

Four-Variable Jacobian Conjecture in a Topological Quantum Model of Intersecting Fields

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Abstract

This paper presents a topological model involving two intersecting curved fields with varying phases that may represent a case of the Jacobian conjecture for four variables. It also suggests connections with other mathematical topics such as Gorenstein Liaison, Tomita-Takesaki modular theory, the Mass gap problem, Reflection positivity, T-duality in string theory, or the Hodge conjecture, all considered within the dual fields model framework.

The mathematical investigation presented in this paper stems from the search for a deterministic atomic model, where two curved fields intersect to create a shared nucleus of matter and its mirror image. This speculative nuclear model is generally described in the paper's concluding section.

1 Introduction

1.1 Antisymmetric system

Two intersecting curved fields that vary with opposite phase, when the right field contracts the left expands and vice versa, form in their intersection two transverse and two vertical subfields. The transverse subfields are mirror antisymmetric, meaning that being mirror reflection of each other, they follow opposite phases: when the right subfield expands the left contract and vice versa.

The curvature of the transverse subfields is half

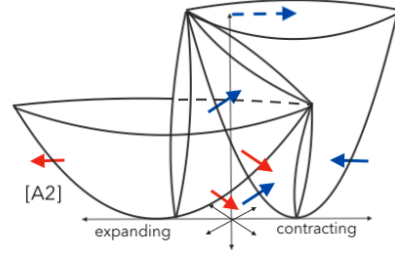


Figure 1: *Antisymmetric system: at moment A2, the right transverse subfield contracts and the left transverse subfield expands; the vertical subfield moves rightward.A4*

positive and half negative, and they are determined by the forces of pressure caused by the inward displacement of the negative curvature of the contracting field, and by the outward displacement of the positive curvature of the expanding field.

These forces of pressure are represented by four eigenvectors with eigenvalue 1 or -1 , all pointing toward right or toward the left.

An inversion of the system A2, equivalent to a 180 degrees rotation, given by the change of sign of the four positive eigenvectors, is operated when the right-hand contracting field expands and the left-hand expanding field contracts. Then, the right contracting transversal subfield at A2 is mapped to the left contracting transversal subfield at A4, and the left expanding transversal subfield at A2 is mapped to the right expanding transversal subfield of A4.

This operation can be considered as an involution, which is a type of automorphism. The system at A2

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maps to itself at $A3$ and viceversa.

The complex conjugate function that describes the continuous evolution from $A2$ to $A4$ and from $A4$ to $A2$ is its own inverse.

The yet non-transformed right contracting transverse subfield at time $T1$ can be interchanged with the transformed left contracting transversal subfield at $T2$, and the non-transformed left expanding transversal subfield at $T1$ can be interchanged with the transformed right expanding transverse subfield at $T2$. In that way, the left and right transversal subfields exhibit chiral mirror symmetry at different times.

This occurs as half of the system $A2$ (and the system $A4$) follows a purely imaginary time dimension, delayed with respect to the real time dimension that follows the other half of the system.

In this context, a time dimension is considered to be a necessary reference to measure the periodic fluctuation of space, which can be represented by an axis in the coordinate system.

The top vertical subfield will move left or right, toward the side of the intersecting field that contracts.

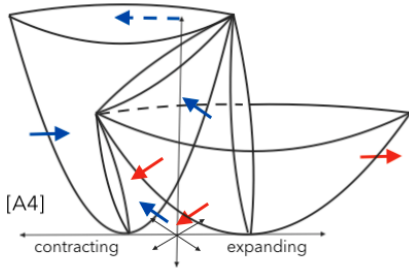


Figure 2: *Antisymmetric system: at moment $A4$, the right transverse subfield expands and the left transverse subfield contracts; the vertical subfield moves leftward.*

Both left and right transverse subfields are here described by the same spatial dimensions. These spatial dimensions cannot be the same that are used to describe the intersecting fields, because the Y coordinate of the transverse subfields will be considered a diagonal axis from the point of view of the coordinate system taken as reference to describe the intersecting

fields.

