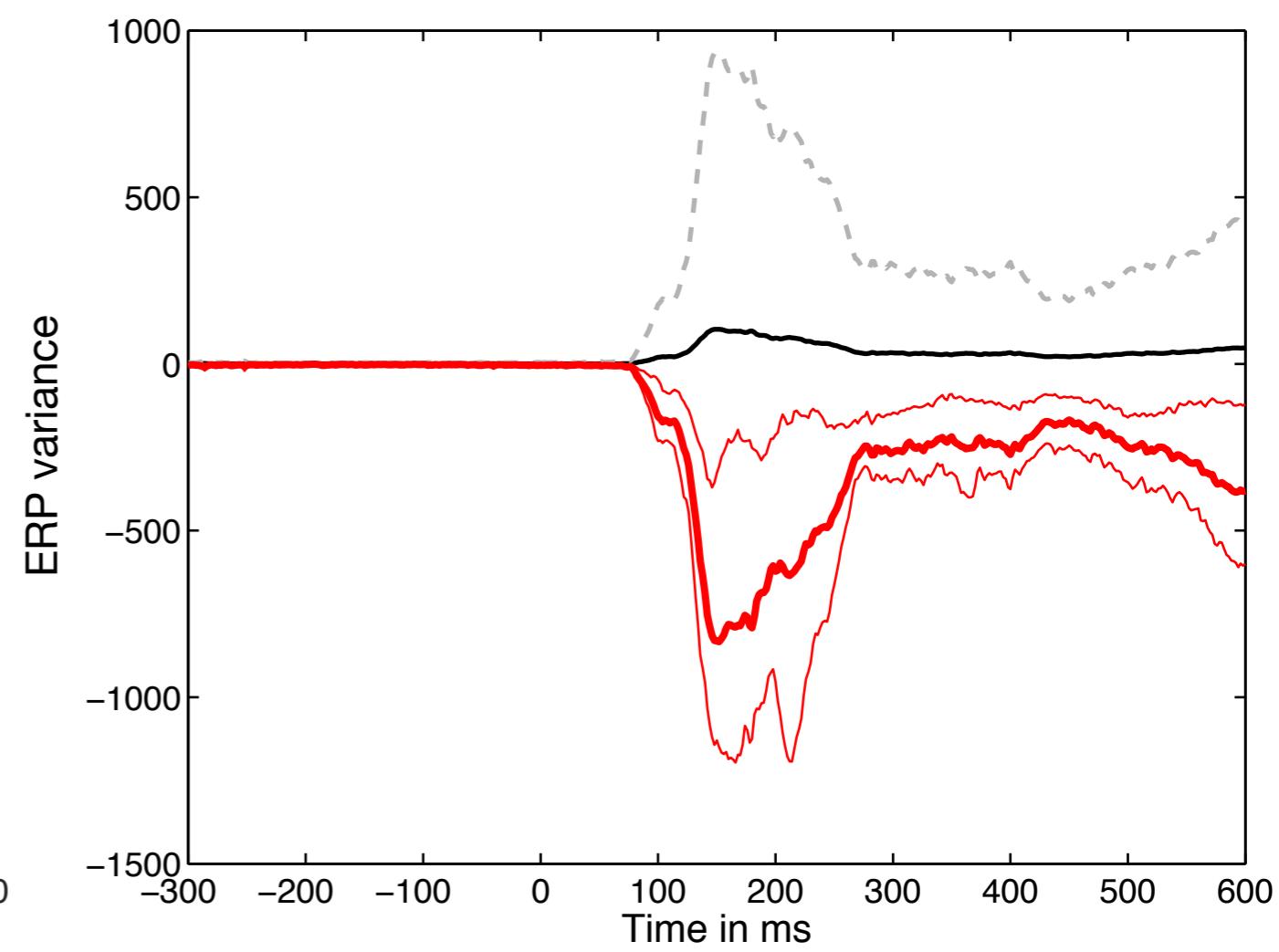
# Solving case studies: percentile bootstrap

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case 1: effect of outliers



case 2: difference in spread



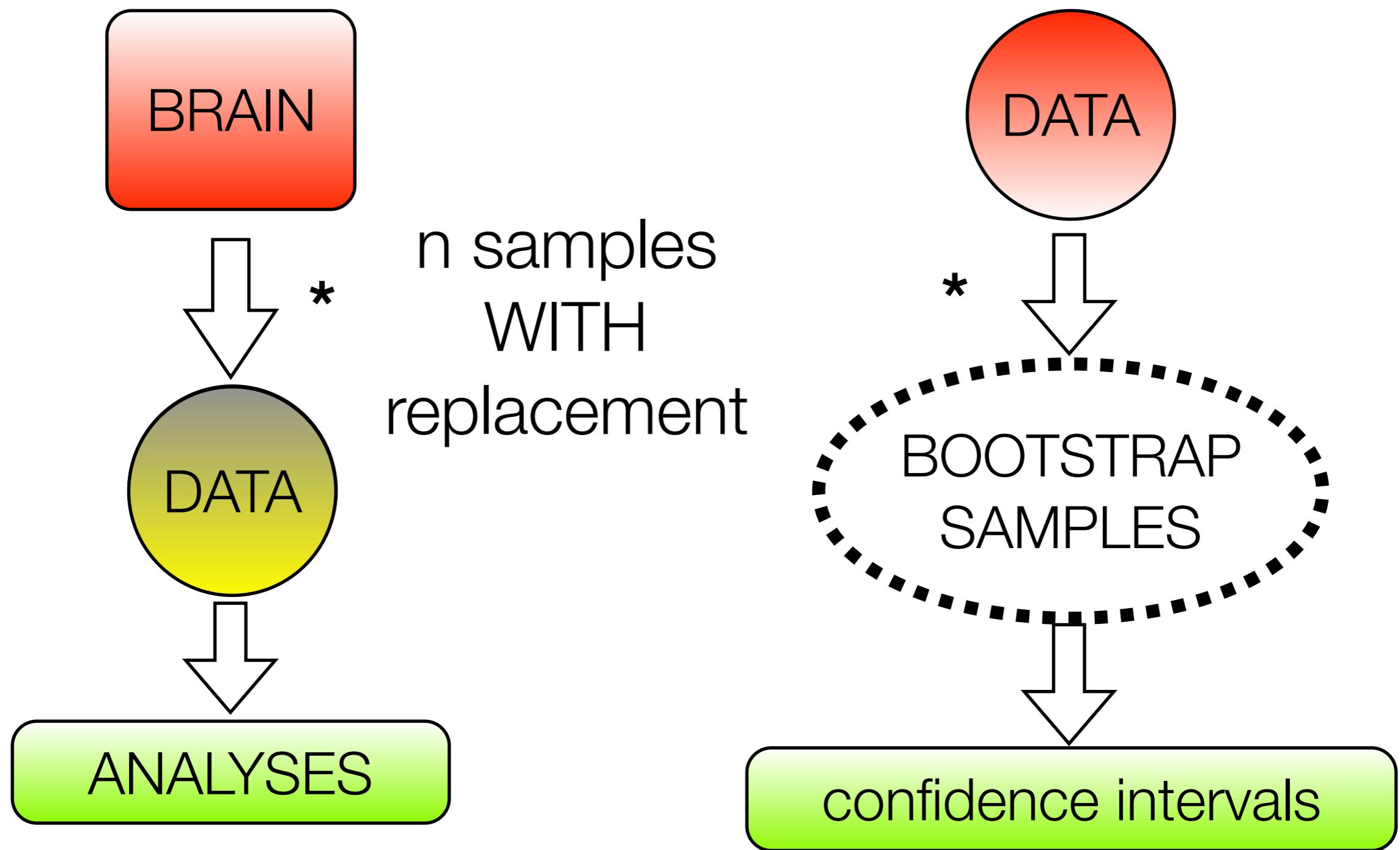
# Bootstrap: central idea

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- “The bootstrap is a computer-based method for assigning measures of accuracy to statistical estimates.” Efron & Tibshirani, An introduction to the bootstrap, 1993
- “The central idea is that it may sometimes be better to draw conclusions about the characteristics of a population strictly from the sample at hand, rather than by making perhaps unrealistic assumptions about the population.” Mooney & Duval, Bootstrapping, 1993

# Bootstrap: philosophy

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# Percentile bootstrap: general recipe

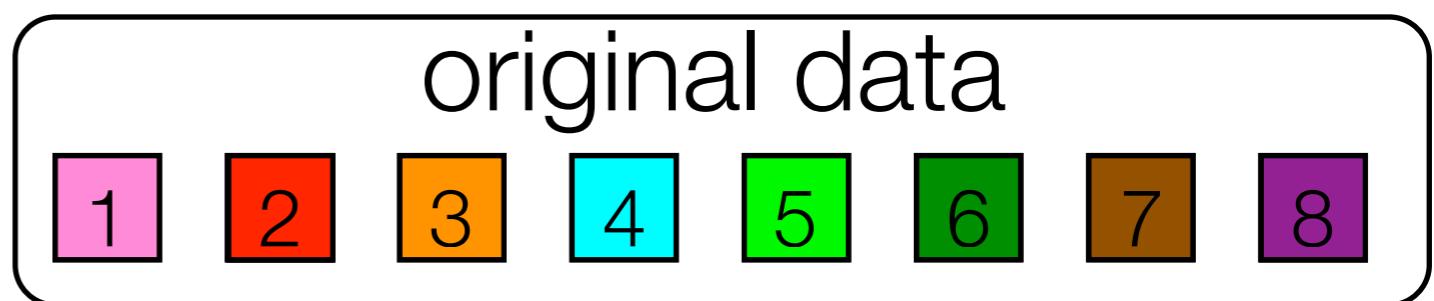
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- sample =  $X_1, \dots, X_n$
- resample  $n$  observations with replacement
- compute estimate
- repeat  $B$  times
- with  $B$  large enough the  $B$  estimates provide a good approximation of the distribution of the estimate of the sample

The percentile bootstrap method allows the bootstrap estimate of the sampling distribution to conform to any shape the data suggest, taking into account the variance and the skewness of the sample.

# Percentile bootstrap: general recipe

(1) sample WITH replacement n observations



(2) compute estimate  
e.g. sum, trimmed mean

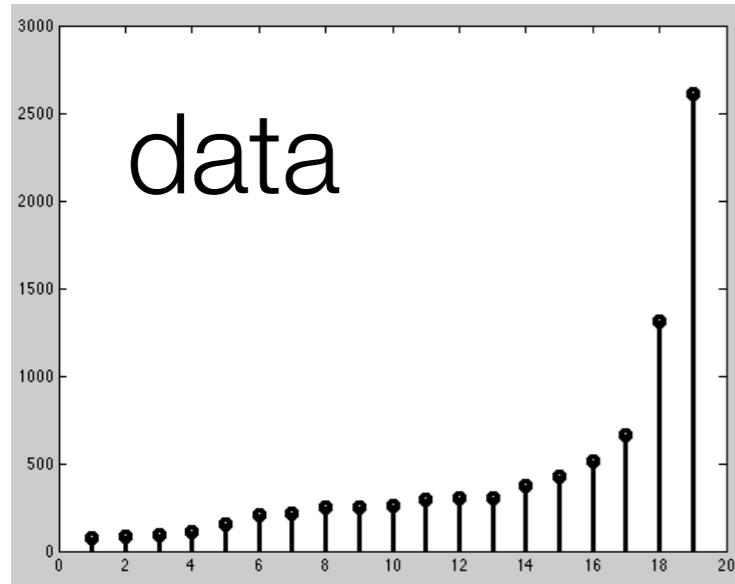


- (3) repeat (1) & (2) b times  
(4) sort the b estimates\*  
(5) get 1-alpha confidence interval

$\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6 \dots \Sigma_b$

# Percentile bootstrap estimate of mean

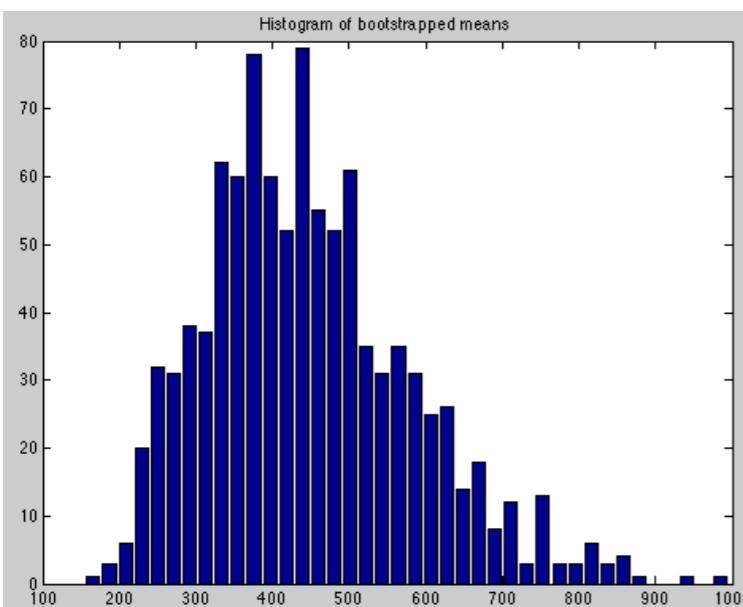
% self-awareness data, Wilcox, 2005, p58



Sample with  
replacement b times

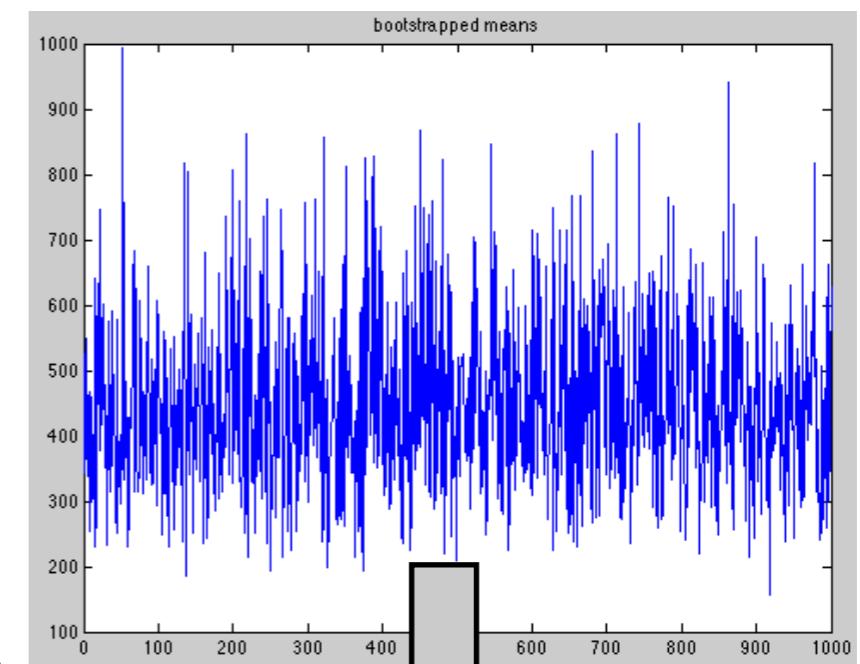
compute estimate

Distribution of bootstrapped  
estimates of the mean

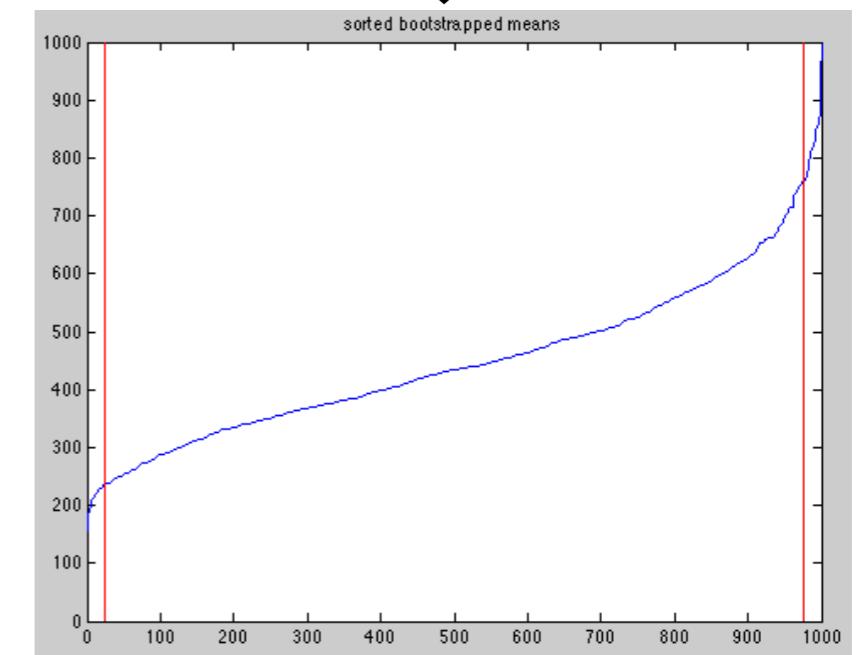


get PDF

Bootstrapped estimates



Sort & get CI

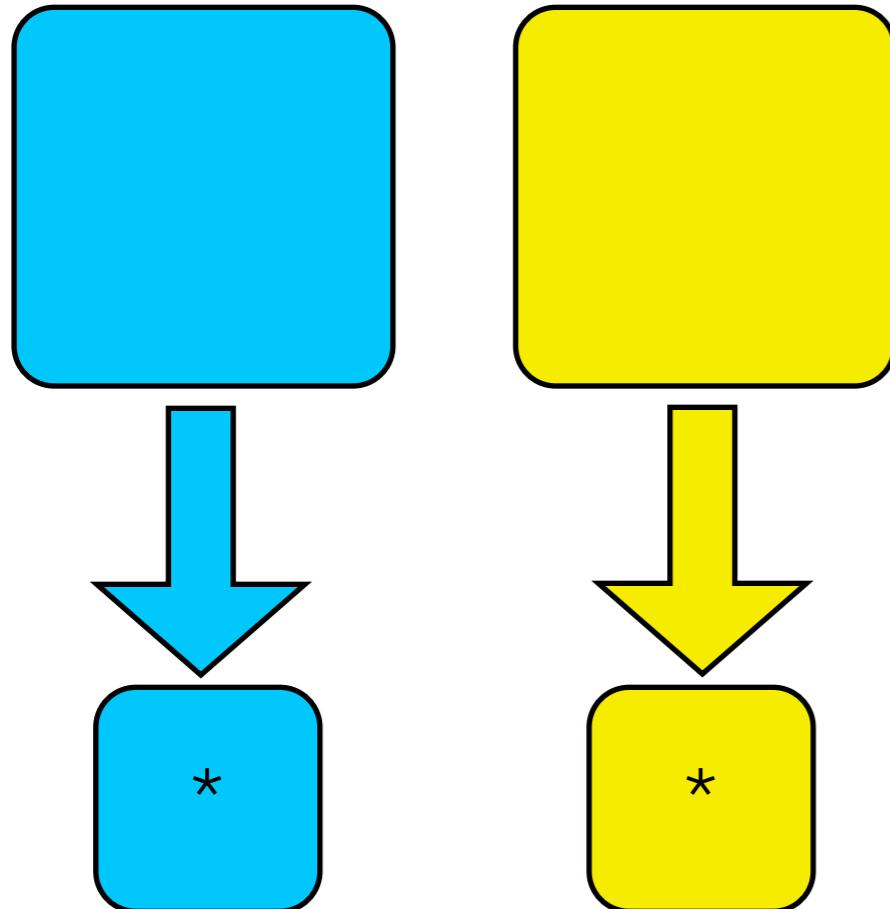


# resampling strategies: follow the data acquisition process

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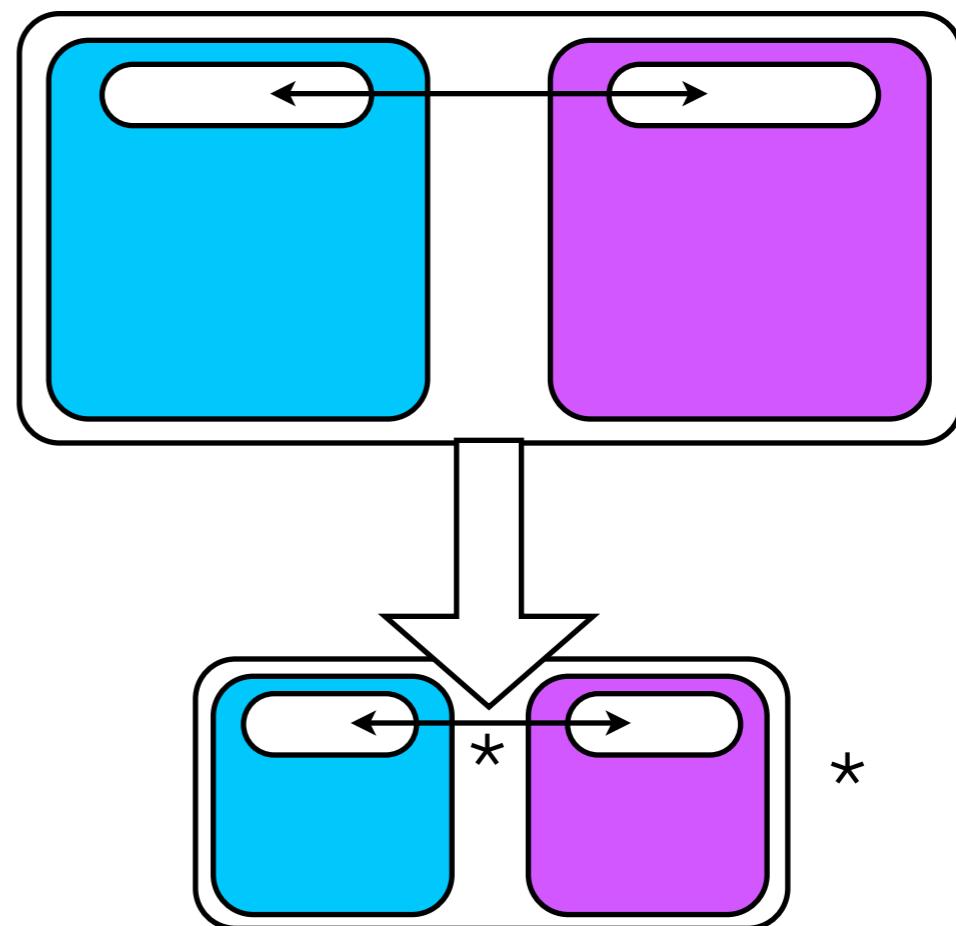
## independent sets:

- 2 conditions in single-subject analyses
- 2 groups of subjects, e.g. patients vs. controls



## dependent sets:

- 2 conditions in repeated-measure group analyses
- correlations
- linear regression



erp\_workshop\_4\_percentile\_bootstrap.m

# robust measures of central tendency

---

- mean

$$\sum(X_i - c)^2 \quad \sum(X_i - c) = 0 \quad c = \bar{X}$$

- median

$$\sum|X_i - c|$$

- trimmed mean

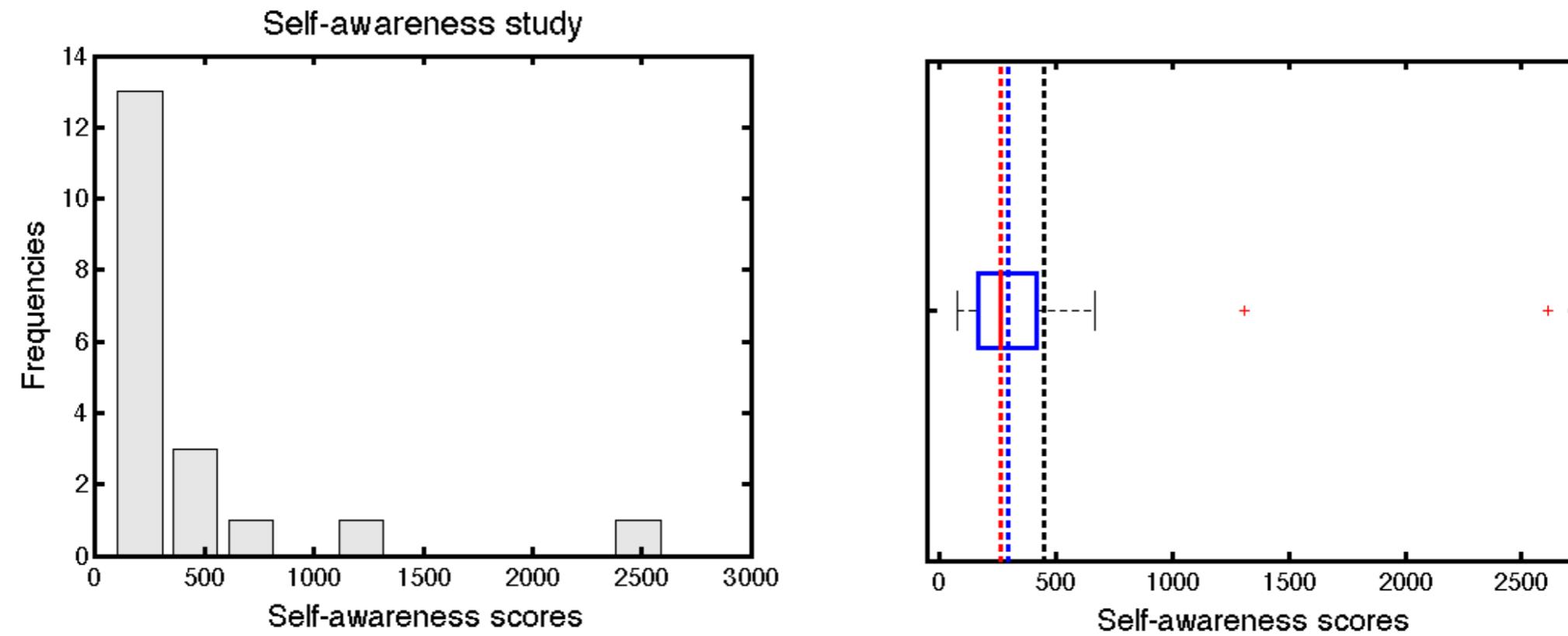
- Winsorized mean

- M estimators

- ...

$$t = \frac{\bar{X}_n - \mu}{s/\sqrt{n}}$$

# Why do we need robust estimators?



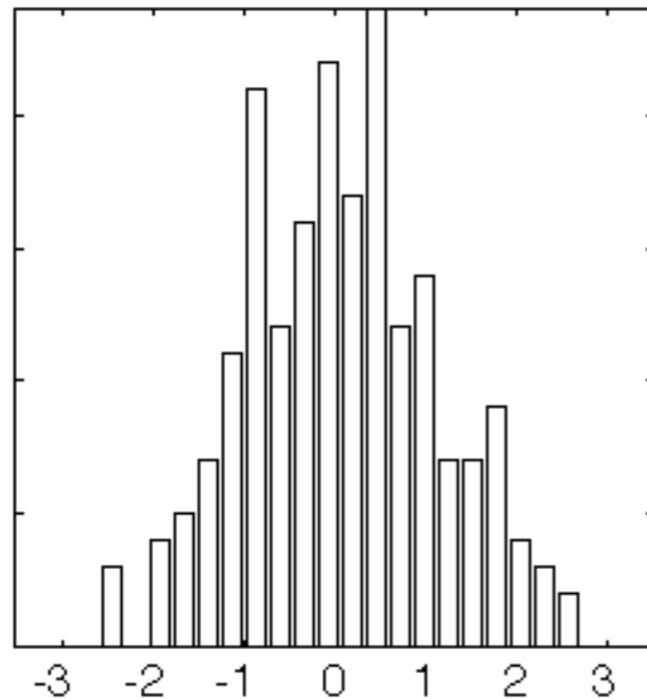
The mean and the variance are very sensitive to small departures from normality, so that tests relying on them (t-tests & ANOVAs) can perform poorly.

Wilcox, R. R., & Keselman, H. J. (2003). Modern robust data analysis methods: measures of central tendency. *Psychol Methods*, 8(3), 254-274.

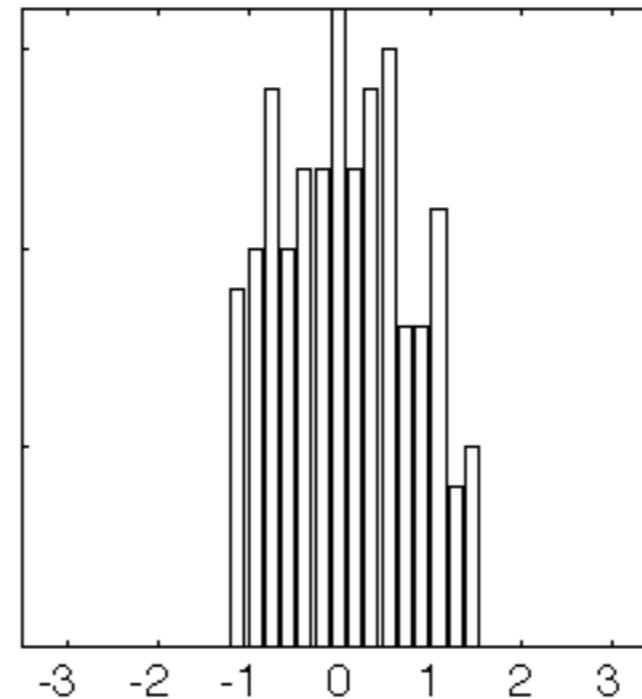
# Trimmed means

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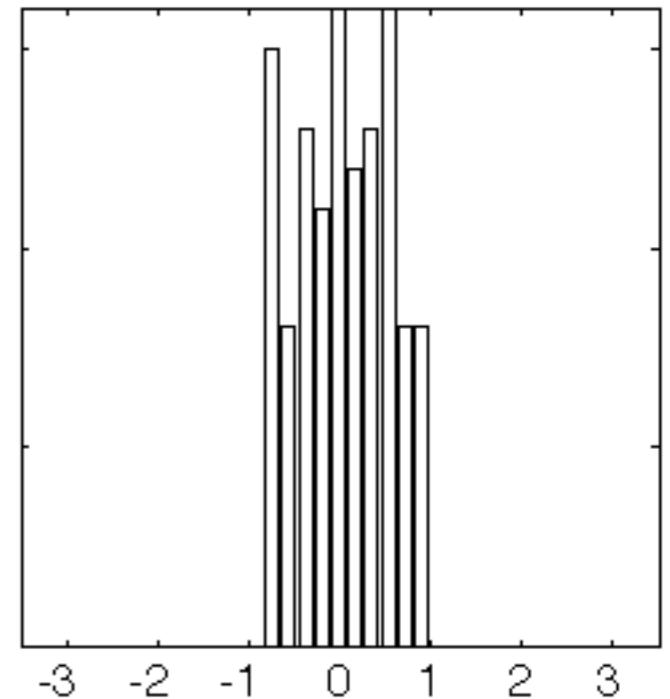
original  
distribution



10% trimmed  
distribution



20% trimmed  
distribution



- 20% trimmed means provide high power under normality and high power in the presence of outliers

Rand Wilcox, 2012, Introduction to Robust Estimation and Hypothesis Testing, Elsevier

**ERP application:** Rousselet, Husk, Bennett & Sekuler, 2008, *Journal of Vision*.

# Robust estimators of central tendency

---

- tmERP = trimmean(EEG.data,...)
- mdERP = median(EEG.data,...)
- in LIMO EEG:
  - mdERP = limo\_median(EEG.data,...)
  - tmERP = limo\_trimmed\_mean(EEG.data,...)
  - hdERP = limo\_harrell\_davis(EEG.data,...)

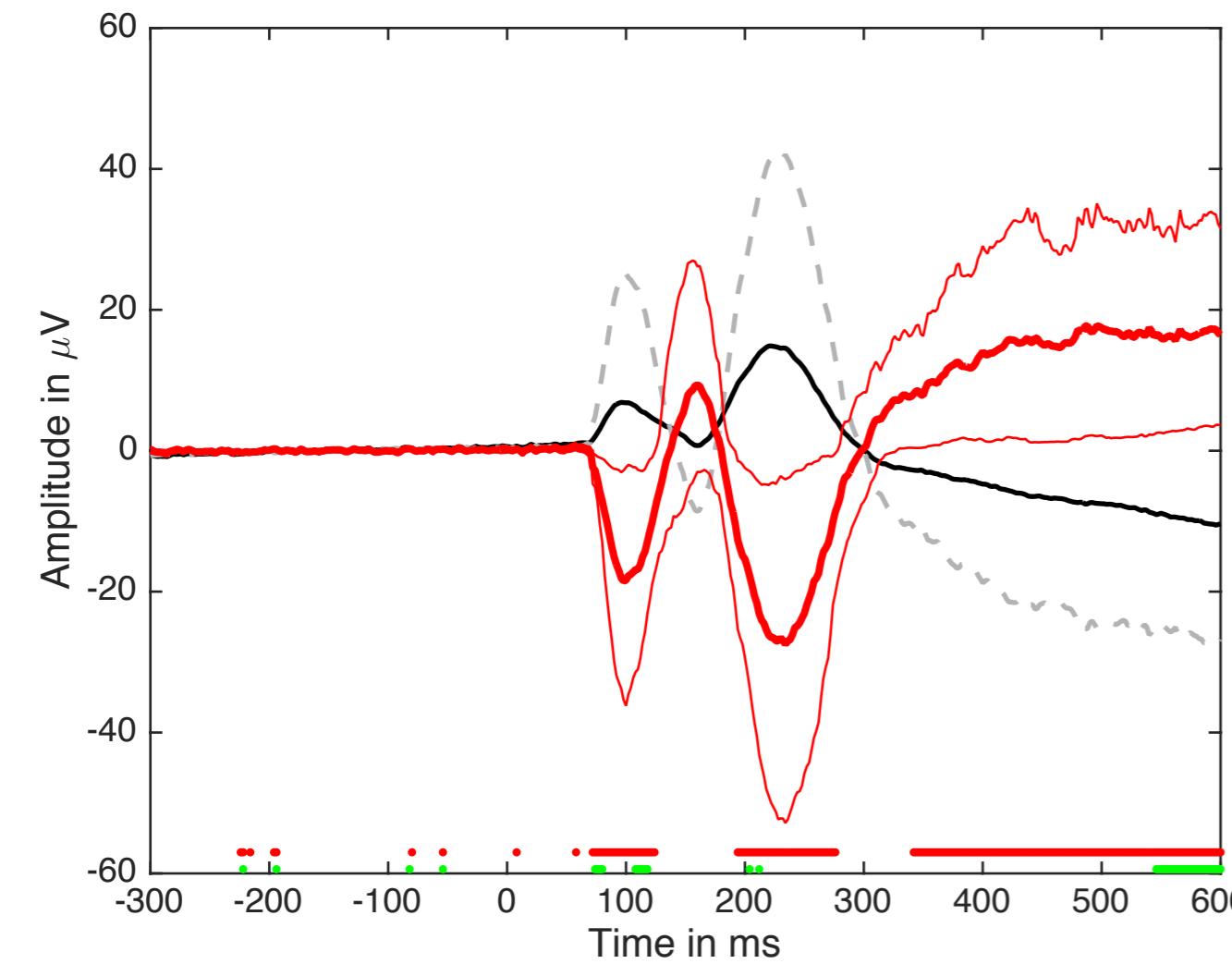
# Robust estimators of dispersion

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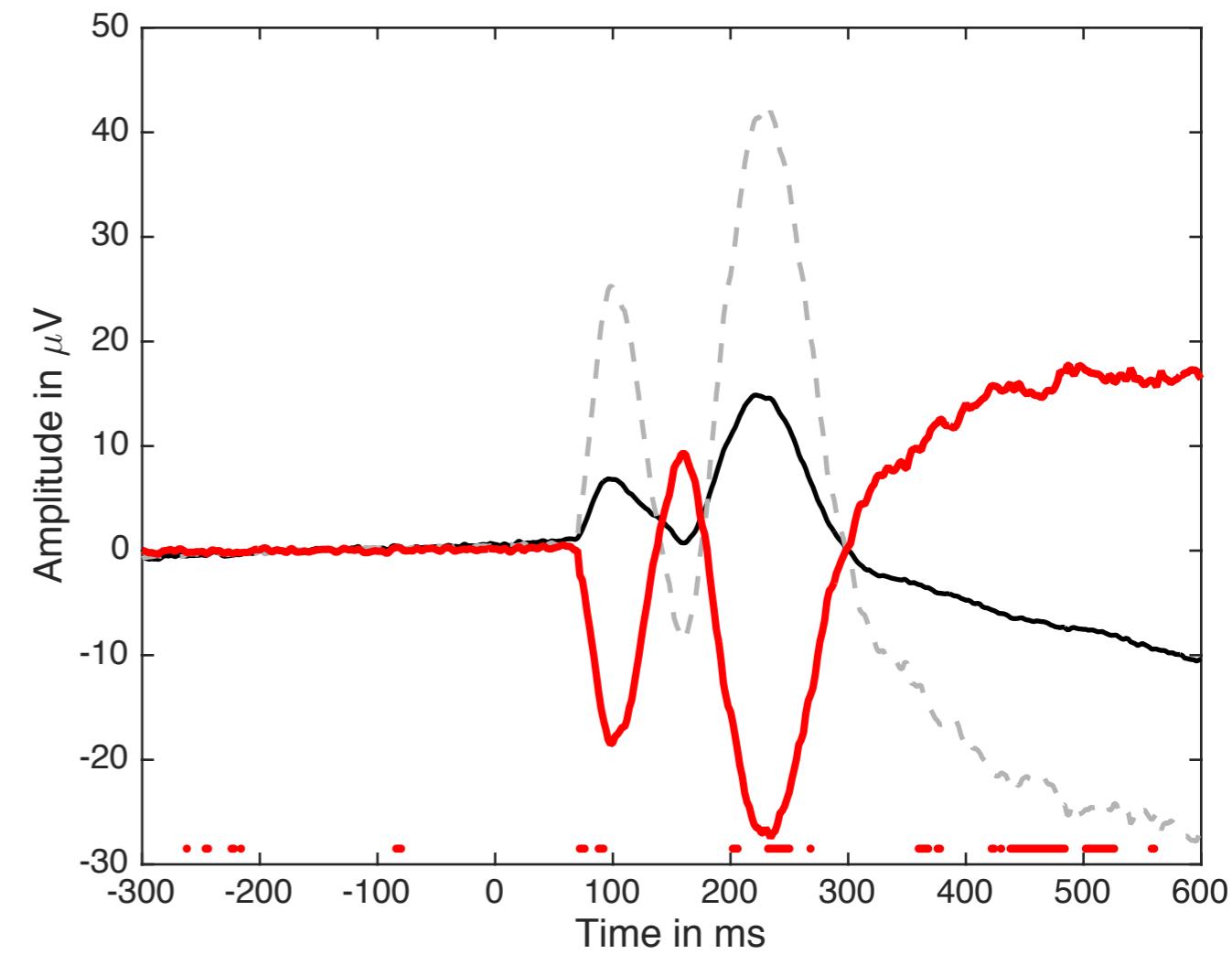
- `madERP = mad(EEG.data,...)`
- in LIMO EEG:
  - `wERP = limo_winvar(EEG.data,...)`
  - `sehdERP = limo_bootse(EEG.data,...)`

# Solving case study 1: robust estimators

t-test vs. percentile bootstrap



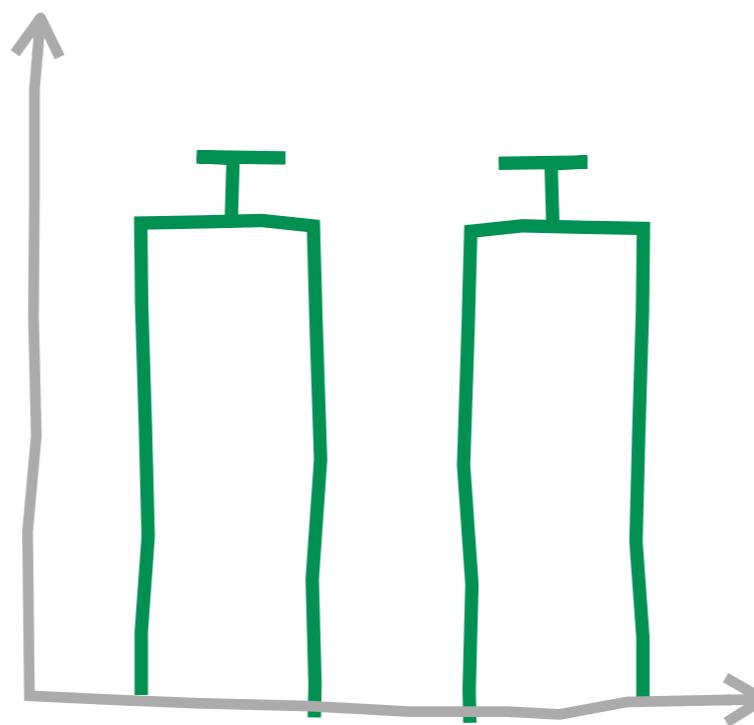
t-test on 20% trimmed means



erp\_workshop\_5\_robust\_estimators.m

# No effect? At all?

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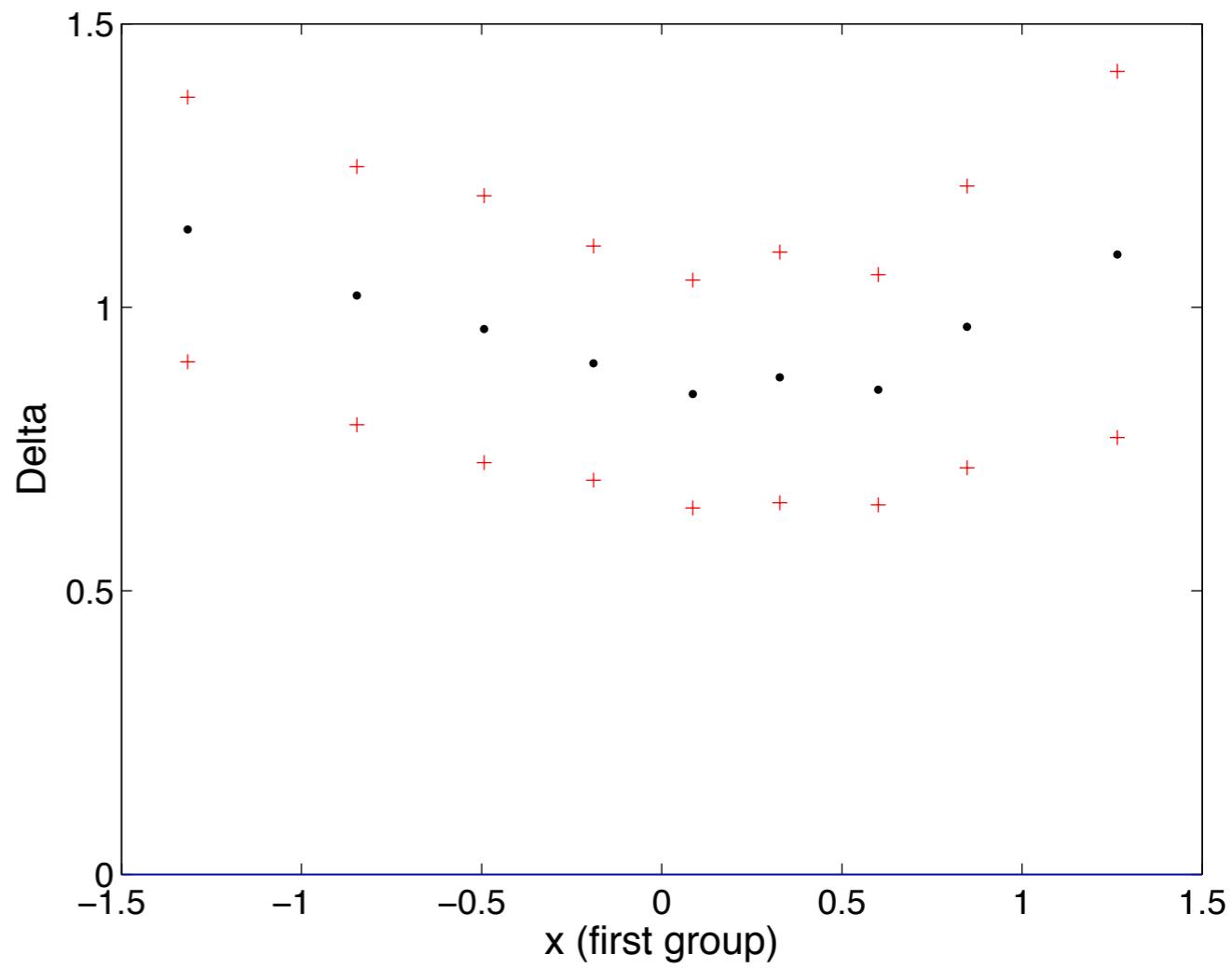
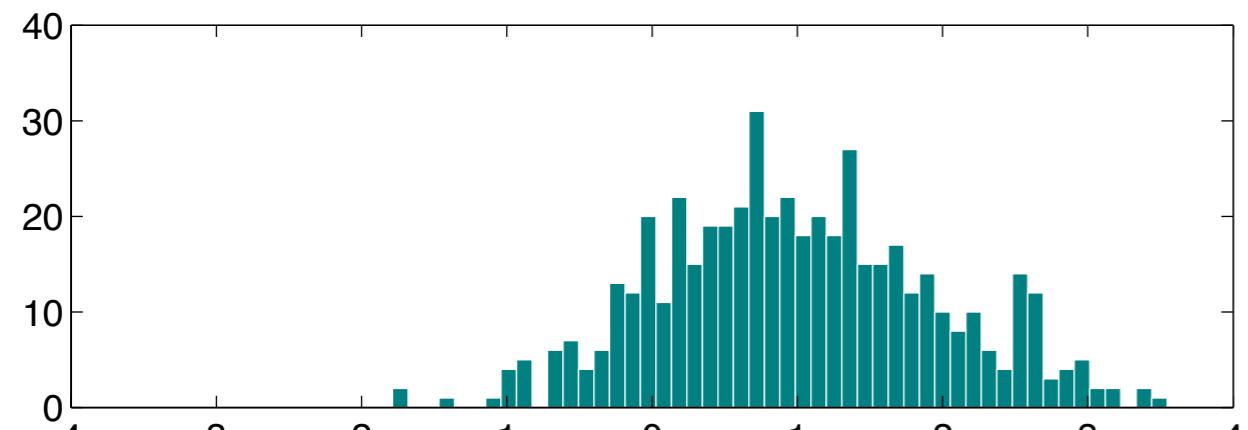
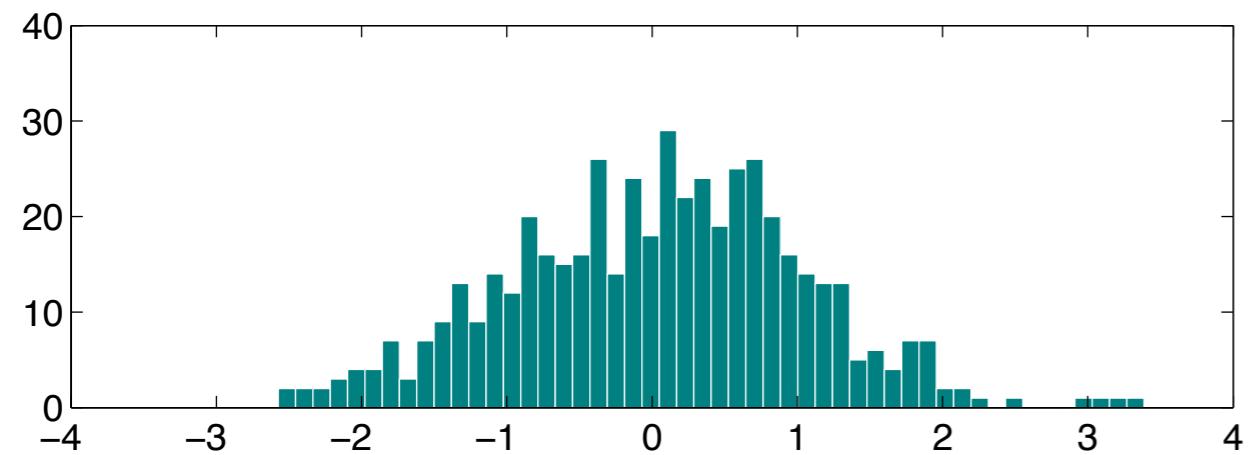