Misleading the different coordinate systems without considering the higher spatial dimensions of the transverse subfields will result in a relativistic space-time metric that appears elongated. This effect would be observed when measuring the transversal subfields from the location of the intersecting fields, using their referential coordinates system.

Given that each eigenvector has two possible directions, pointing toward right or left, the antisymmetric system formed by $A2$ and $A4$ can be described by a complex conjugate function of two variables, and a pair of 2×2 complex matrices whose elements are the mentioned eigenvectors:

$$A_2 \left\{ \begin{array}{cc} \nearrow & \searrow \\ \searrow & \nearrow \end{array} \right\} \quad A_4 \left\{ \begin{array}{cc} \nwarrow & \swarrow \\ \swarrow & \nwarrow \end{array} \right\}$$

Figure 3: *Pair of 2×2 complex conjugate matrices whose elements are eigenvectors with eigenvalue 1 or -1 , related to the antisymmetric system.*

1.2 Symmetric system

In the symmetric system represented by $A1$ and $A3$, the two intersecting fields oscillate in unison: they contract and expand simultaneously.

The transversal subfields exhibit chiral mirror symmetry at the same time. If the system $A1$ were inverted in the same way previously seen for the antisymmetric system, equivalent to a 180-degree rotation in the horizontal plane, both left and right transverse subfields would be interchangeable.

However, in the symmetric system, the positive and negative eigenvectors point upward and downward, respectively. The continuous inversion of system $A1$ performed at $A3$ implies that all four positive eigenvectors pointing upward revert their sign, becoming negative and pointing downward. In this context, the inversion does not represent a 180-degree rotation of the vertical plane, as the curvatures do not get inverted. However, the inversion operated at $A3$ has consequences similar to that type of planar rotation. All the pressure forces at $A2$ are caused by the

negative curvatures of both intersecting fields moving inward, and all the pressure forces at $A4$ are caused by the positive curvature of both intersecting fields moving outward. This change in the relevant side of the curvature in terms of pushing force is equivalent to an inversion in curvature.

Considering a mapping that takes the system from a state of expansion at $A1$ to a state of contraction at $A3$, there will exist a reverse mapping that transitions the system from contraction back to expansion. This transformation is a continuous process, and despite differences in shape, the topological structure of the curvatures of the transverse subfields is preserved. Therefore, this mapping can be considered a homeomorphism.

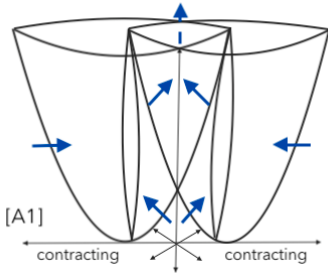


Figure 4: *Symmetric system: at moment A1, both left and right transversal subfields expand. The vertical subfield moves upward while contracting.* $A4$

Observing the dynamics determined by the periodic contraction and expansion of the intersecting fields, it can be observed that at moment $A1$, when both intersecting fields undergo contraction, there is a simultaneous expansion of both transverse subfields. Conversely, at $A3$, when the previously contracting fields begin to expand, both transverse subfields enter a state of contraction.

The linear, continuous evolution from $A1$ to $A3$ and vice versa, can be modeled by a complex function of two variables. The changes of the curvatures of the oscillating spaces and subspaces are indicated by the variation in the eigenvectors' direction. The four eigenvectors can point upward (at $A1$) or downward (at $A3$), depending on their sign, and they can be represented in a pair of 2×2 complex matrices with

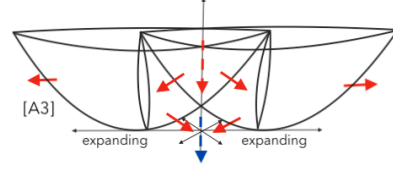


Figure 5: *Symmetric system: at moment A3, both left and right transversal subfields contract. The vertical subfield moves downward while expanding.* $A4$

eigenvalues of either 1 or -1 .