$p > 0.05$ , therefore,  
there is no evidence the two conditions differ

Don't let variability, outliers & skewness hide: ban bar graphs

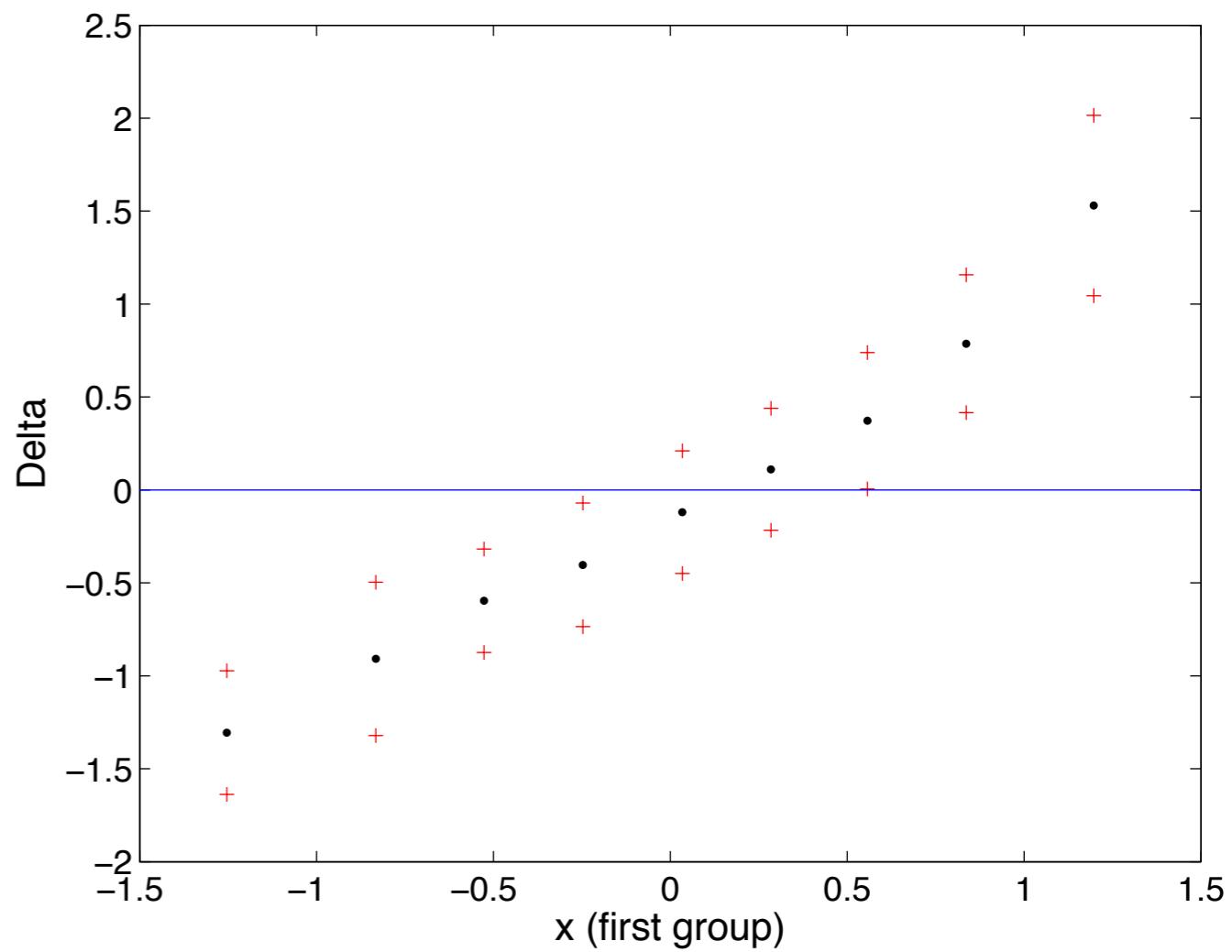
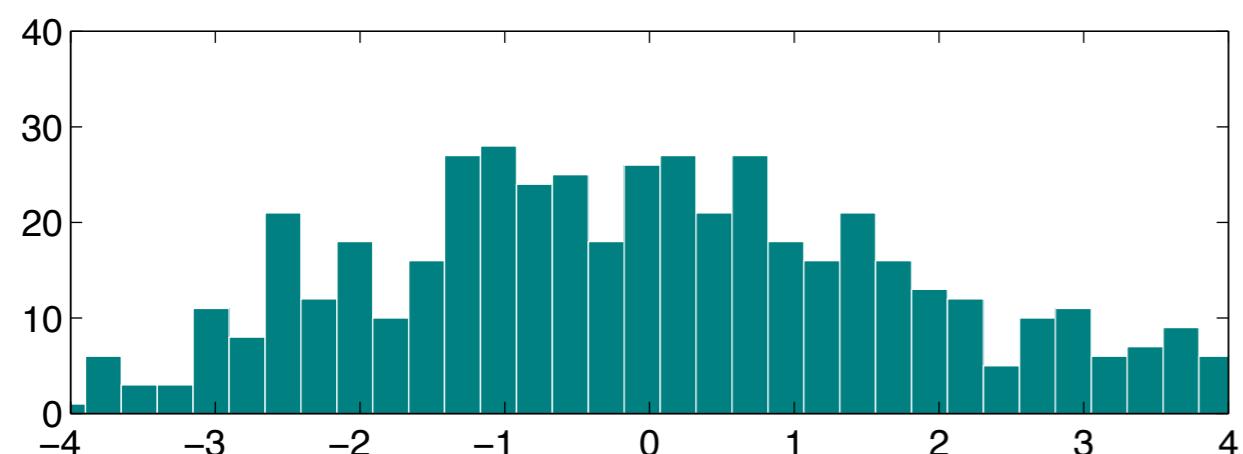
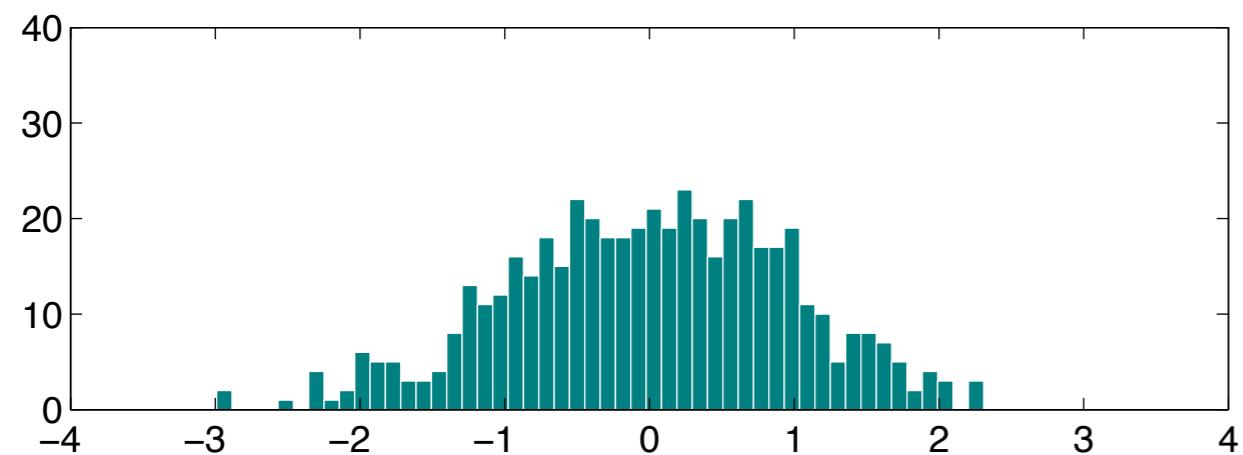


# Compare entire distributions using the **shift** function



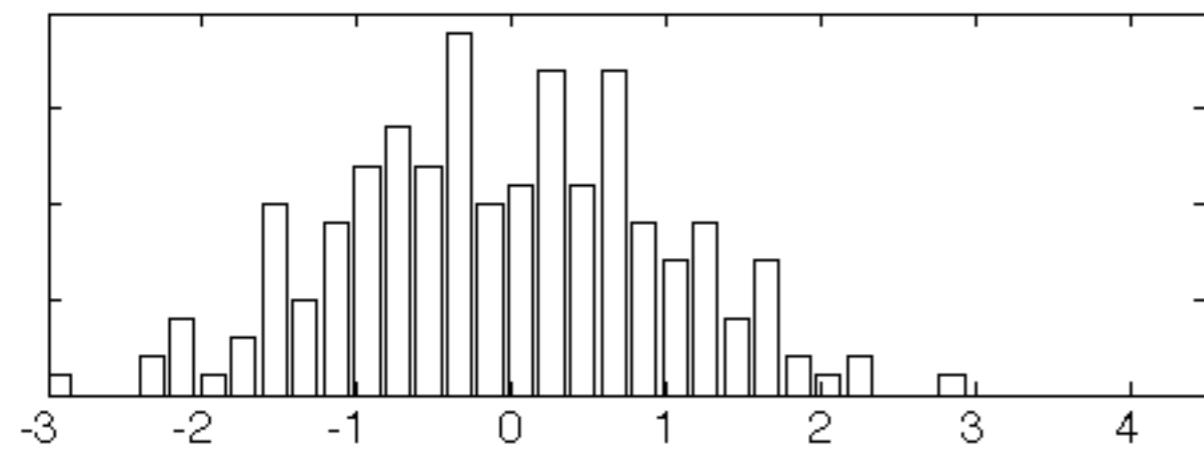
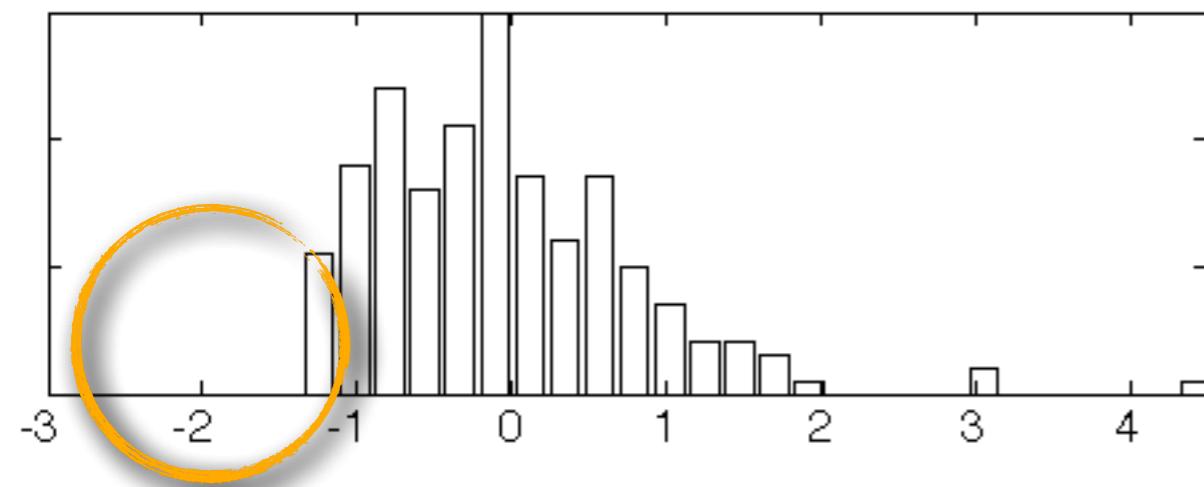
# Compare entire distributions using the **shift** function

---

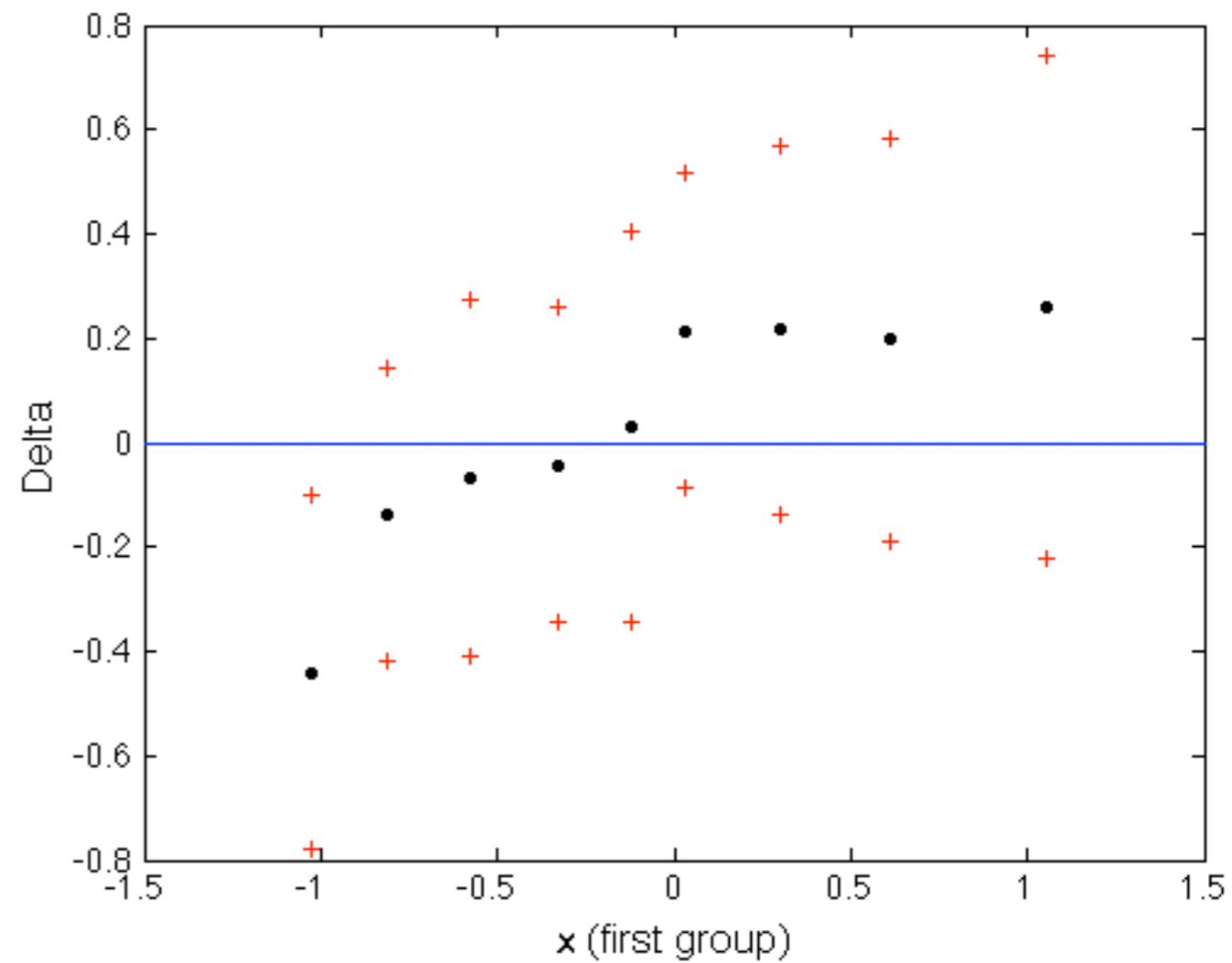


# Compare entire distributions: the **shift** function

mean=0, std=1



95% simultaneous confidence intervals  
for the difference between deciles



Doksum, K. (1974). Empirical Probability Plots and Statistical Inference for Nonlinear Models in the two-Sample Case. *Annals of Statistics*, 2(2), 267–277.

Harrell-Davis estimates of the quantiles

Application to ERP data:

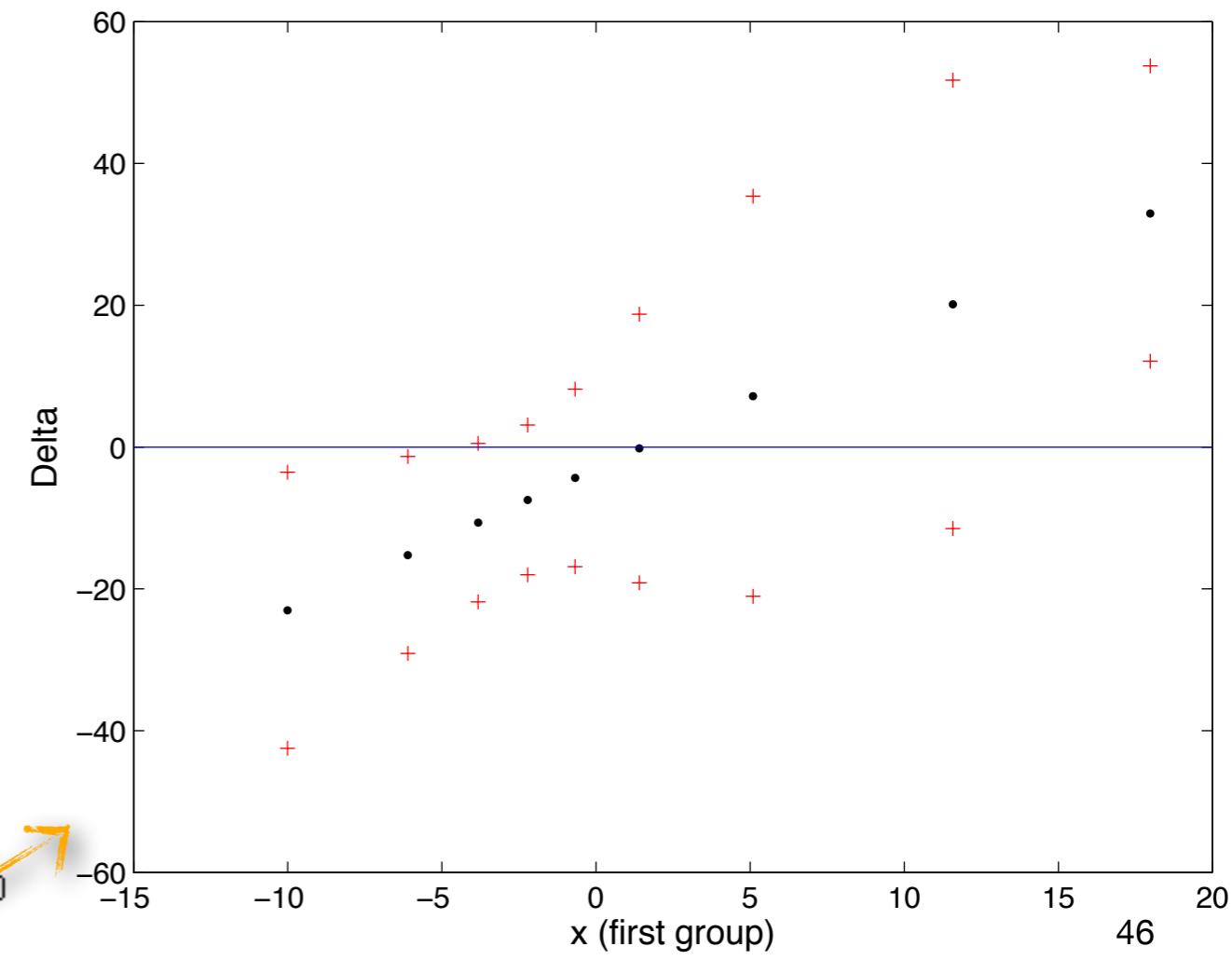
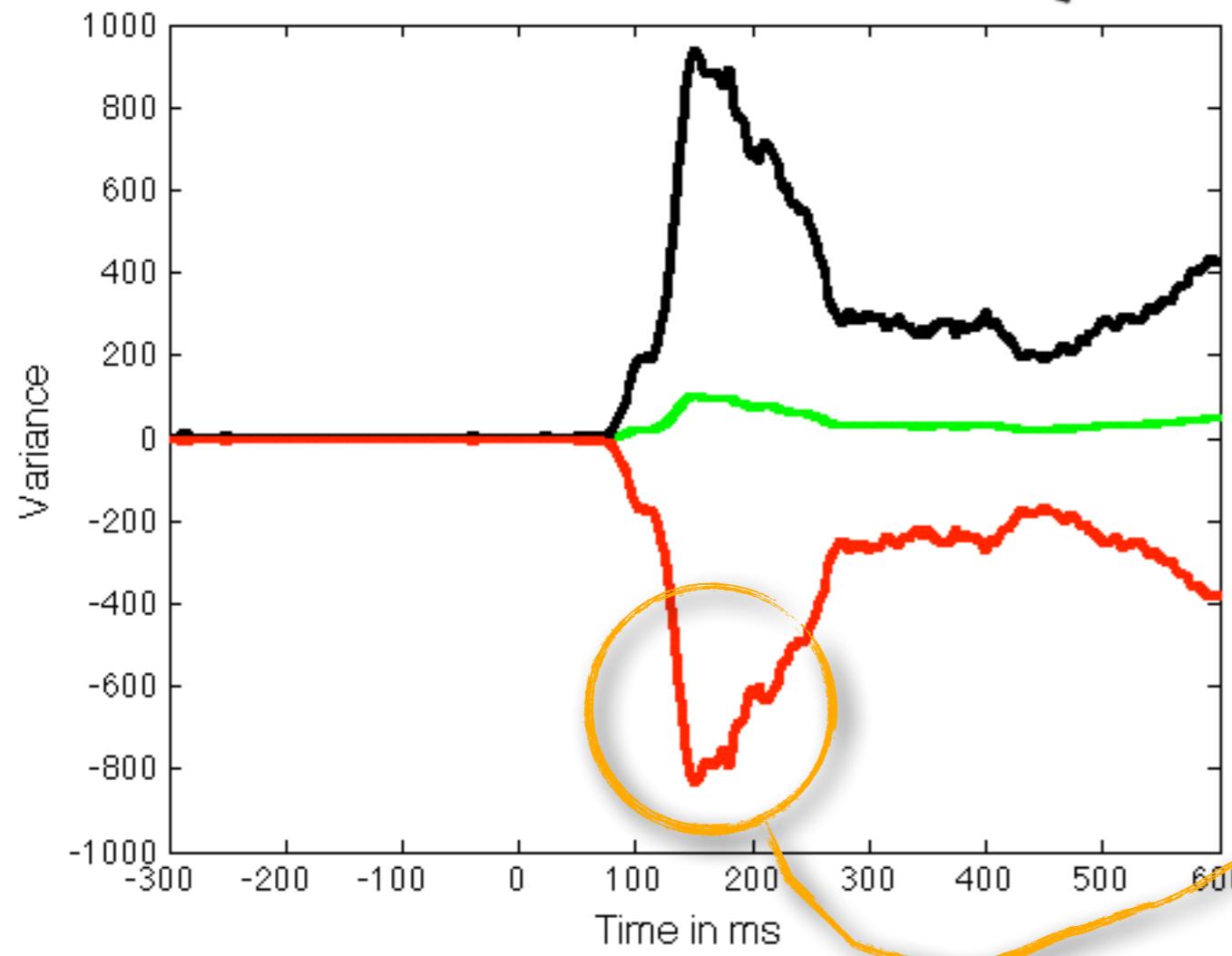
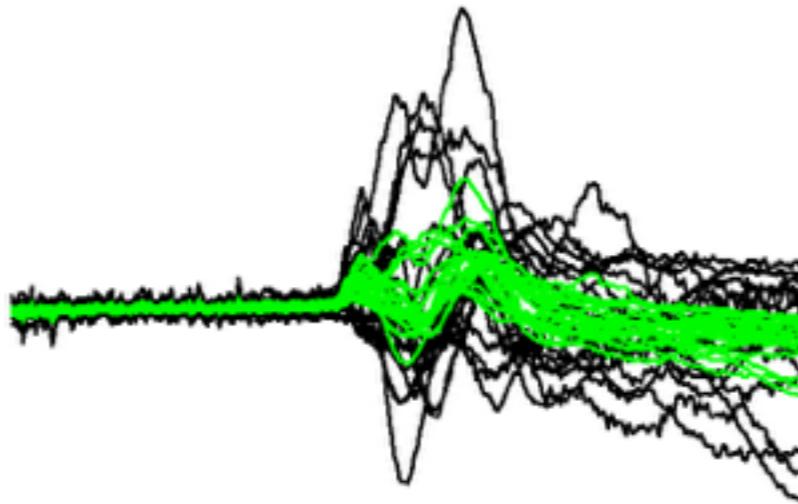
Rousselet GA, Gaspar CM, Wieczorek KP and Pernet CR (2011)

Modelling single-trial ERP reveals modulation of bottom-up face visual processing by top-down task constraints (in some subjects)

Frontiers in Psychology 2:137. doi: 10.3389/fpsyg.2011.00137

# Solving case study 2: shift function

---

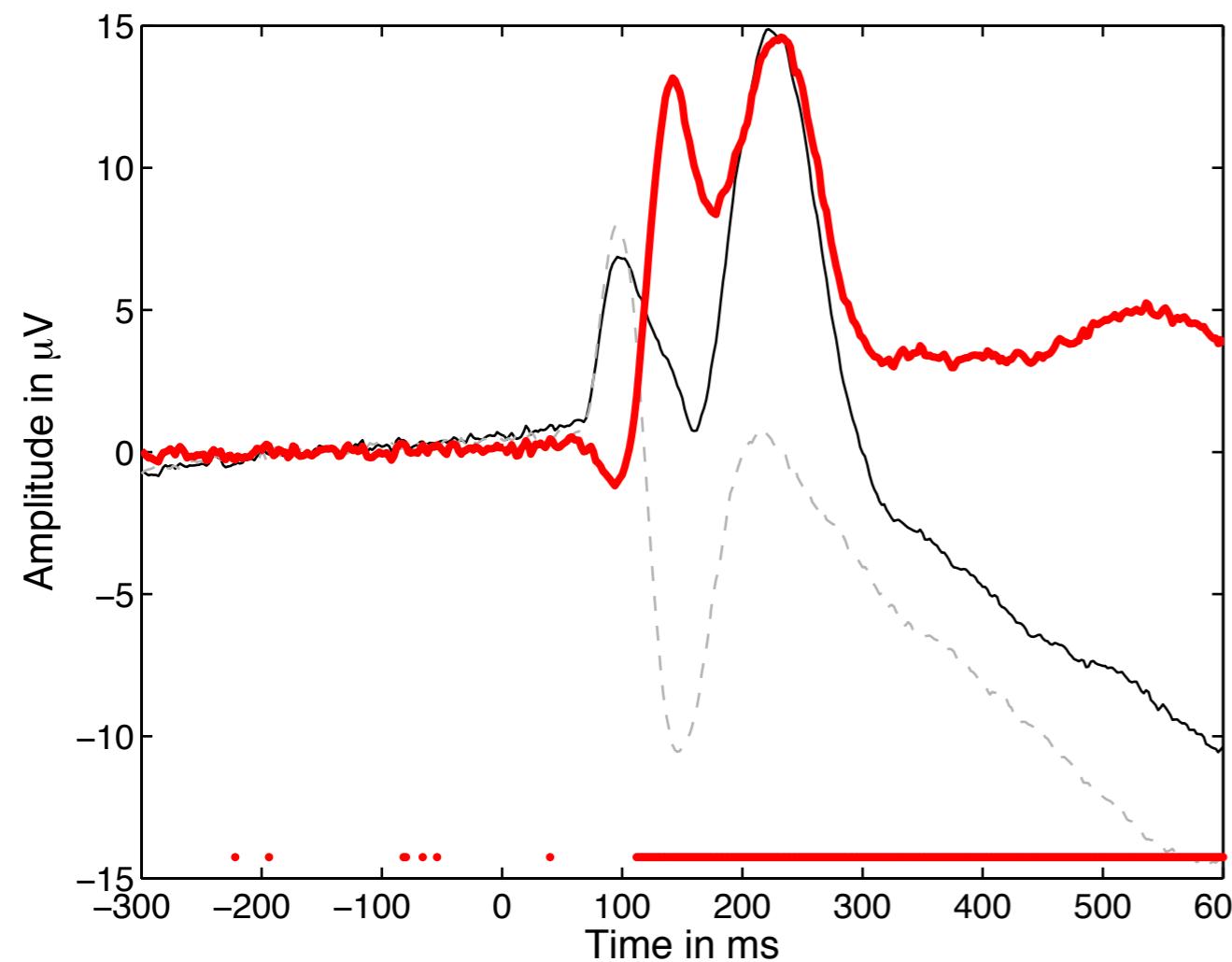


erp\_workshop\_6\_shift\_function.m

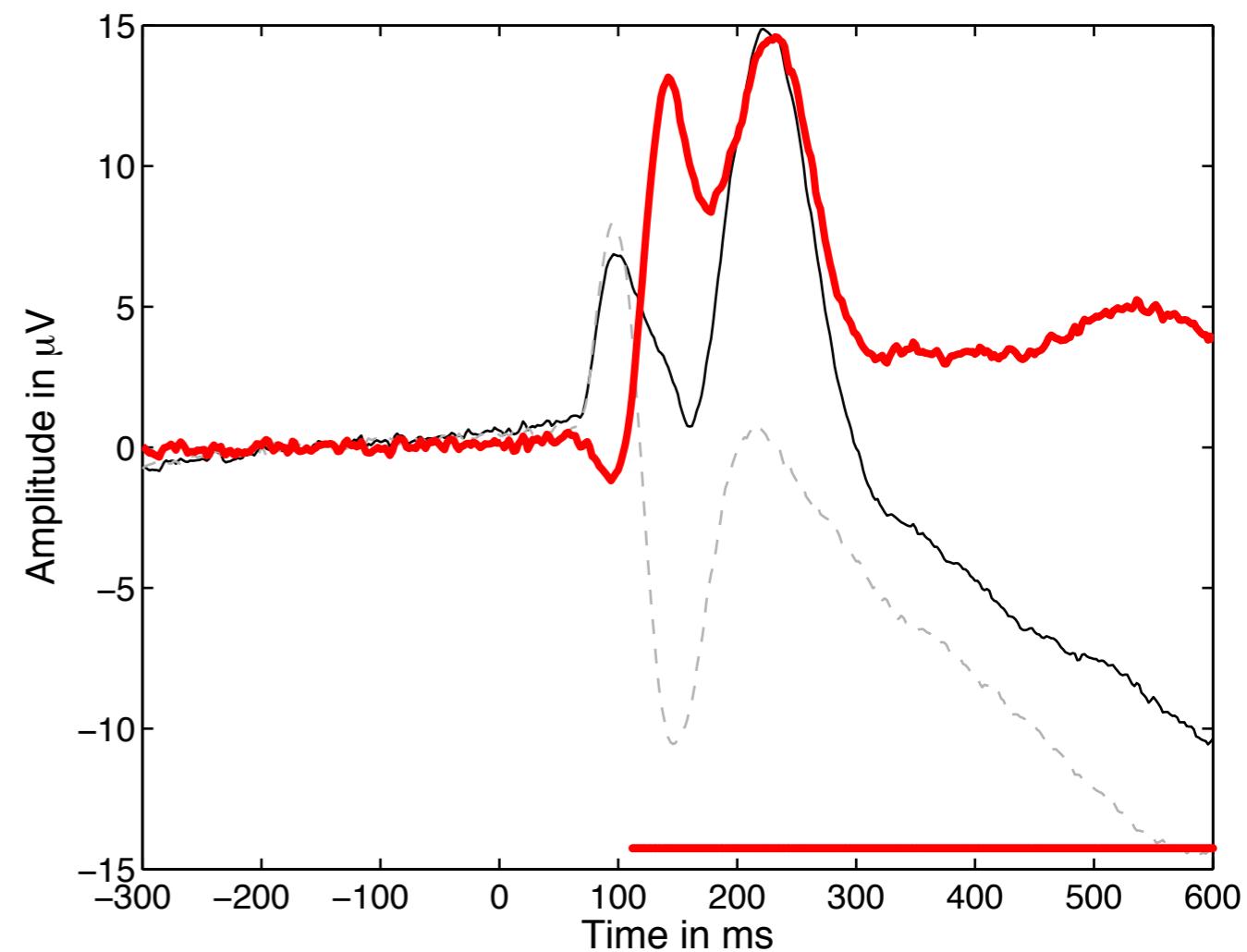
# Control for multiple comparisons: bootstrap-t technique & spatial-temporal clustering

---

univariate thresholds

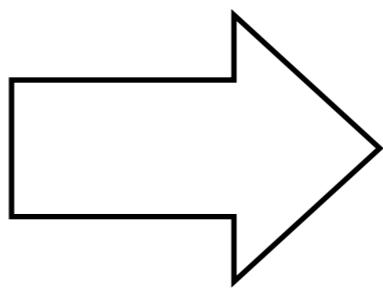
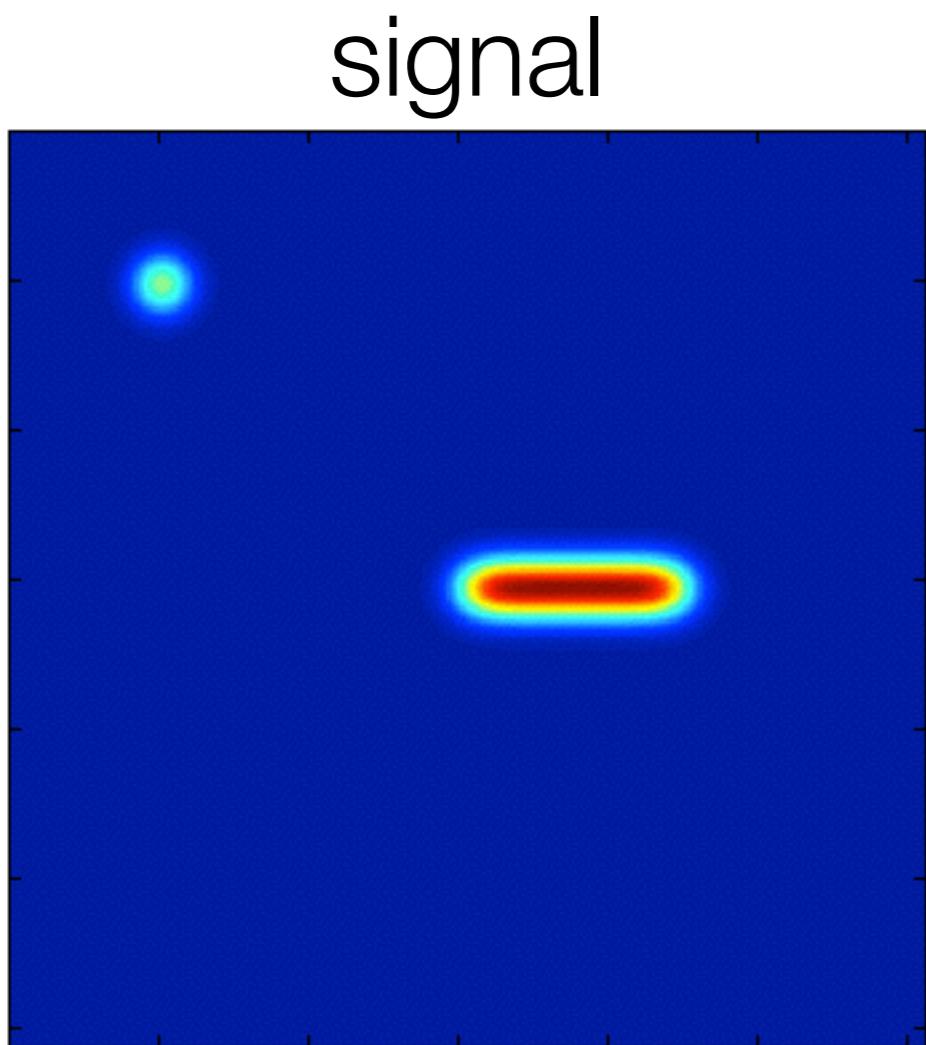


after correction

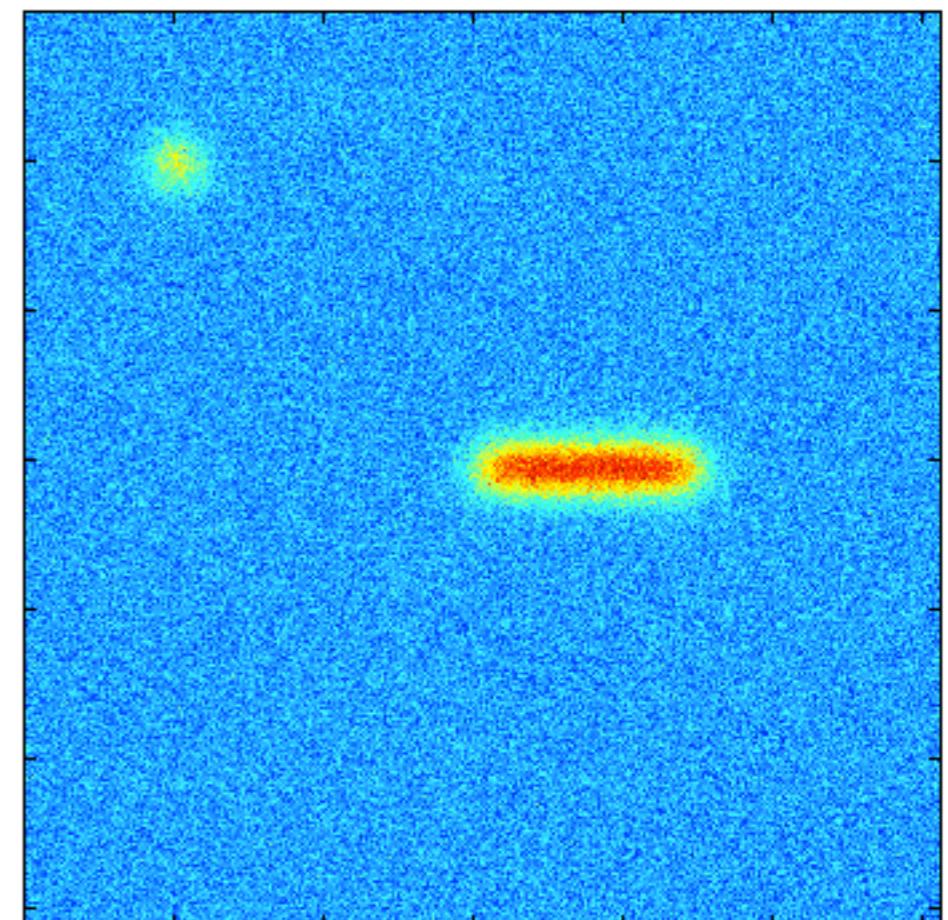


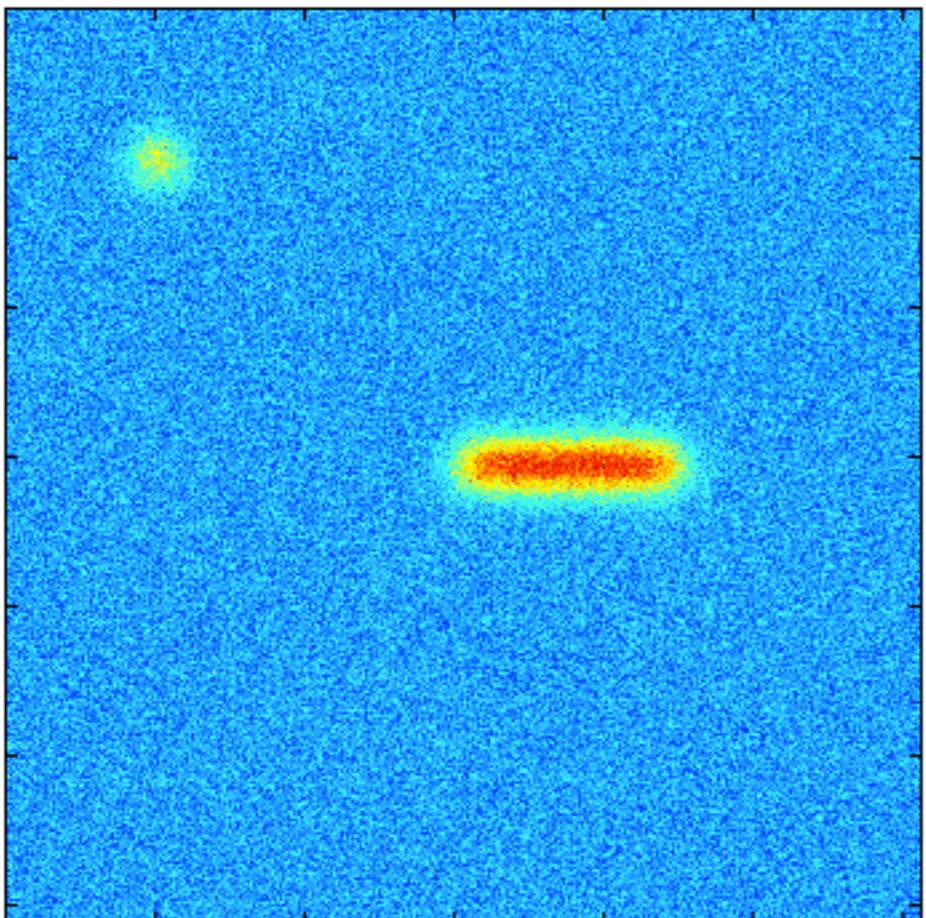
# Control for multiple comparisons

---

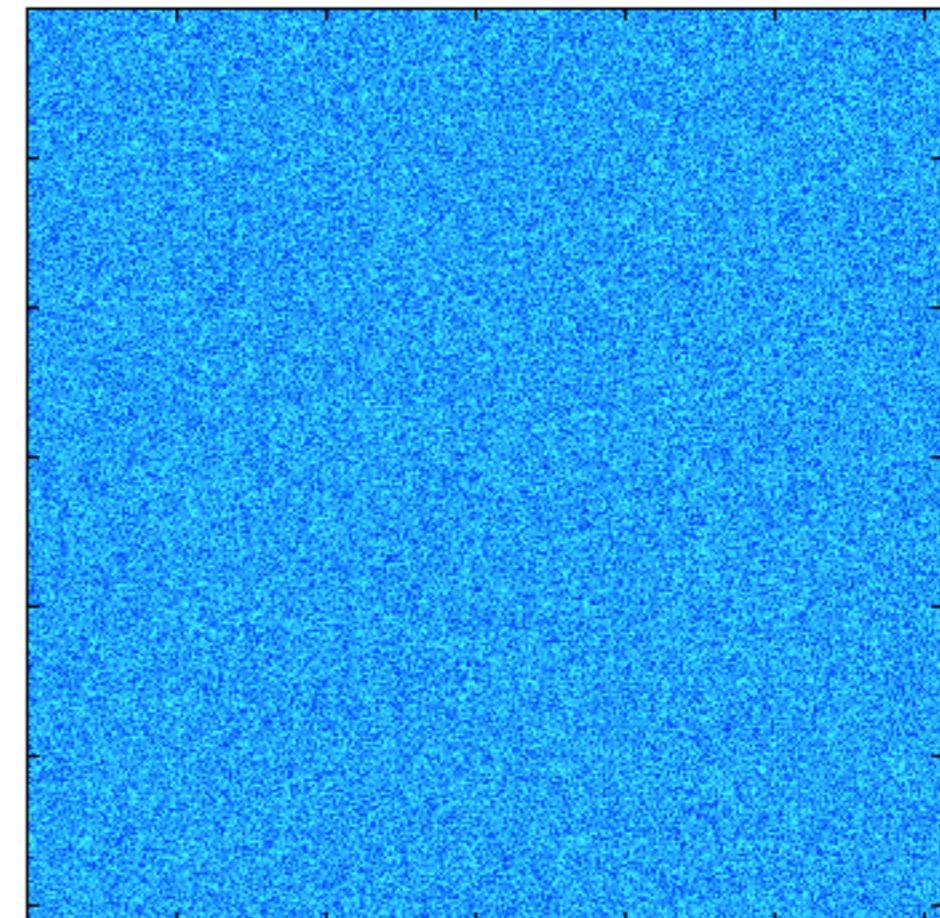
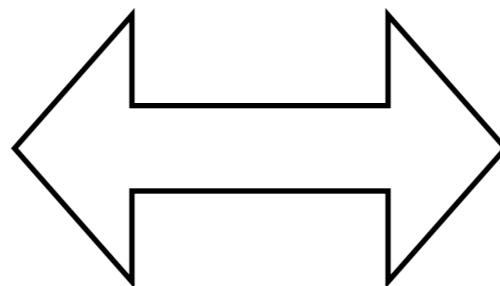


100 trials =  
signal +  
white noise





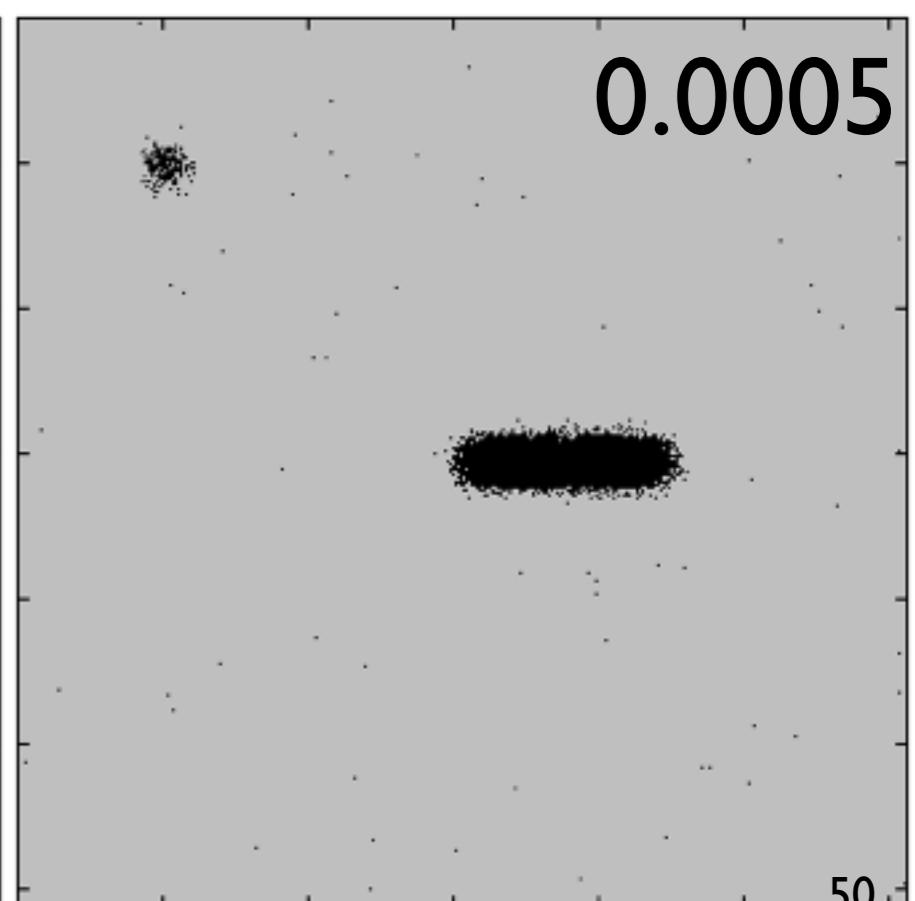
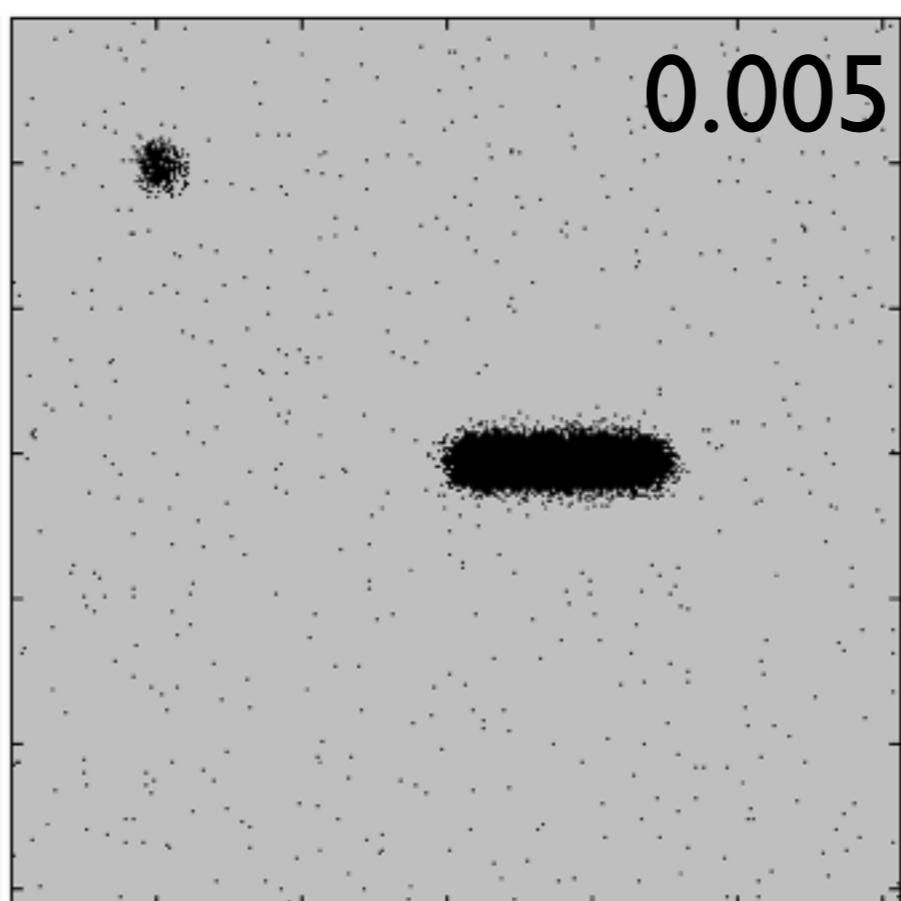
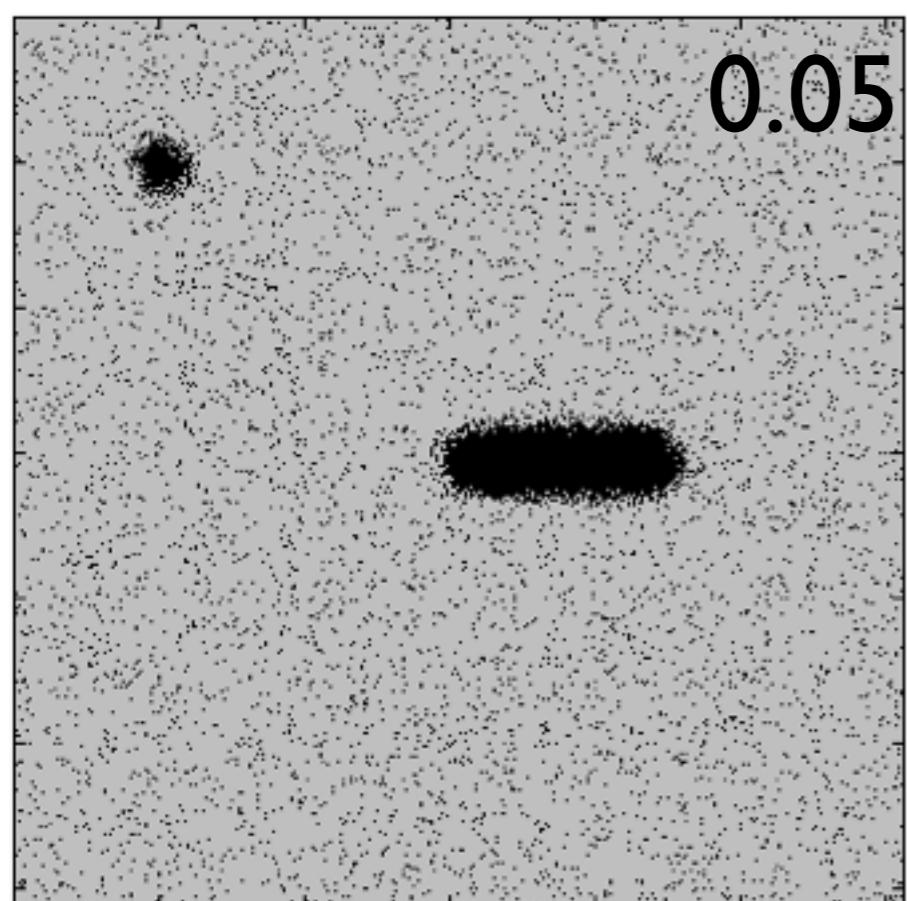
**t-test**



**0.05**

**0.005**

**0.0005**



# max procedure

---

- in each bootstrap loop, take the MAX of the absolute value of your estimator (e.g. mean difference) across all pairwise comparisons.
- then: `maxCI = diffsort(:,U);`
- with `U = round(b.*(1-alpha));`
- compare absolute original difference to this distribution

# False Discovery Rate procedure

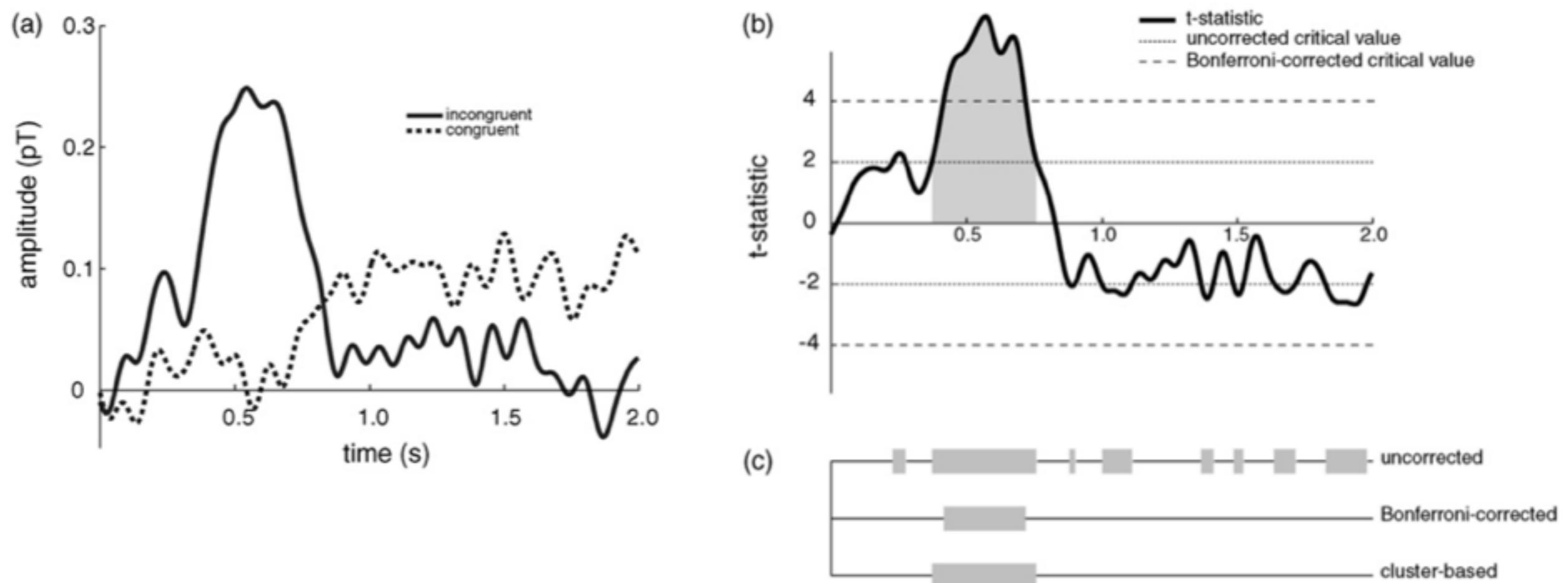
## Procedure:

- Sort all p values (column C1)
- C3
- Create column C2 by computing  $j^*a/N$
- Subtract column C1 from C2 to build column C3
- Find the highest negative index in C3 and find the corresponding p-value in C1 ( $p_{fdr}$ )
- Reject all null hypothesis whose p-value are less than or equal to  $p_{fdr}$

	C1	C2	C3
Index "j"	Actual	$j^*0.05/10$	$C2-C1$
1	0.001	0.005	-0.004
2	0.002	0.01	-0.008
3	0.01	0.015	-0.005
4	0.03	0.02	0.01
5	0.04	0.025	0.015
6	0.045	0.03	0.015
7	0.05	0.035	0.015
8	0.1	0.04	0.06
9	0.2	0.045	0.155
10	0.6	0.05	0.55

<http://www-personal.umich.edu/~nichols/FDR/>

# Control for multiple comparisons

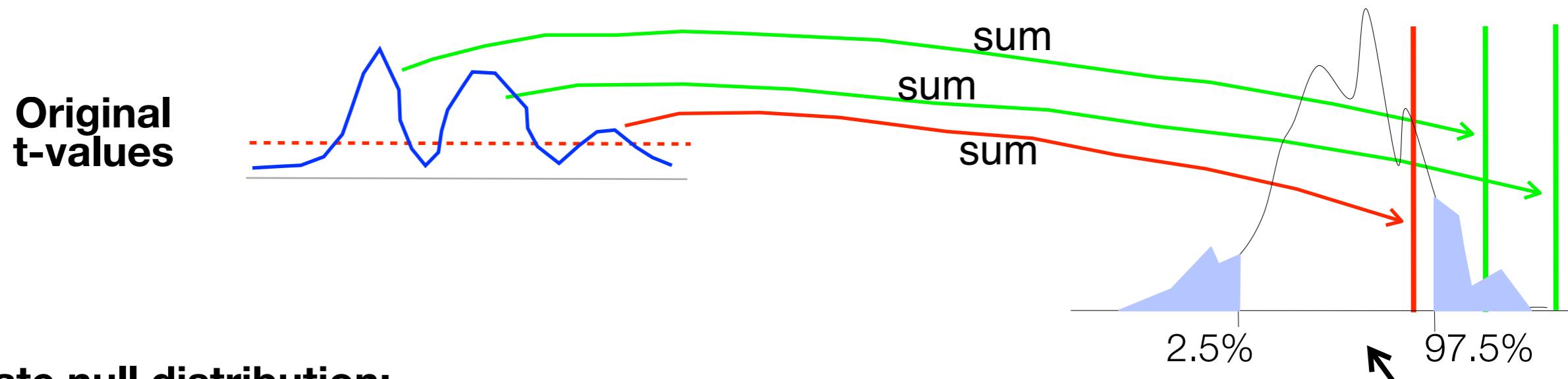


Maris & Oostenveld, J. Neurosci. Methods 2007

Matlab toolboxes: Fieldtrip + LIMO\_EEG

Pernet, Latinus, Nichols & Rousselet (2015) Cluster-based computational methods for mass univariate analyses of event-related brain potentials/fields: A simulation study. Journal of neuroscience methods, 250, 85-93.