Figure 6: *Pair of 2×2 complex conjugate matrices of eigenvectors related to the antisymmetric system.*

1.3 Rotational system

Thus far in the article, the symmetric system $A1 - A3$, and the antisymmetric system $A2 - A4$ have been treated as independent and unrelated systems, linearly described by two distinct functions related to different pairs of matrices.

However, within a rotational framework, the symmetric and antisymmetric systems may in fact turn out to be the same system that undergoes topological transformations with each 90-degree rotation, alternating between symmetric and antisymmetric states. The nonlinear evolution of such a rotational system would need to be described by two interpolated functions: a complex function and its harmonic conjugate solution.

However, in a rotational framework, the symmetric and the antisymmetric systems may turn to be the same system which is topologically transformed after each 90-degrees rotation, becoming periodically symmetric or antisymmetric.

This would imply that the smooth but non-linear evolution of the system given by $A1 + A2 + A3 + A4$, must be described by two interpolated functions, a complex function (related to the symmetric moments

of the system) and its harmonic conjugate solution (related to the antisymmetric moments of the system).

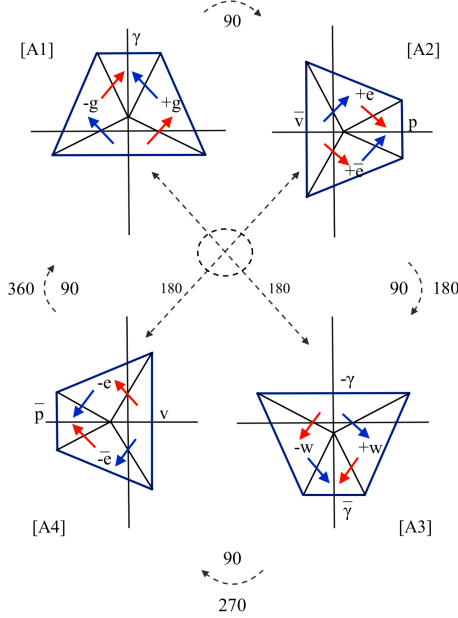


Figure 7: *Rotational interpolation of the vector symmetric and antisymmetric systems represented in a 2D scheme*

The evolution of that interpolated rotational system can be represented by a set of four 2×2 complex rotational matrices of eigenvectors with eigenvalue 1 or -1 :

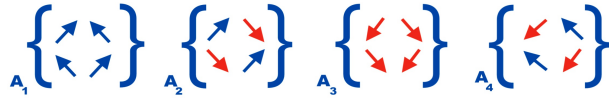


Figure 8: *Set of 2×2 rotational matrices related by pairs to the complex symmetric (A1 and A3) and the conjugate antisymmetric (A2 and A4) systems.*

The rotational system's evolution progresses through four stages, each corresponding to a 90-degree rotation. This evolution is associated with the

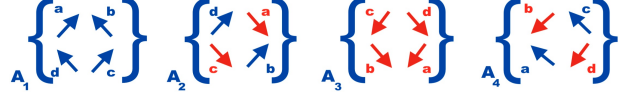


Figure 9: *Actual rotation of the vectors in the context of the rotational matrices, changing their position after each 90-degrees rotation.*

periodic interpolation of symmetric and antisymmetric moments of the system. In each stage, only two of the four eigenvectors, which signify the variation in the curvature of half of the system, change their sign according to their eigenvalues of 1 or -1 .

This variation in half the eigenvectors can be interpreted as a half-order derivative. The full first-order derivative is realized when all four eigenvectors change sign after two consecutive 90-degree rotations.

However, by assigning specific letters to the eigenvectors, it can be observed that all four eigenvectors change their direction at each stage, effectively displacing after each 90-degree rotation.