# Cluster correction for multiple comparisons

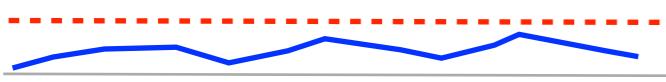


**estimate null distribution:**

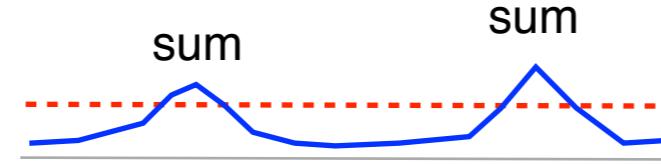
- permutation of trials across two conditions
- sampling with replacement of mean centred distributions

max  
x 1000

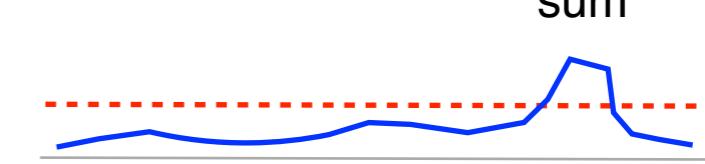
bootstrap difference 1



bootstrap difference 2

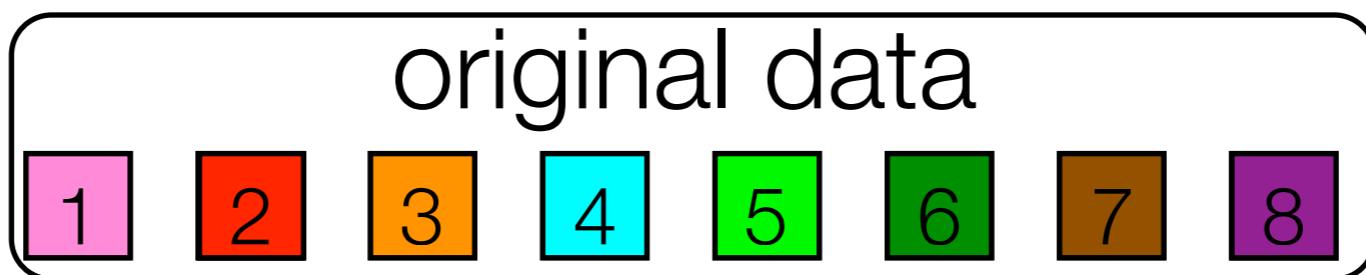


bootstrap difference 3...



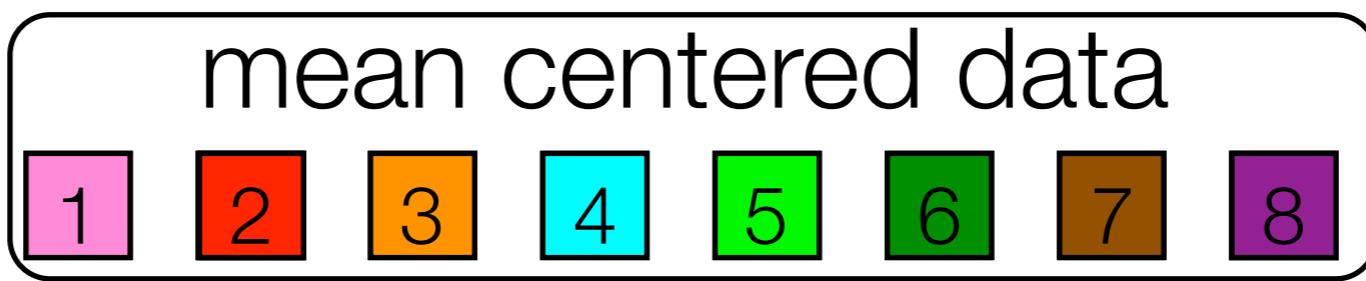
# bootstrap-t method (percentile-t technique)

(1) t-test



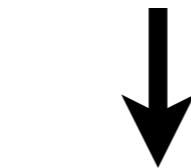
-> t value

(2) center data



- mean([S1...S8])

(3) sample WITH  
replacement n  
centered  
observations



**t\* value**

(4) t-test

(5) repeat (3) & (4) b times

$t^*_1 \ t^*_2 \ t^*_3 \ t^*_4 \ t^*_5 \dots \ t^*_b$

(6) sort the b estimates\*

(7) get 1-alpha confidence interval

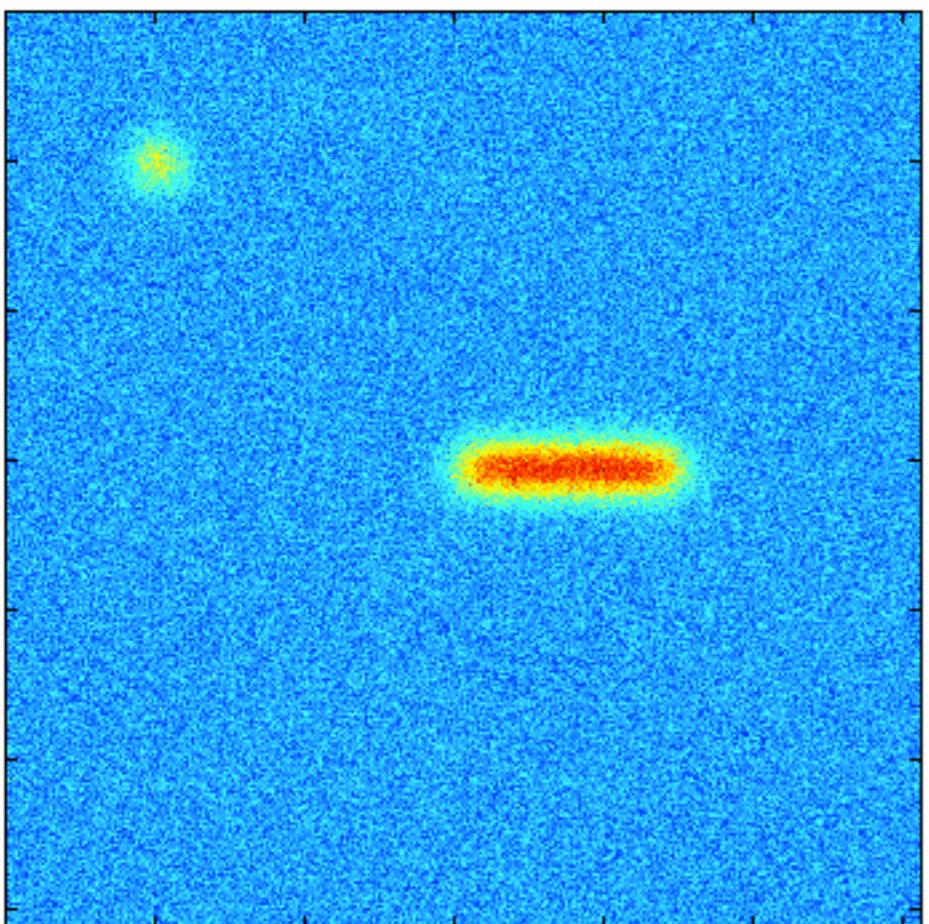
(8) compare t to t\* threshold

$$\left( \bar{X} - T_{(u)}^* \frac{s}{\sqrt{n}}, \bar{X} - T_{(l)}^* \frac{s}{\sqrt{n}} \right)$$

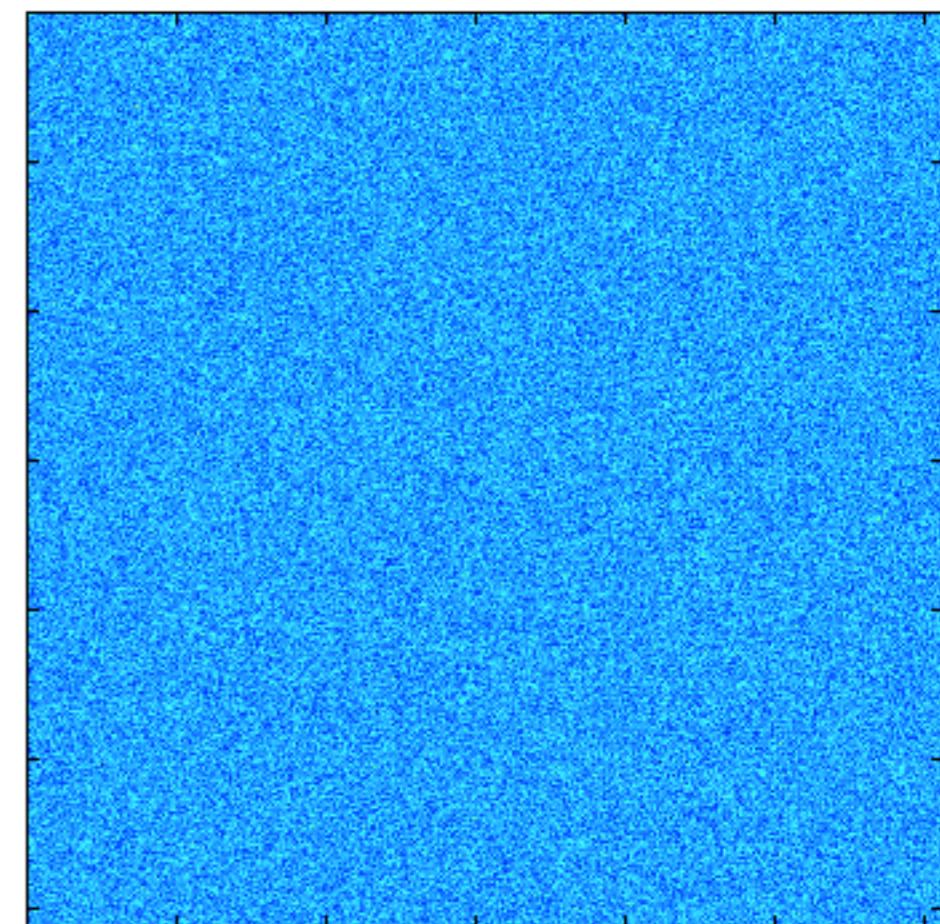
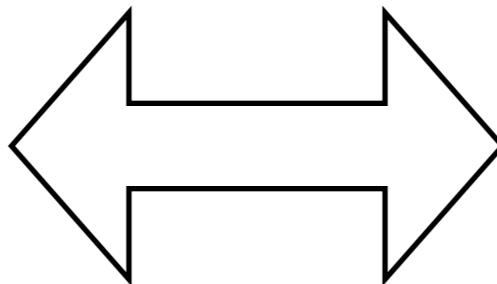
# Which bootstrap method to use?

---

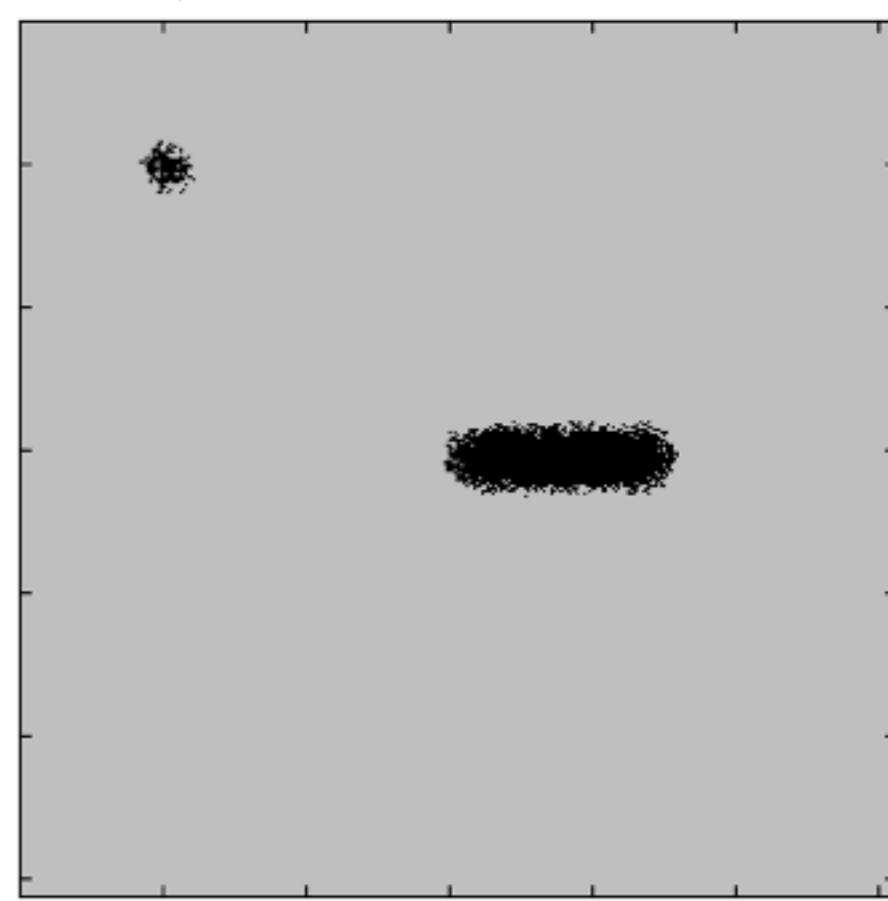
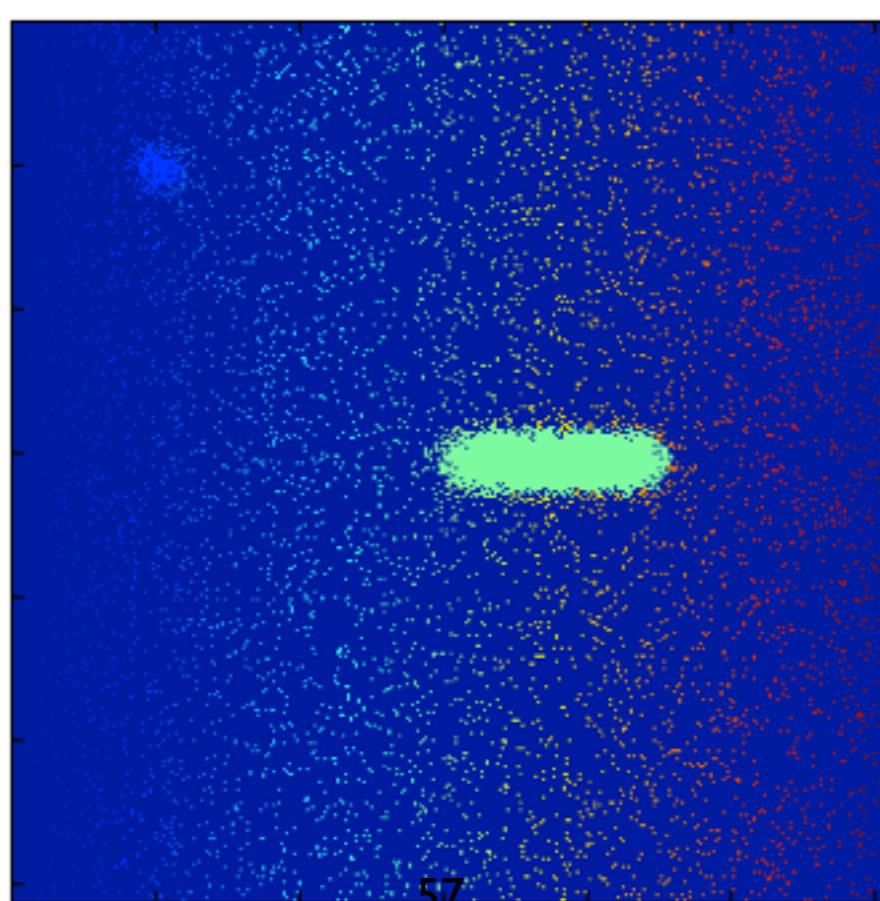
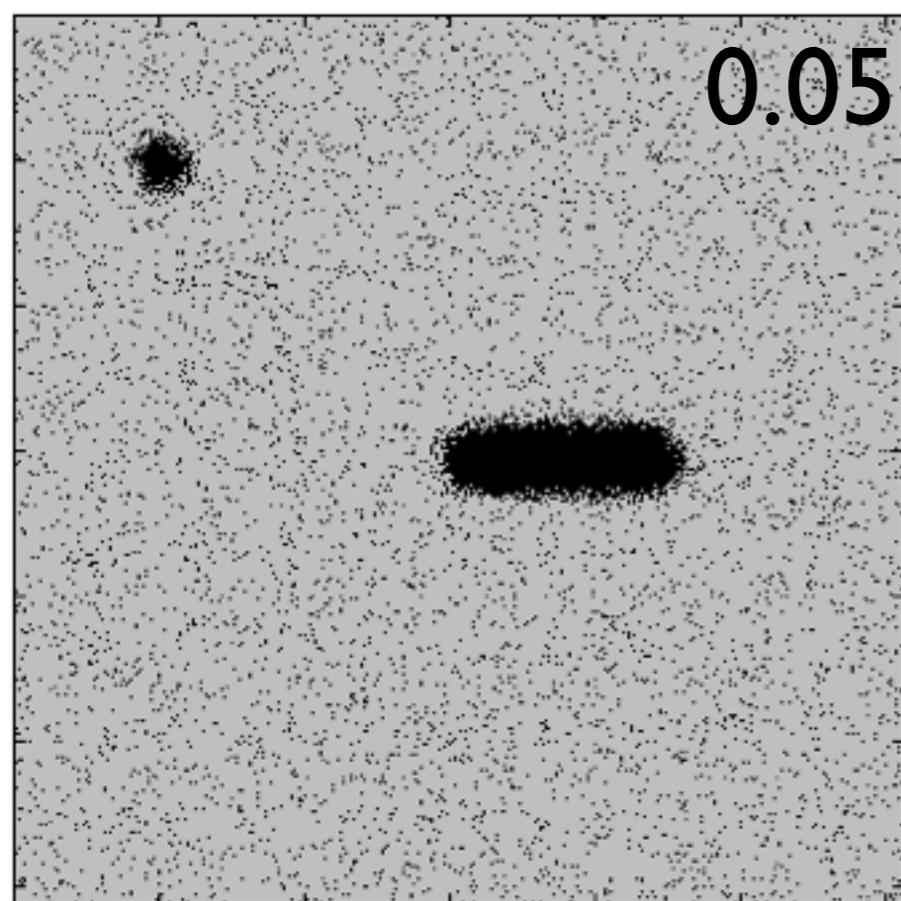
- How do we build good confidence intervals?
  - choose estimator
  - choose technique
- Rand Wilcox recommends:
  - trimming  $\geq 20\%$ : percentile technique
  - trimming  $< 20\%$ : percentile-t technique
  - M-estimators: percentile technique