The four eigenvectors actually rotate in the complex plane, altering their position and direction four times, each time by 90 degrees. In this context, the function describing the system's evolution can be interpreted as a function of four variables. These four variables represent the four potential directions or positions in the complex plane that the eigenvectors can assume during the system's non linear rotational evolution.

When only two of the four eigenvectors change their sign, the system's evolution will be represented by a function of two variables.

On the other hand, the set of transformation matrices $A1$, $A2$, $A3$, $A4$ that describe the evolution of the rotational system result from the operations of transposition, complex conjugation (as the sum of two partial complex conjugations), and inversion:

- **A1** (0-degrees rotation) represents the eigenvectors in the symmetric system, when both intersecting fields contract. The transversal subspaces have mirror symmetry at the same moment and the top vertical contracting subfield experiences a double force of compression while

ascending. $A1$ can be taken as the identity matrix.

- **A2** (90-degrees rotation) represents the eigenvectors when half of the system has delayed its phase, introducing a purely imaginary time dimension. The transversal subfields have mirror antisymmetry (the left expands while the right contracts), and the vertical subfield moves right. $A2$ represents the transposition of $A1$. The change of sign of half of eigenvectors implies a partial complex conjugation of $A1$, and its $\frac{1}{2}$ order derivative.
- **A3** (180-degrees rotation) represents the partial conjugation of $A2$, changing sign the two eigenvectors that had not changed sign at $A2$). As its four eigenvectors have already commuted their sign with respect to $A1$, $A3$ represents the negative reflection of $A1$ and its whole first order ($\frac{1}{2} + \frac{1}{2}$) derivative derivative; ($A3$ represents the $\frac{1}{2}$ order derivative of $A2$).
- **A4** (270-degrees rotation with respect to $A1$, 180-degrees with respect to $A2$, and 90-degrees with respect to $A3$) represents the transpose of $A3$, the $\frac{1}{2}$ order antiderivative of $A3$, the second transposition of $A1$, and the first order ($\frac{1}{2} + \frac{1}{2}$) derivative of $A2$; $A4$ is also the negative mirror reflection of $A2$, having commuted sign its four eigenvectors.
- An additional 90-degrees rotation produces $A1$ which represents the positive reflection of $A3$, a $\frac{1}{2}$ order antiderivative of $A4$, and the first order ($\frac{1}{2} + \frac{1}{2}$) antiderivative of $A3$.

2 Jacobian conjecture

The Jacobian conjecture [1] formulated by Keller in 1939 states that if a polynomial map from an n -dimensional space to itself has Jacobian determinant which is a non-zero constant, then, the function has a polynomial inverse.

Expressed in terms of vectorial functions, it would state that if a vector-valued function map from an n -dimensional space to itself has Jacobian determinant

which is a non-zero constant, then the function has a vector-valued inverse.

The Jacobian determinant is a measure of how much a transformation stretches or shrinks the space it maps to, and it is defined for continuous transformations. The Jacobian conjecture applies to maps between homeomorphic spaces.

In the context of the rotational system, the transformations are continuous but not in a linear way. The smooth continuity passes through the interpolation of the complex and the conjugate function spaces after each 90-degrees rotation.

In that context, the topological structure of the transverse subfields is preserved, being automorphic, even when their size is not identical as it happens in the symmetric system when the contracting subfields map the expanding subfields. Their curvatures are always half positive and half negative, as they are formed by the inner curvature of an intersecting field and by the outer curvature of the other intersecting field.

In the antisymmetric system, the topological structure of the automorphic vertical subfield that maps to itself when moving leftward or rightward is also preserved because it's always formed by a negative curvature formed by the inner curvatures of both left and right intersecting fields.