t-test

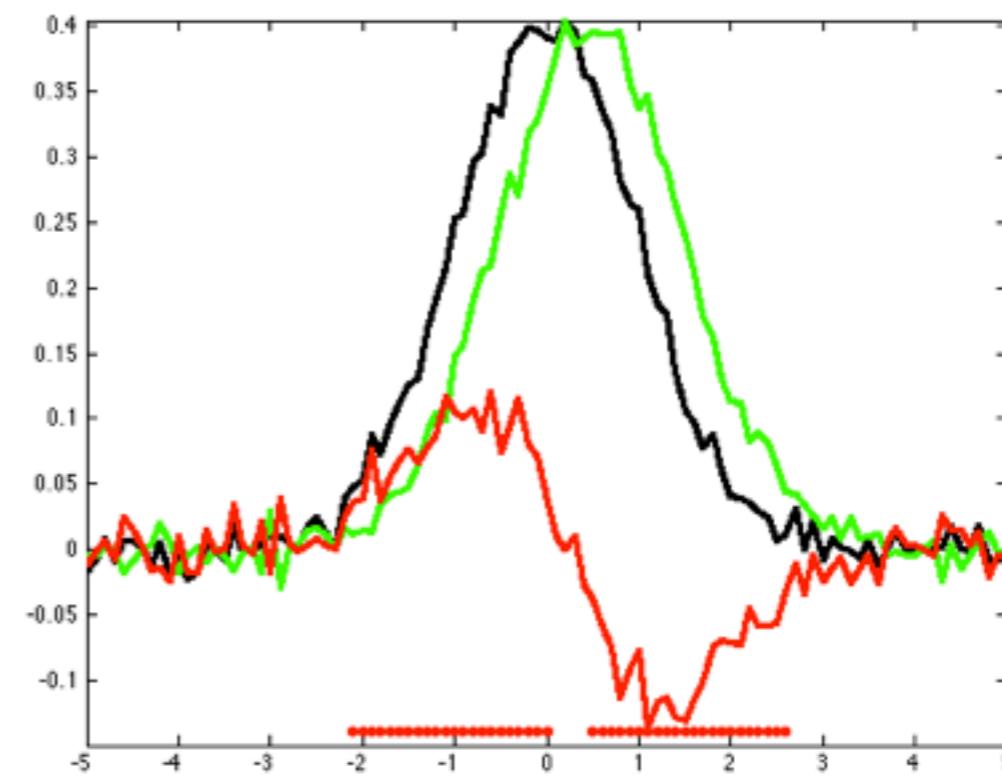
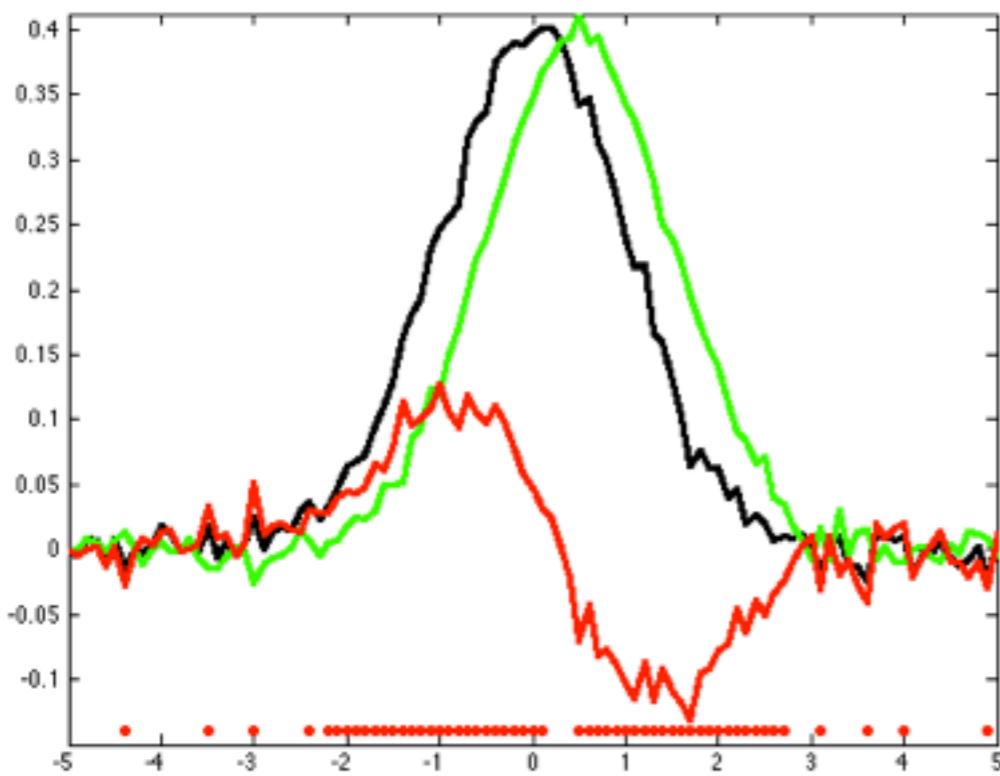
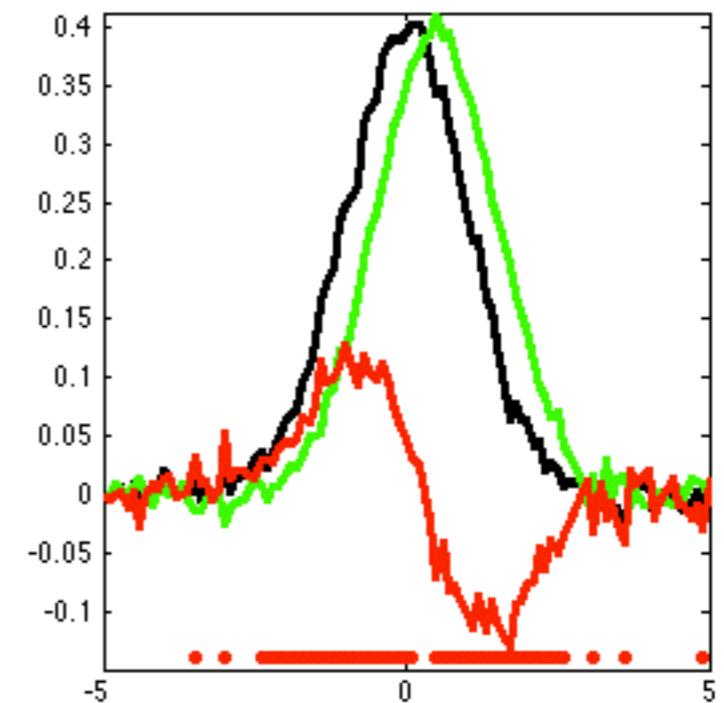
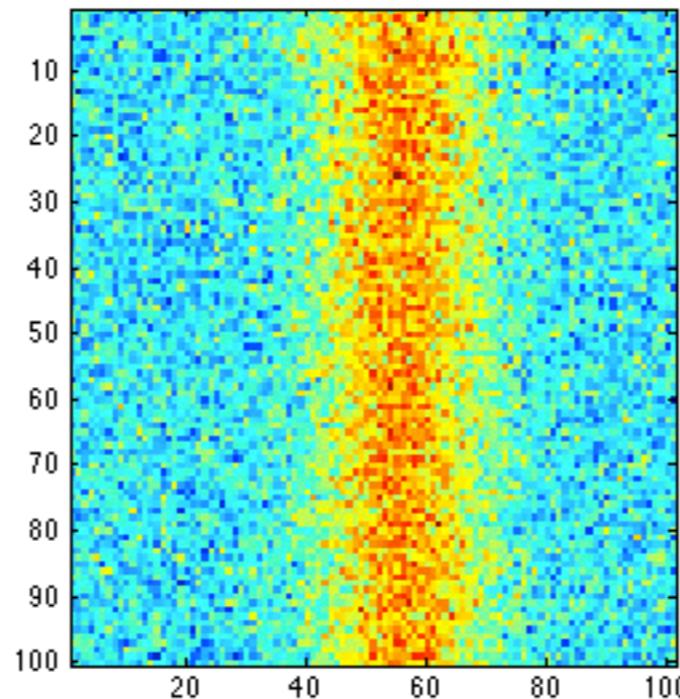
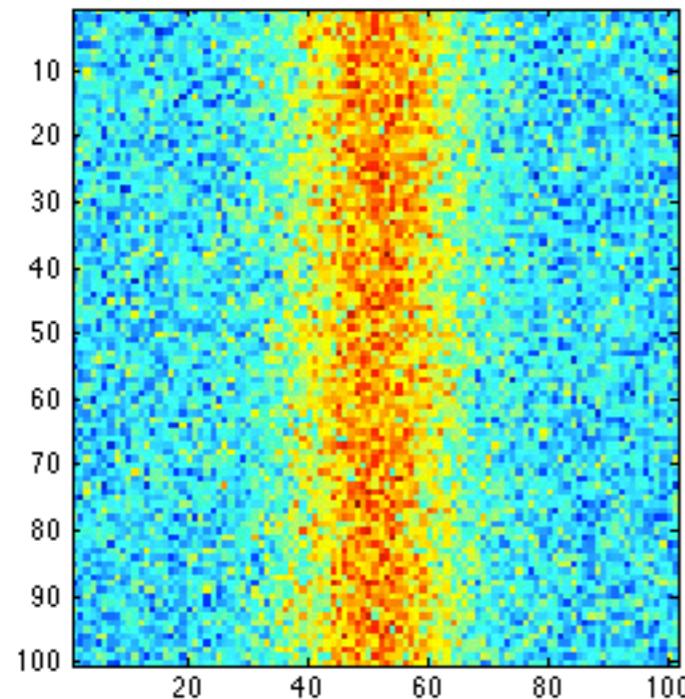


17044 clusters  $\xrightarrow{\hspace{1cm}}$  2 clusters



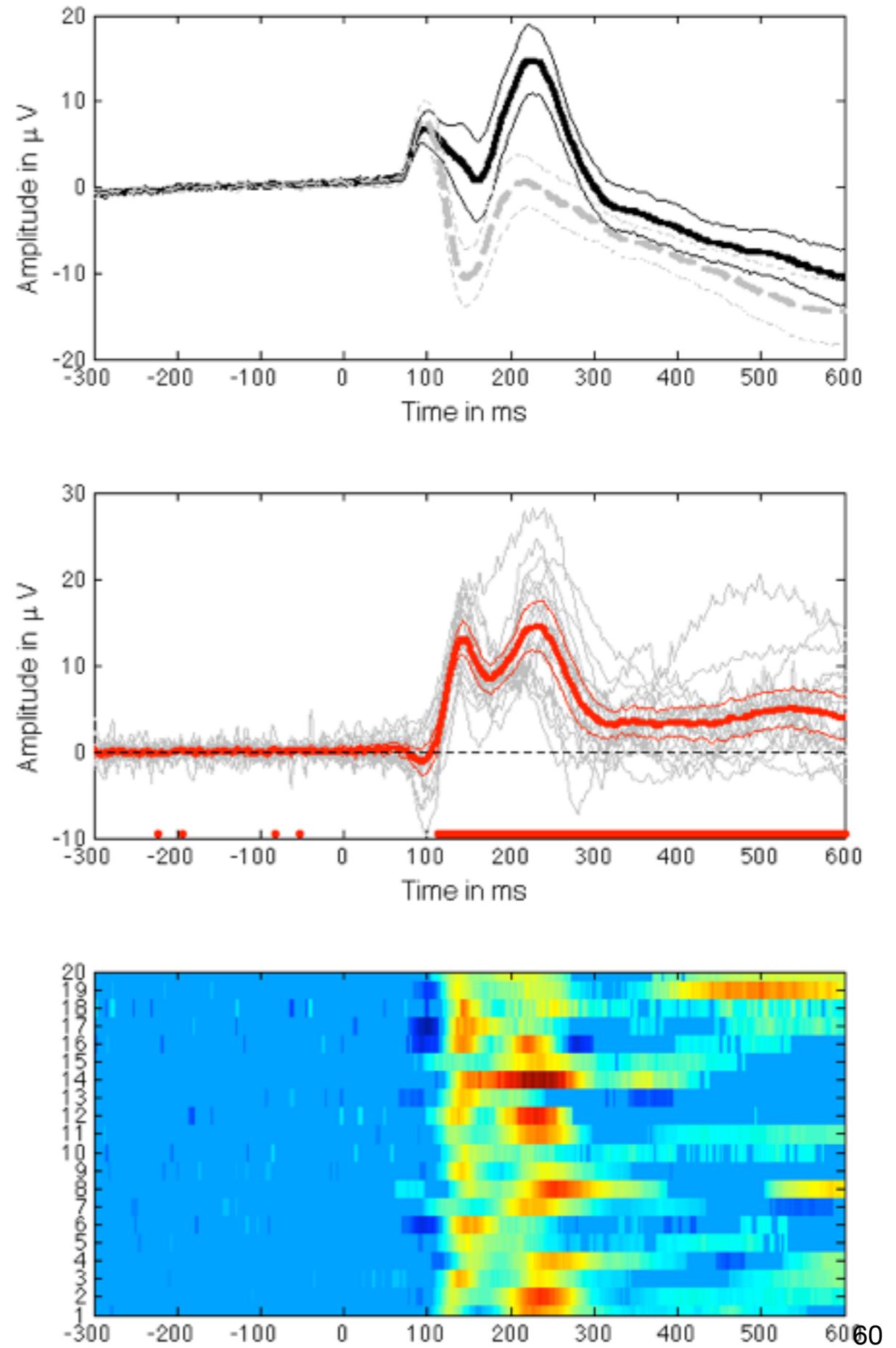
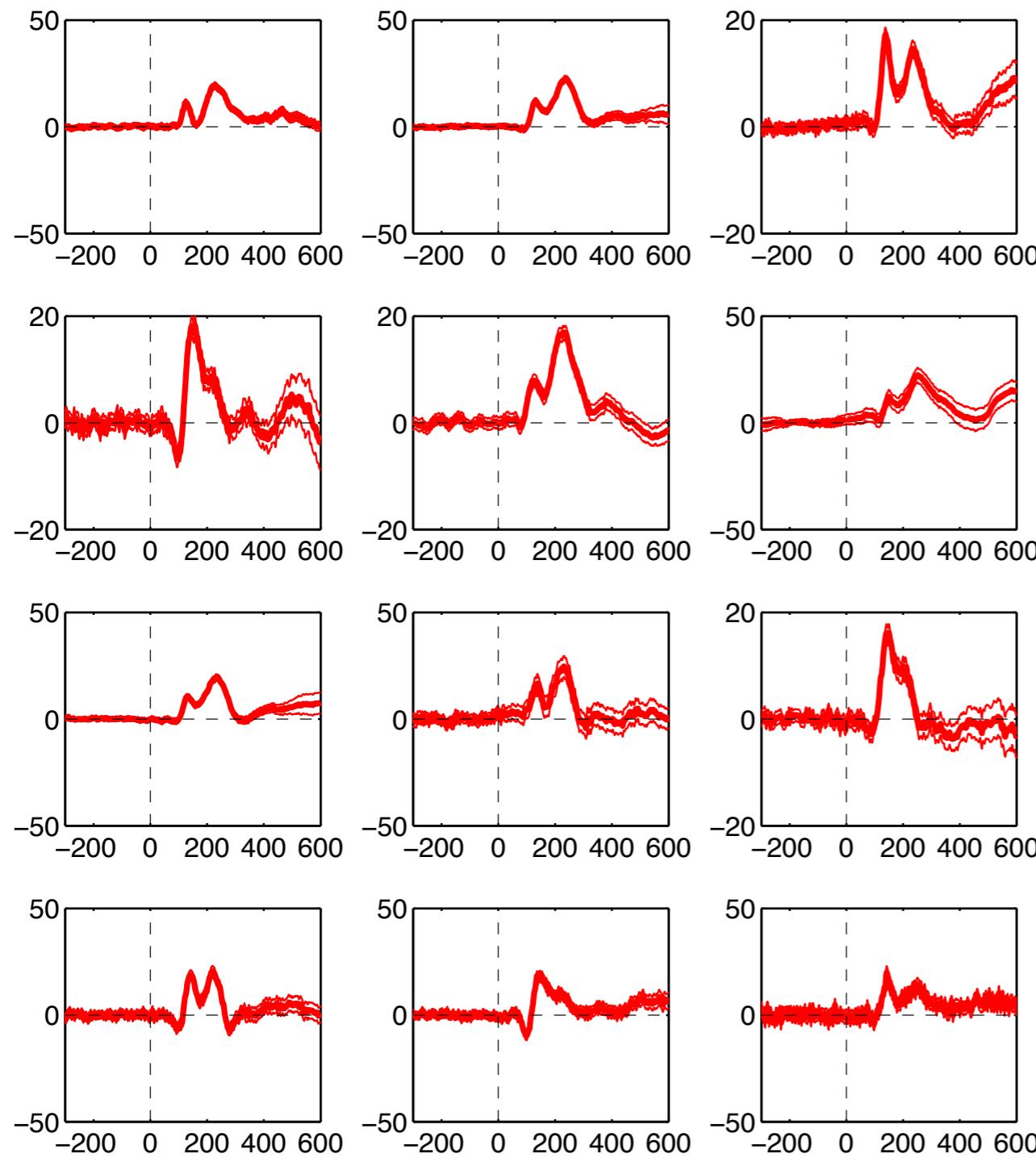
# 1D cluster test

# cmc\_example.m



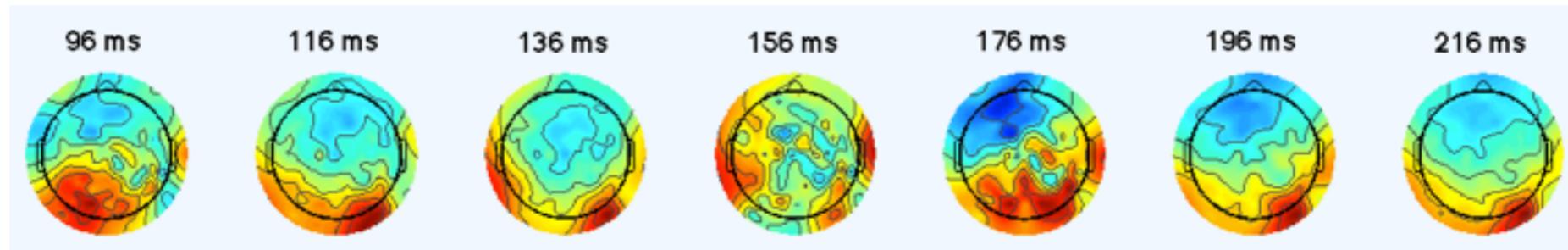
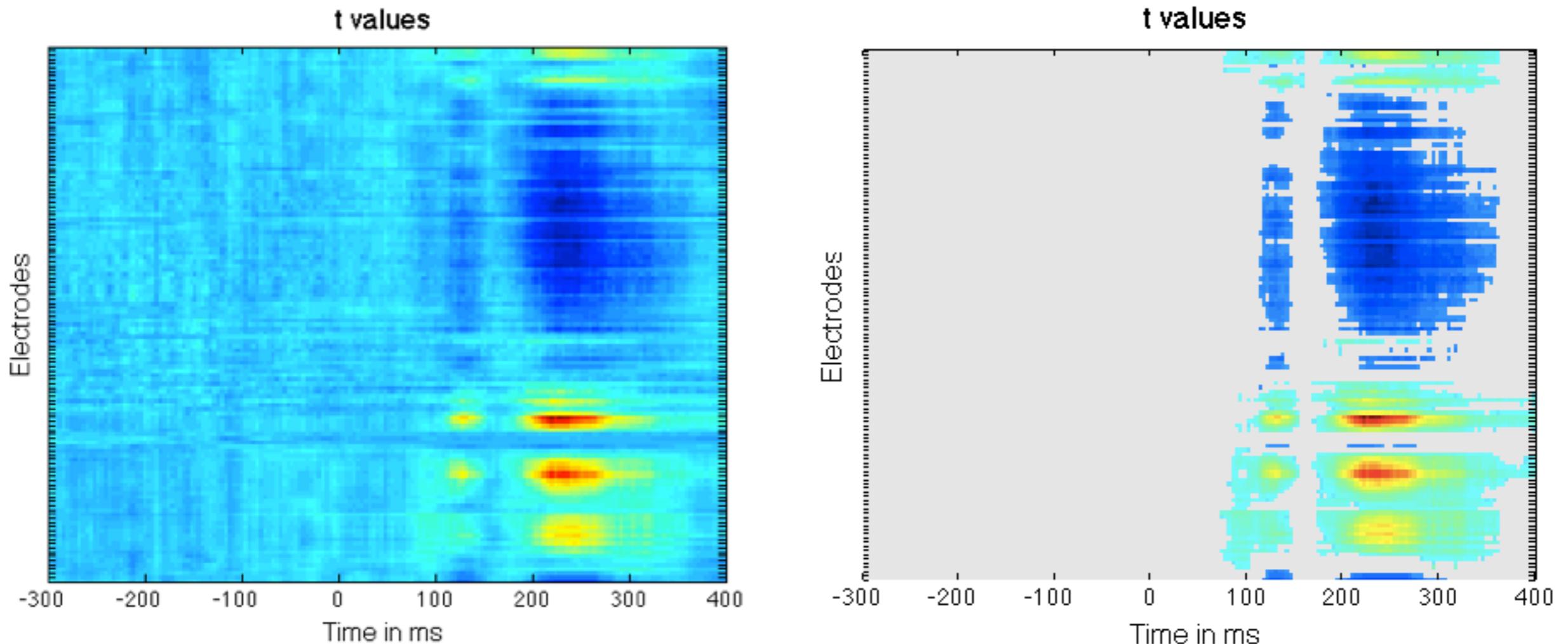
erp\_workshop\_7\_cmc\_group.m

# Single-subject analyses



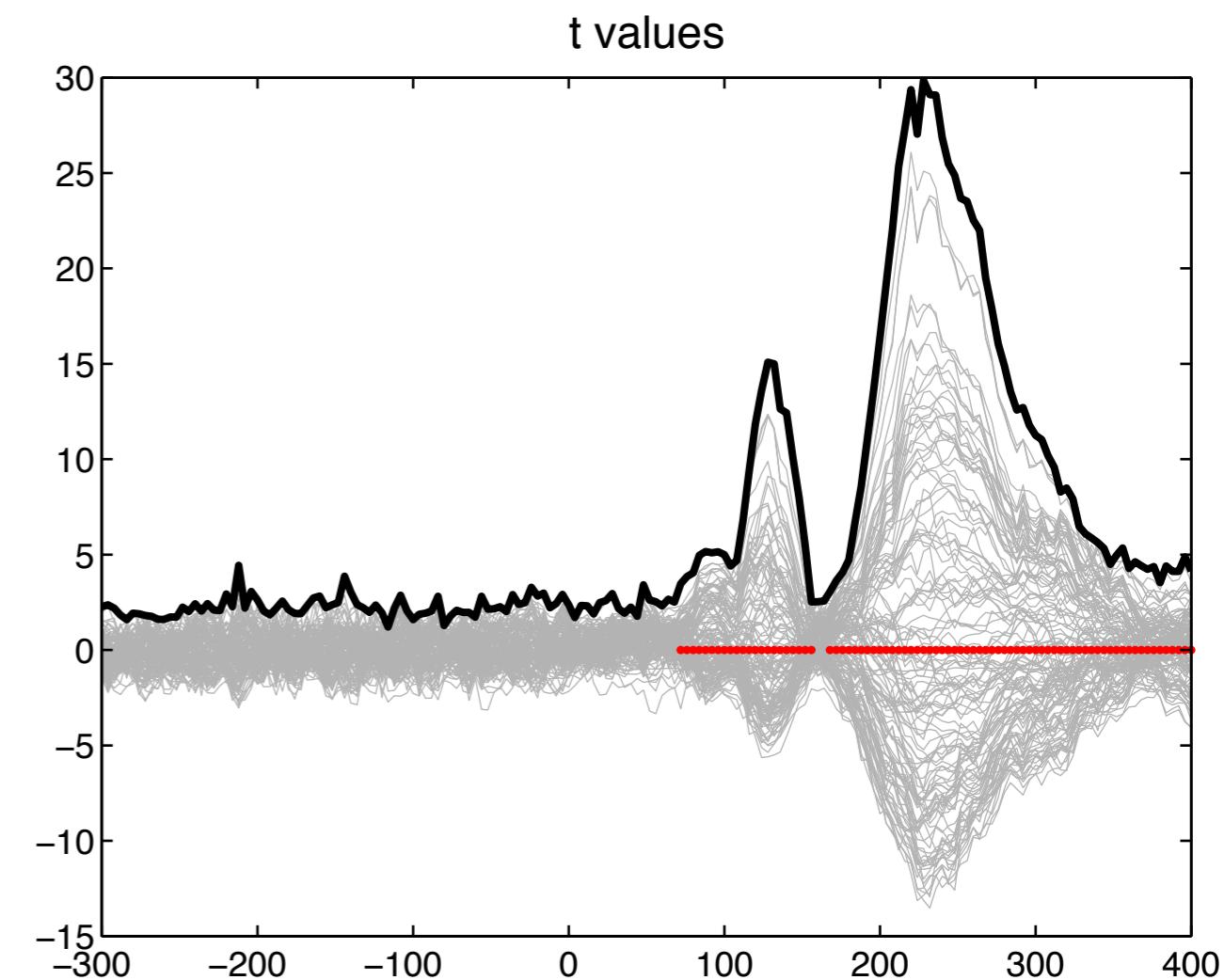
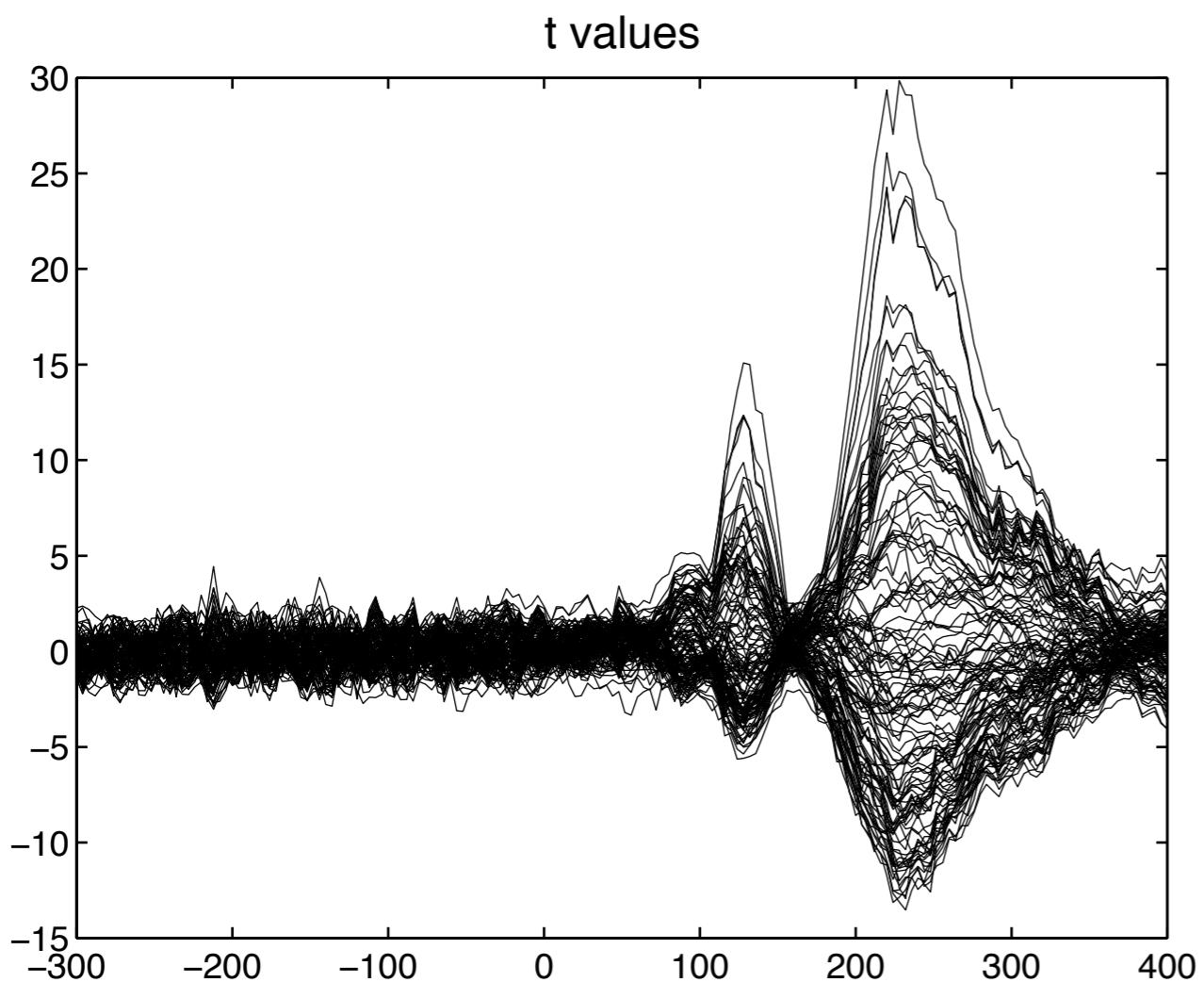
erp\_workshop\_8\_single.m

# Single-subject analyses: all electrodes



# Single-subject analyses: all electrodes

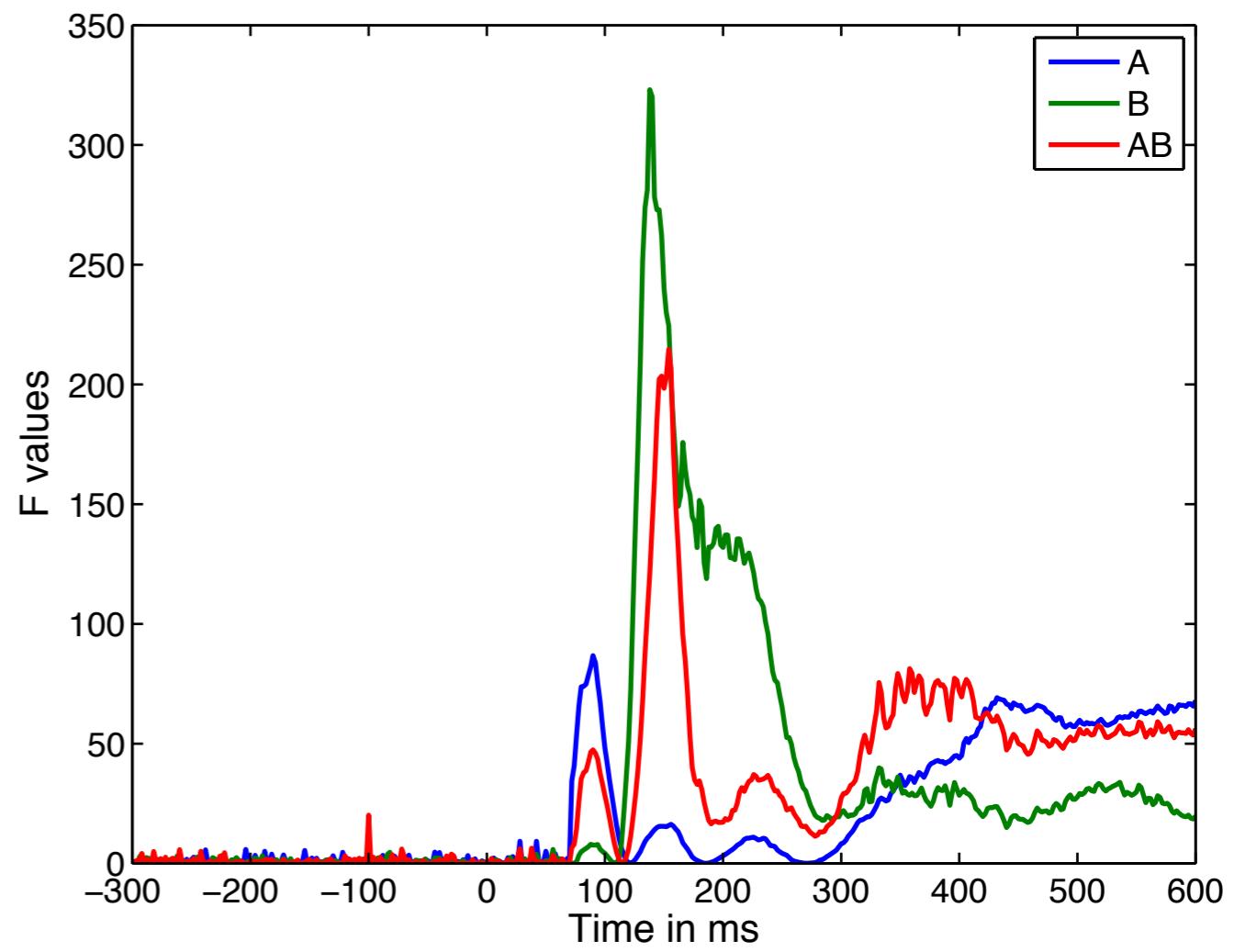
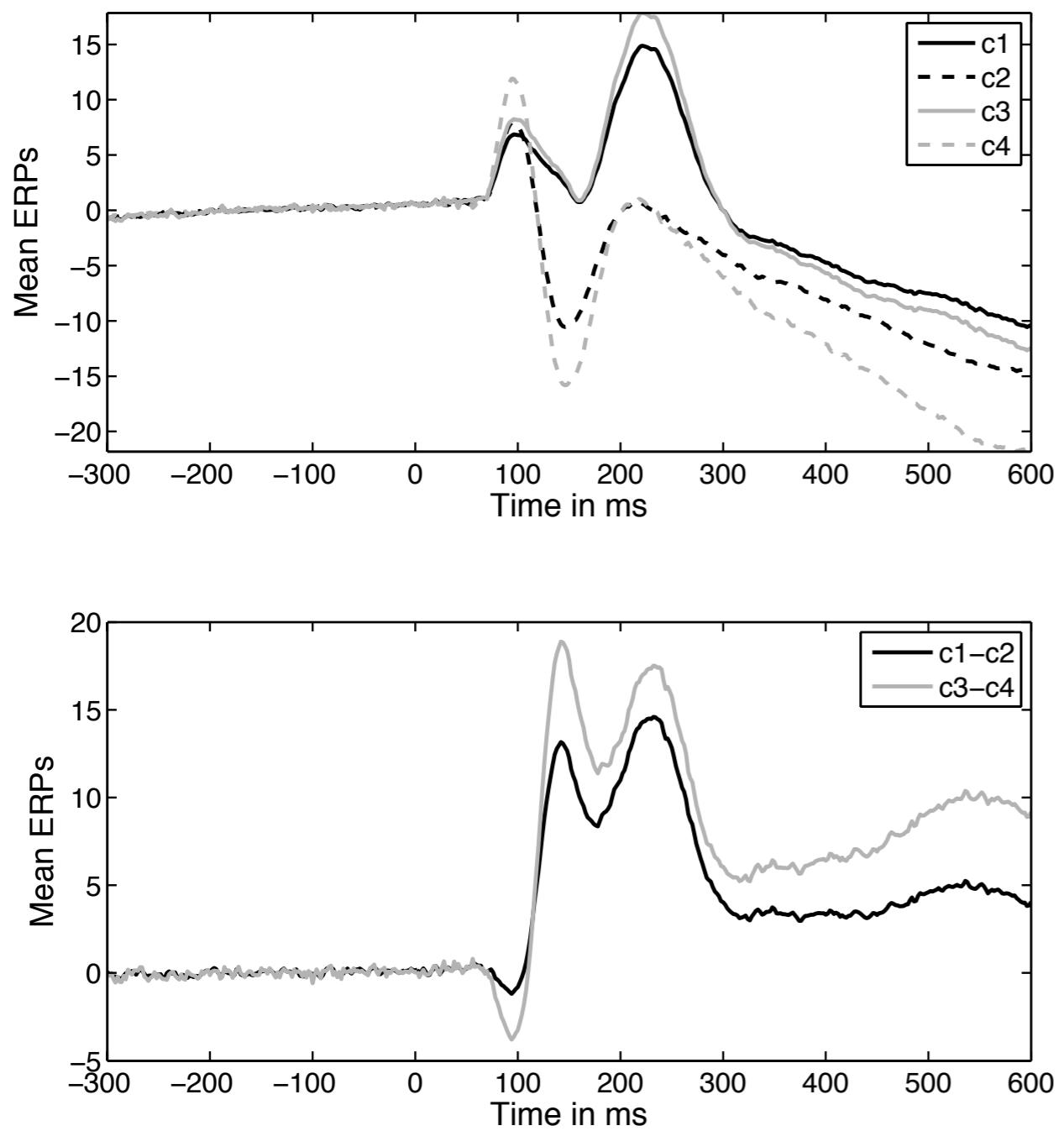
---



erp\_workshop\_9\_cmc\_single.m

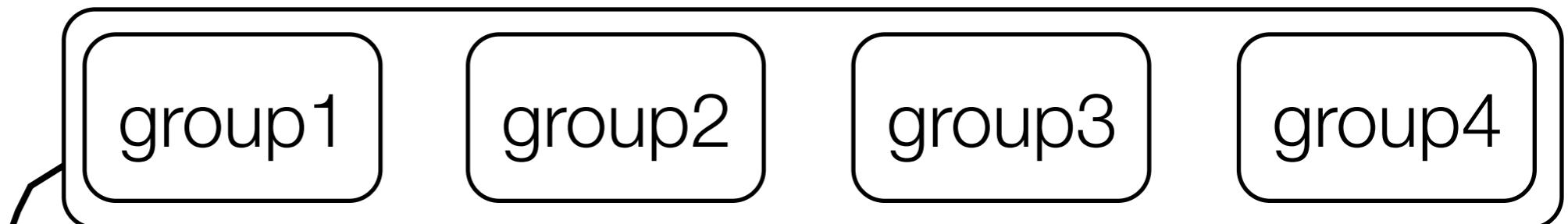
# Group analyses: ANOVA

---



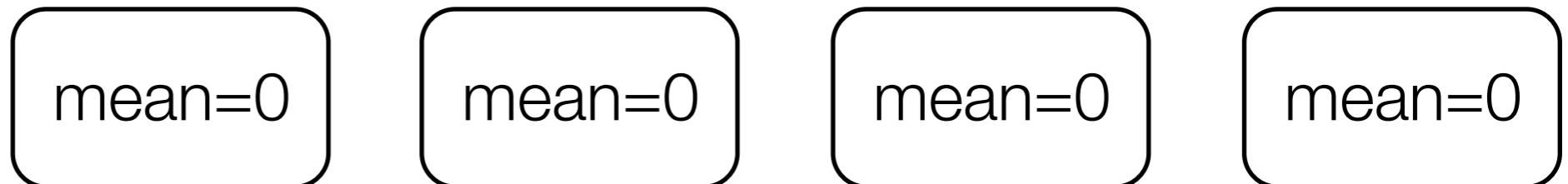
# bootstrap-F technique

data

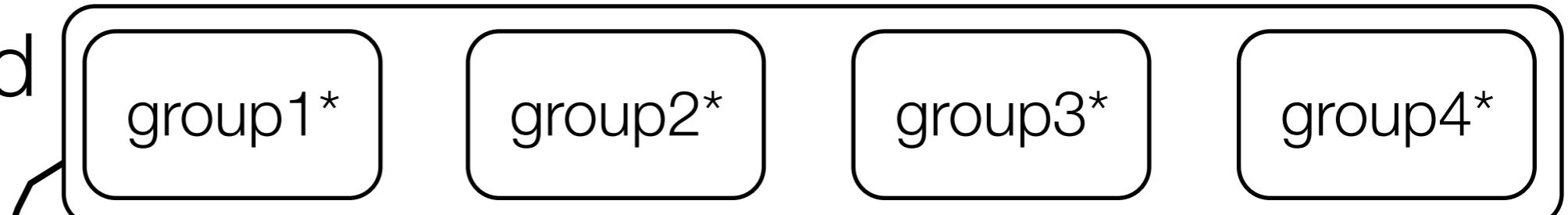


→ ANOVA: F

centred  
data



bootstrapped  
data

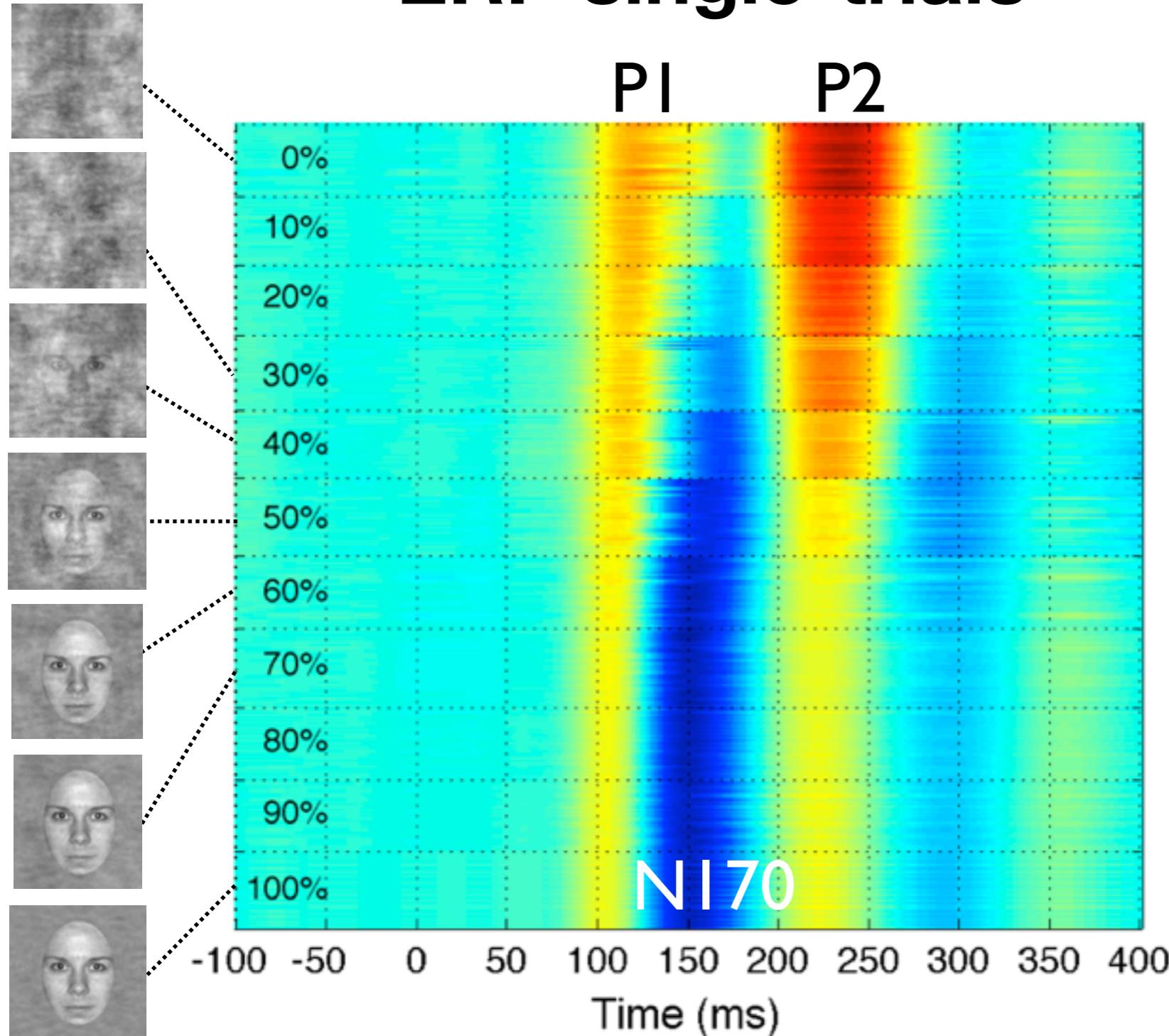


→ ANOVA: F\*

repeat b times => data driven bootstrapped F table

erp\_workshop\_10\_anova.m

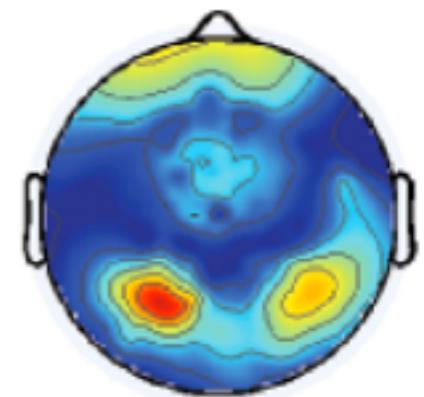
# ERP single-trials

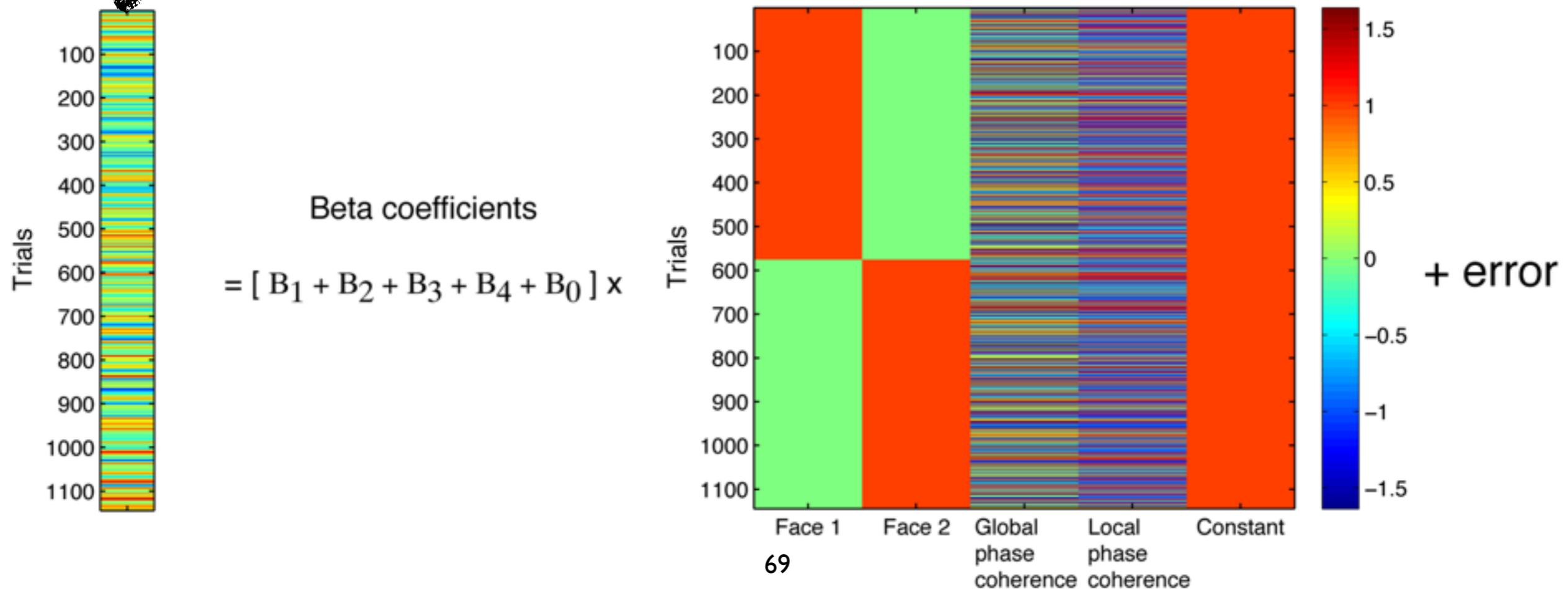
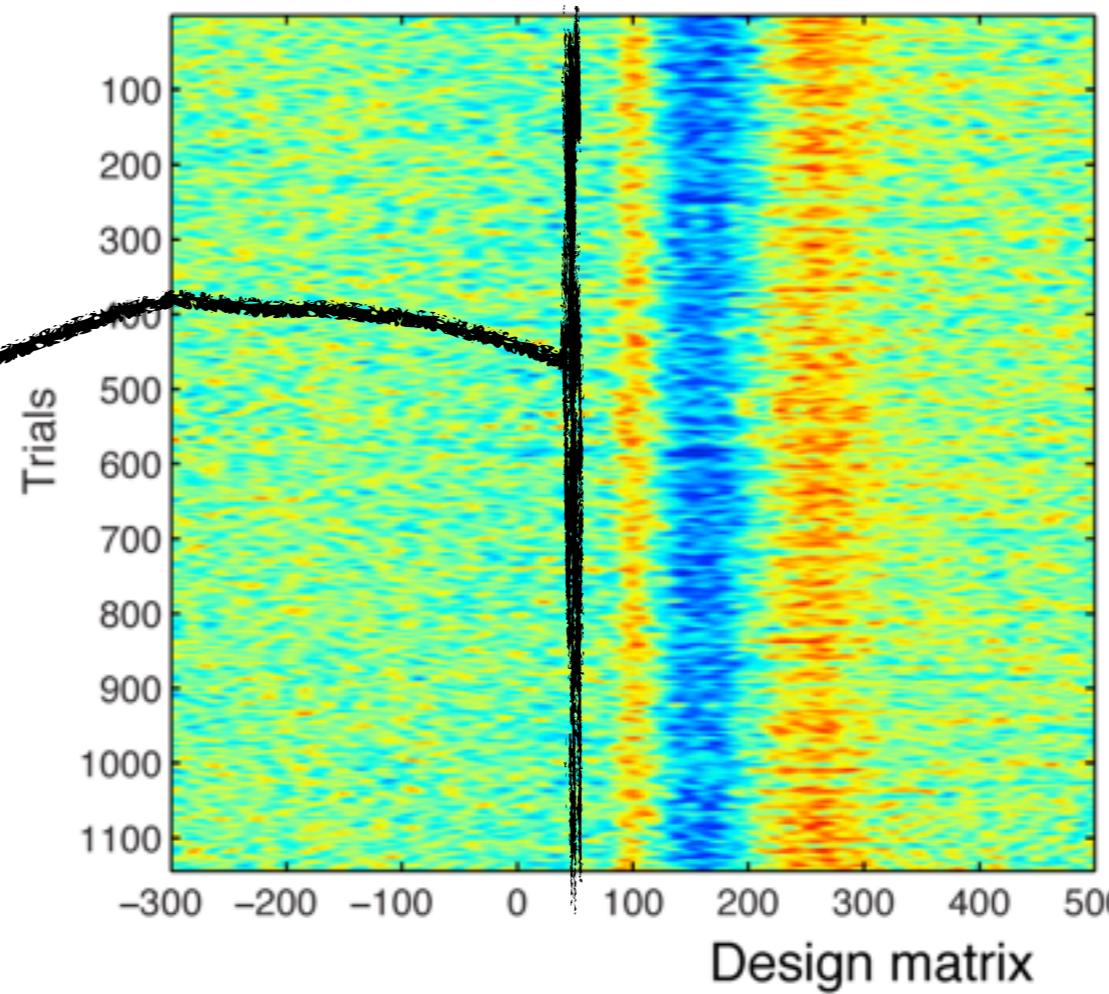


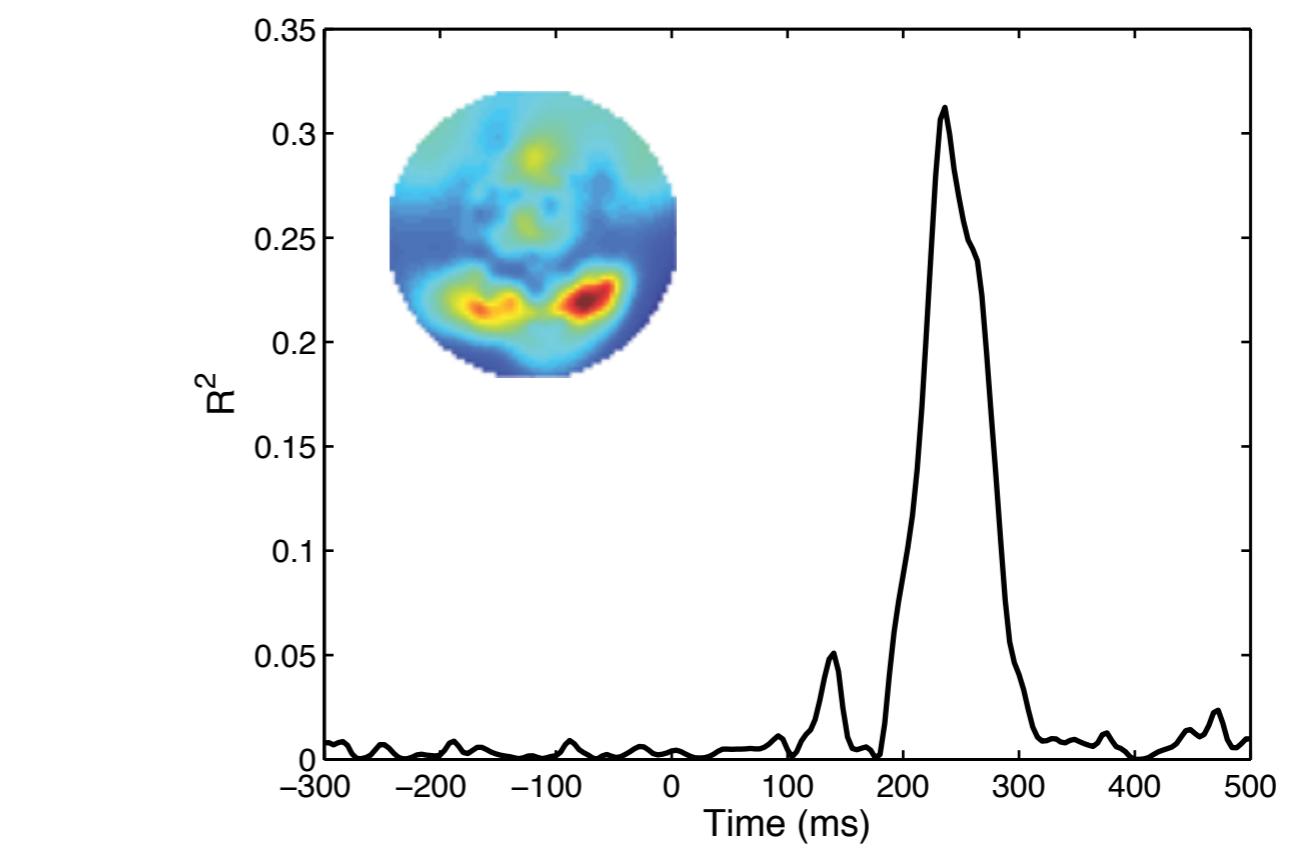
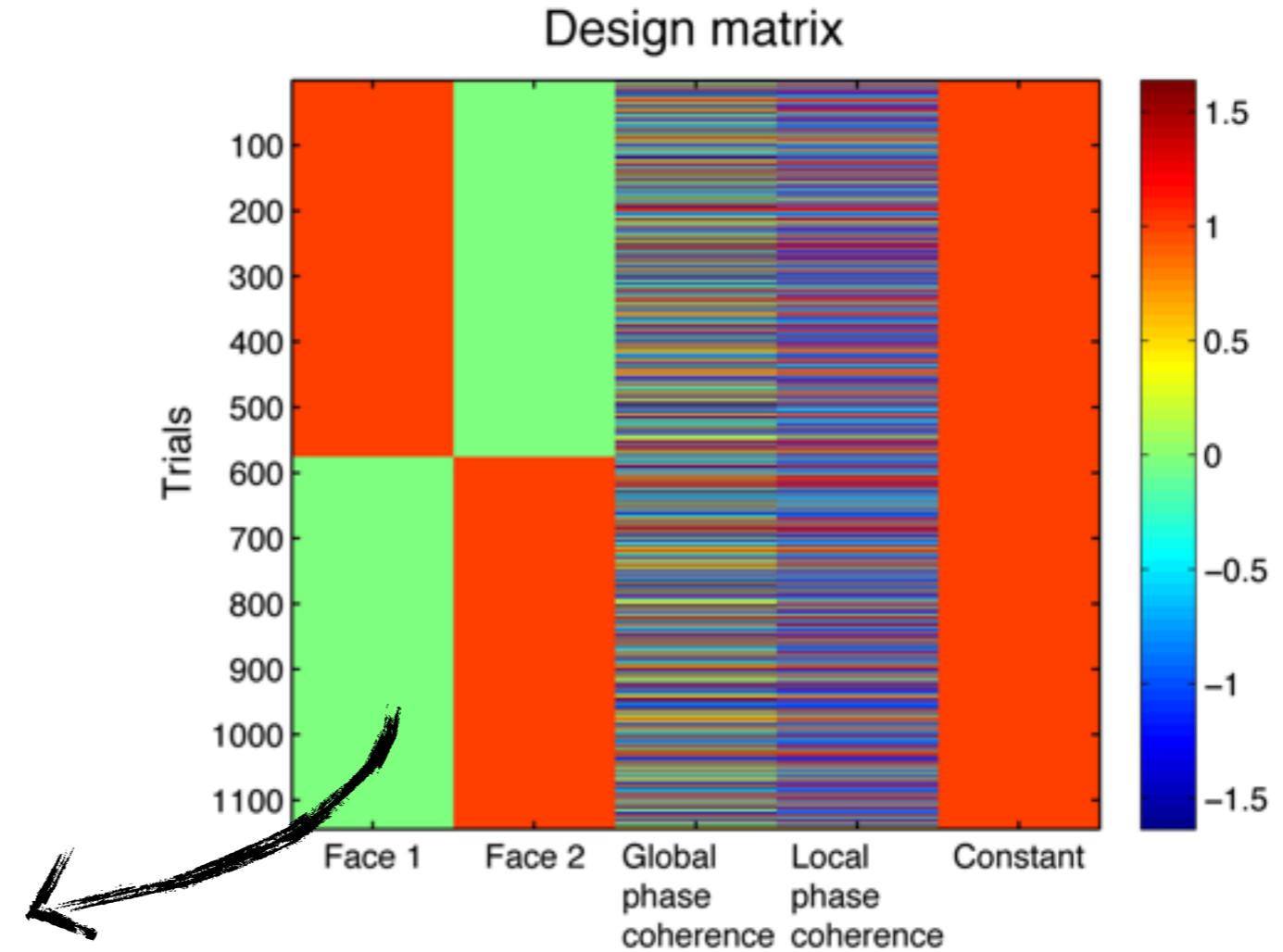
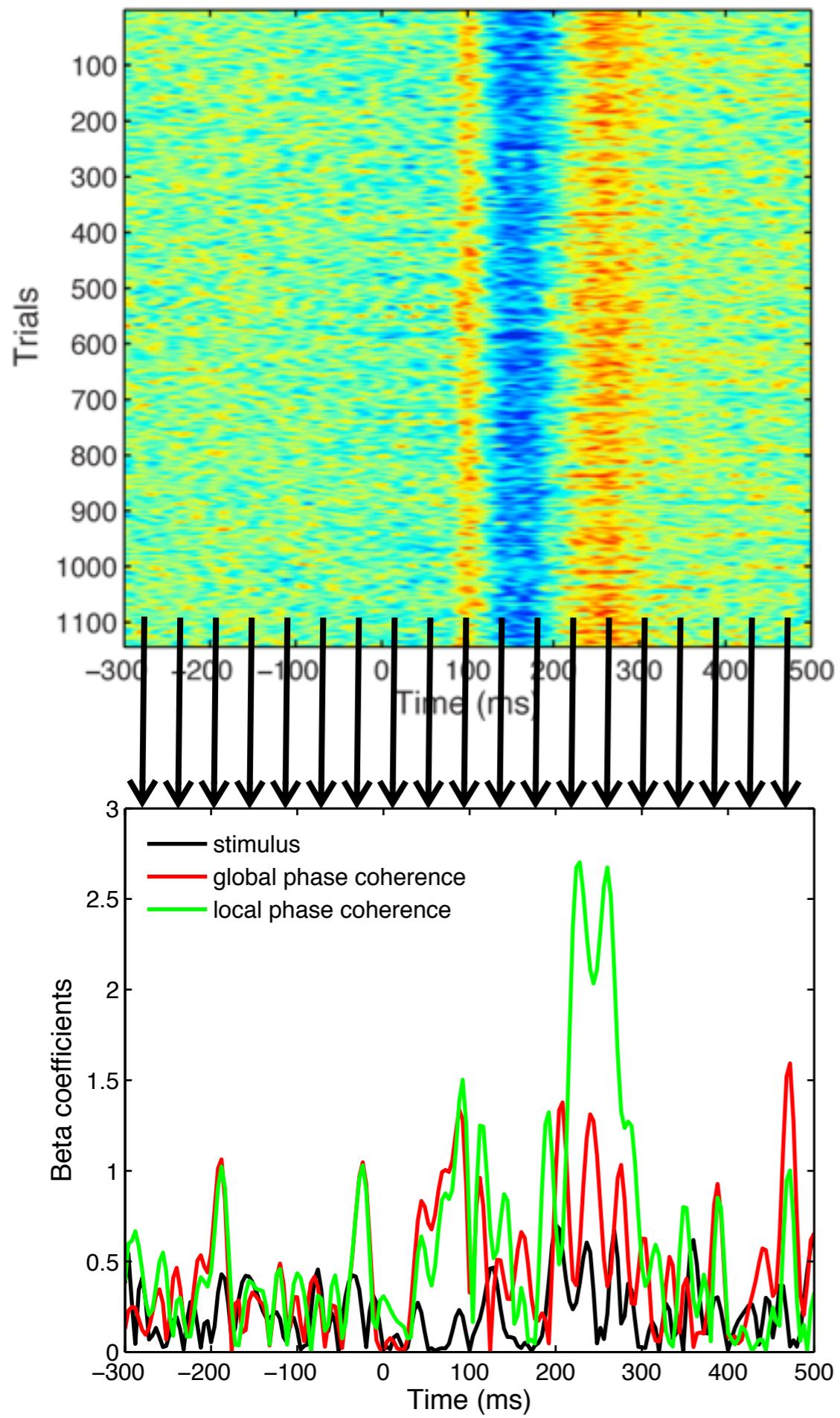
**Phase noise  
sensitivity:**

single subject

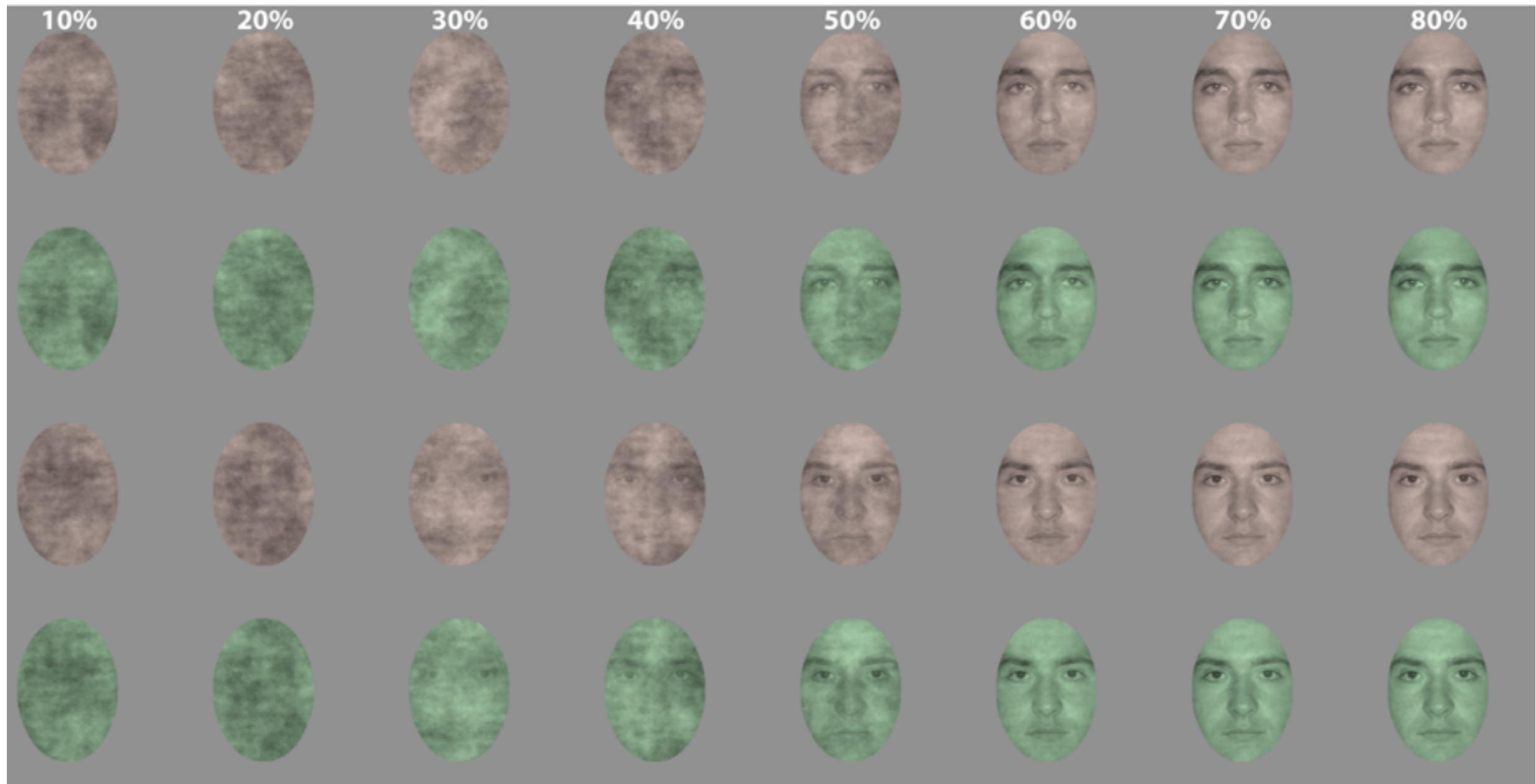
posterior  
electrode







# task effects on noise sensitivity?

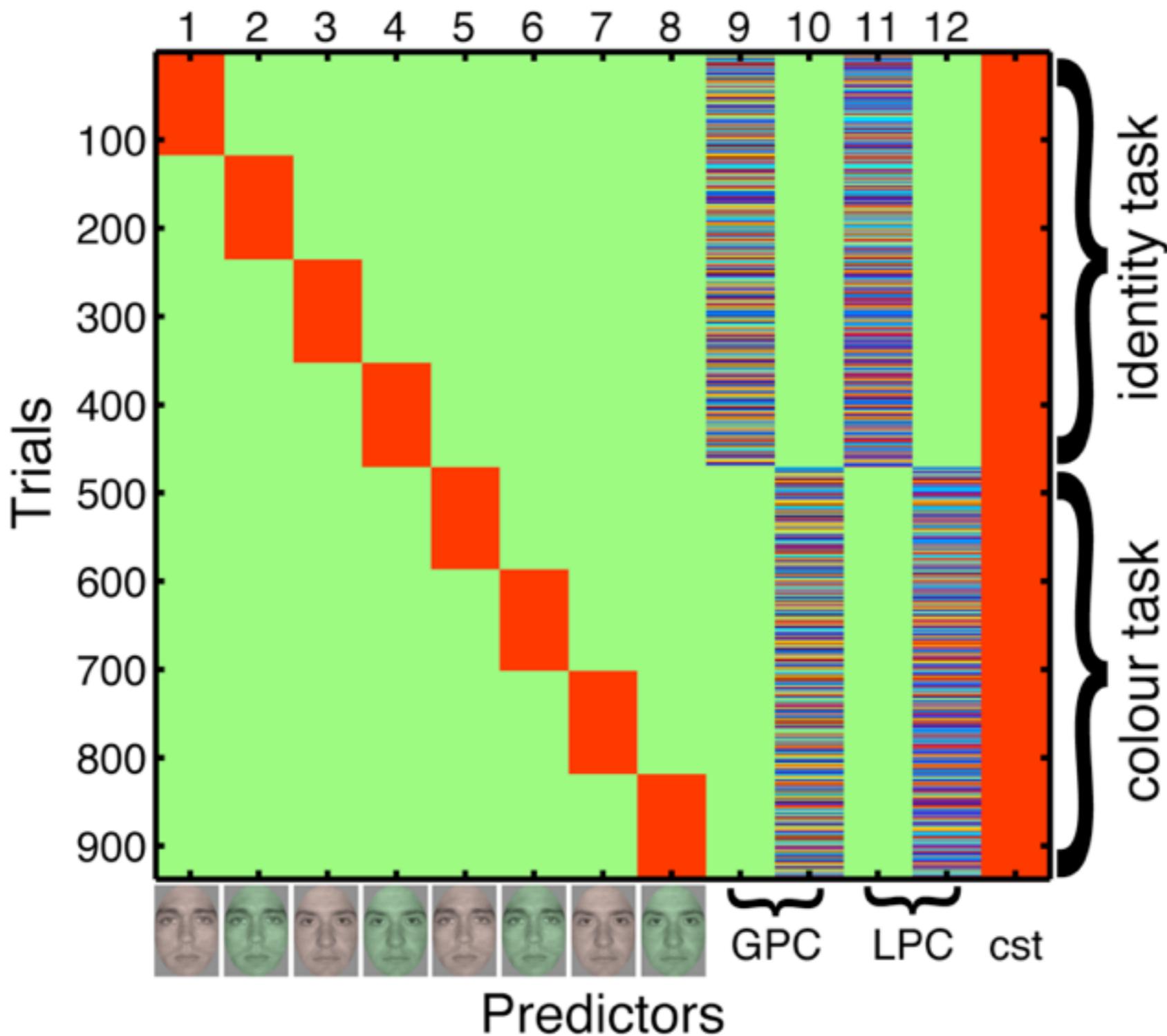


2 tasks - block design, 13 young subjects  
Rousselet, Gaspar, Wiegert & Pernet (2011). *Frontiers in Psychology* 2(137)

how many subjects?  
shift function

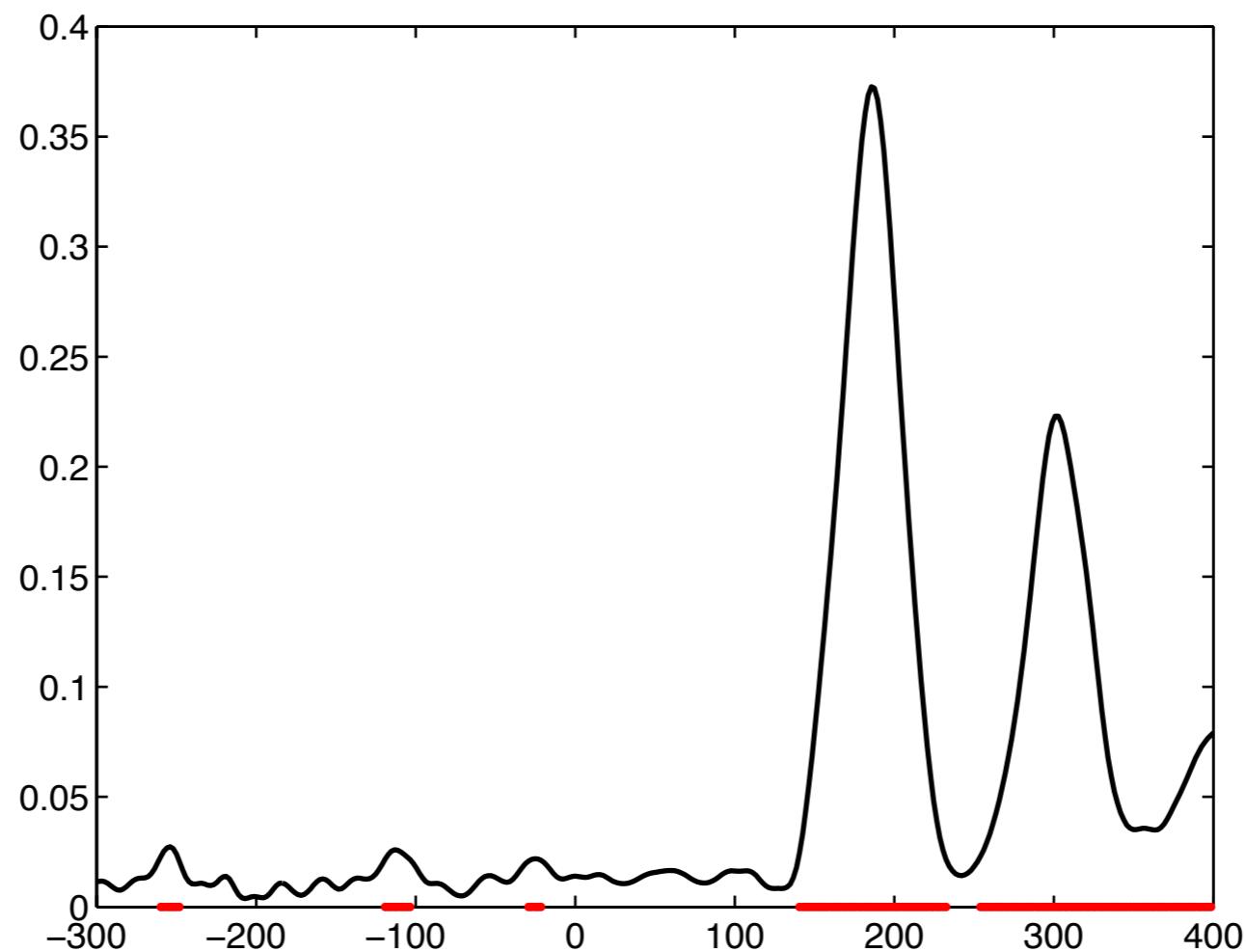
# design matrix

---



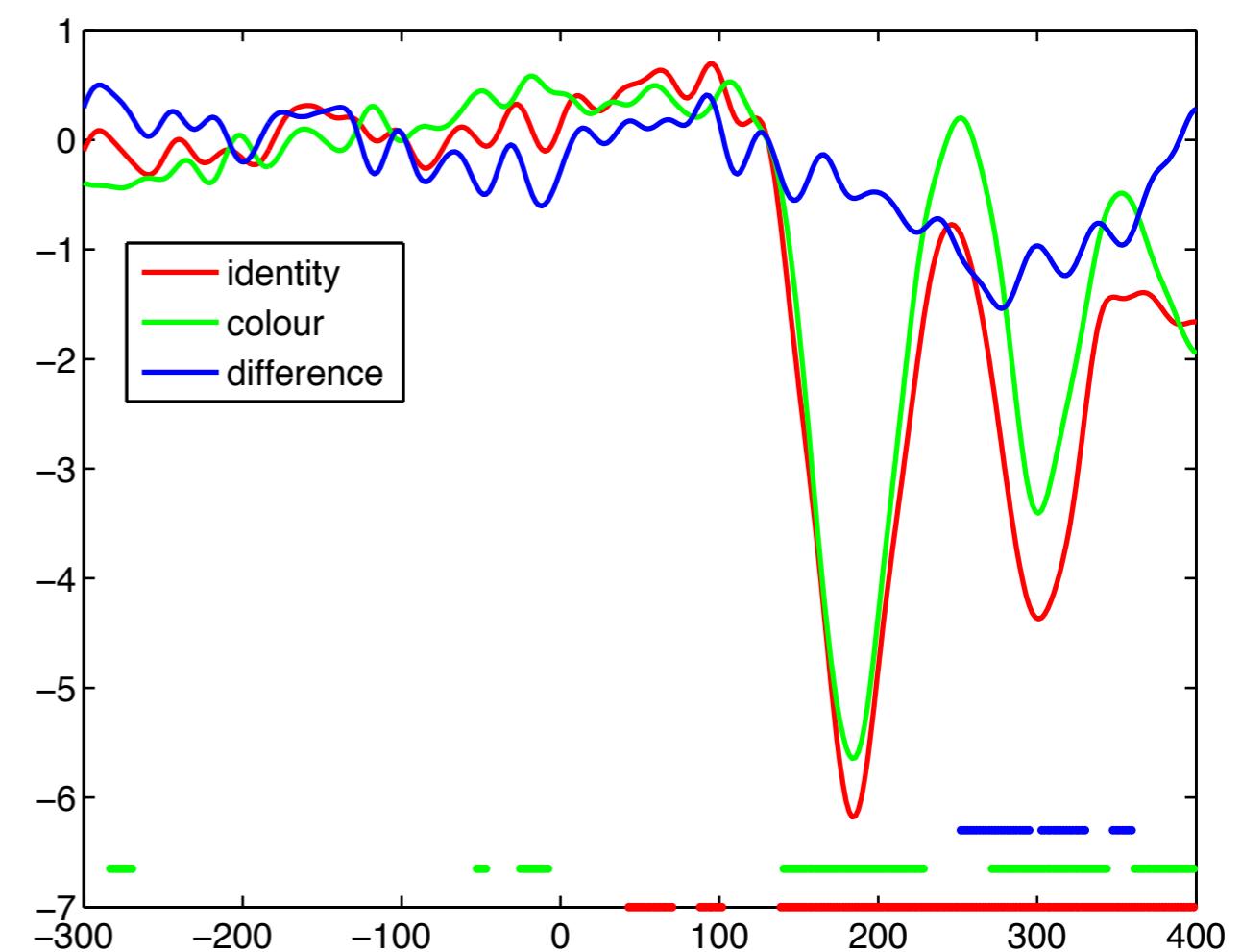
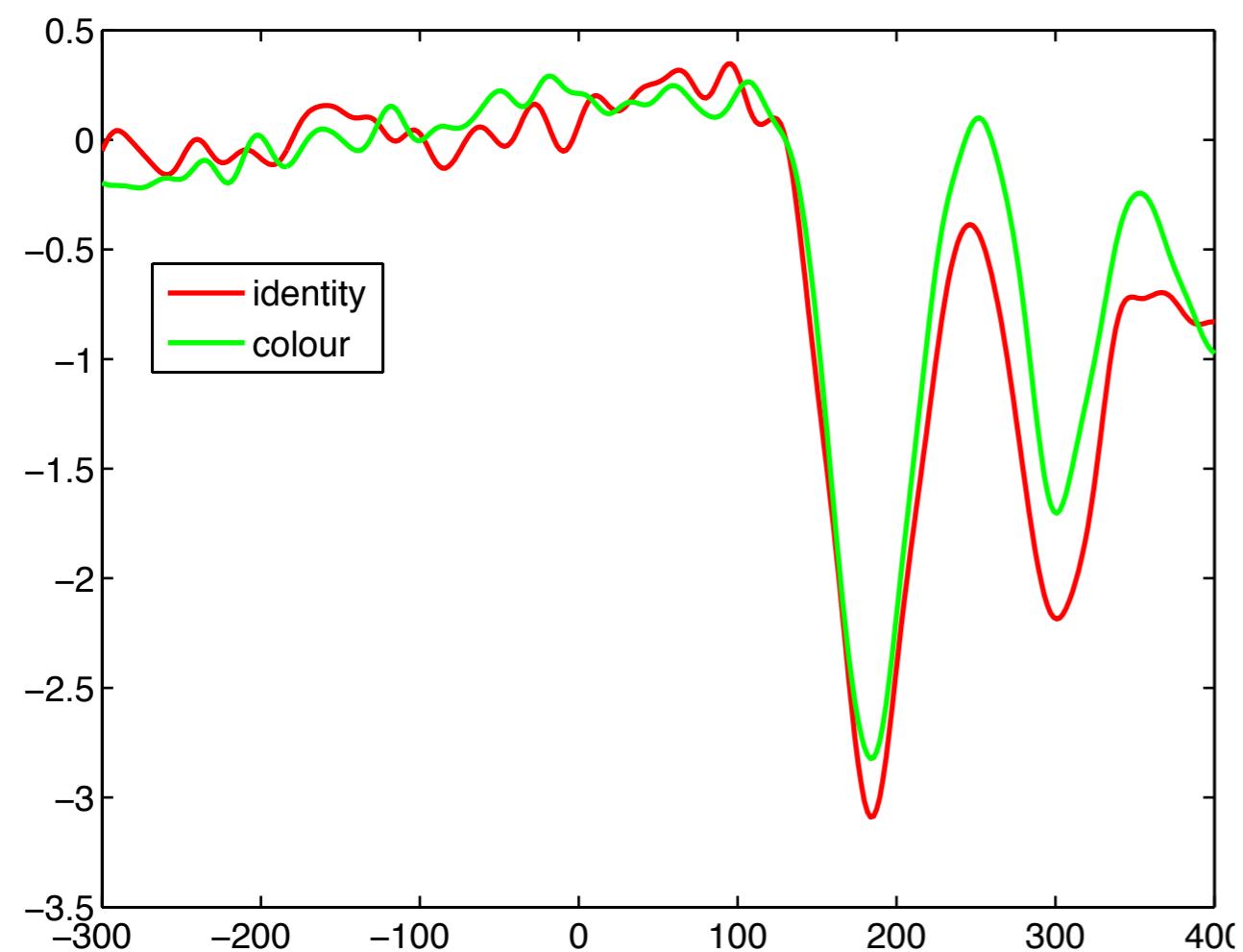
# Global model $R^2$

---



# Beta coefficients & linear contrasts

---



erp\_workshop\_11\_ancova.m

# Questions?

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Cyril Pernet <cyril.pernet@ed.ac.uk> <- LIMO EEG mastermind