The top vertical subfield in the antisymmetric system that moves upward while contracting (when both intersecting fields contract), or downward while expanding (when both intersecting fields expand), has a negative curvature. However, the inverted subfield that at moment $A3$ exists at the convex side of the system, mapping the top vertical subfield that exists in the concave side at $A1$, has a double positive curvature.

In that sense, the topological structure of the concave vertical subfield of $A1$ is not preserved in the inverse convex subfield of $A3$. That would represent that the top vertical subfield is not mapped when the inversion is operated at $A3$.

Still in that case, the ascending contracting vertical subfield with negative curvature of $A1$ can be considered being mapped to itself at $A3$, when it descends while expanding. However, in that case, the mirror reflection property of the vertical subfield would not

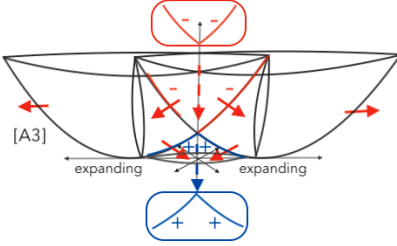


Figure 10: *Antisymmetric system: at moment A3, when the top vertical subfield descends while expanding it has negative curvatures; but its mirror counterpart at the convex side has positive curvatures.*

be being considered by the Jacobian conjecture.

An additional complexity could be introduced if the intersecting fields periodically synchronize and desynchronize their phases of fluctuation while the whole system rotates.

3 Gorenstein Liaison

In algebraic geometry, the Gorenstein theory [2] establishes that two modules in projective space are linked if they are isomorphic.

These isomorphic modules contain curved with a same deficiency. The curvature singularity reflects a lack of regularity. For instance, a curve exhibiting a change in its sign, inverting its direction, represents a defect in that curve.

The deficiency module of a curve is isomorphic to the deficiency module of any other curve with the same deficiency. Consequently, if two curves share the same deficiency, then the modules that encompass them in the projective space will also exhibit the same deficiency module.

This establishes that the two modules are "linked" by their deficiency in a Gorenstein sense.

To be linked in projective space, it suffices for two modules to preserve the algebraic structure imparted by the deficiency in the curves.

Under the model of intersecting fields proposed in this article, the modules that contain the curves of the Gorenstein theory are interpreted as the trans-

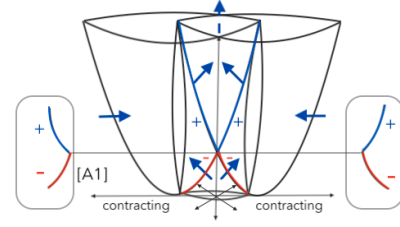


Figure 11: *The left and right modulus, represented by the mirror transversal subfields, encompass curvatures that exhibit a same irregularity, being half + and half - from the point of intersection between the left and right fields.*

verse subfields in both the symmetric and antisymmetric systems.

The projective space scenario is represented by the left and right intersecting curved fields. These fields have a different weight in regard to the creation of the singularity in the curvature of the transversal subfields.

In that way, the left transverse subfield, embedded in the left intersecting field, exhibits a negative curvature up to a point of inflection, where that curvature inverts its sign becoming positive. That point of singularity is the point of intersection of the left and right fields.

That deficient by irregular positive sign of the curvature is given because it corresponds to the outer side of the curvature of the right intersecting field. In that context, the right field has a relevant or determining weight or in the inversion of the sign that causes the singularity.

The same occurs in regard to the right transverse subspace, embedded in the right intersecting field. Its curvature exhibits the same dual structure as the left transversal subfield, being half positive and half negative. In this case, the relevant weight in the curvature singularity will be attributed to the left intersecting field.

In this context, both left and right modular transverse subspaces are isomorphic because the structure of their deficiency is preserved under the map to each other that occurs during the evolution of the projecting arena, creating a Gorenstein-style linkage, as a

short of entanglement.

Considered in these terms, the Gorenstein modular linkage can be connected to the Jacobian conjecture previously exposed, in the framework of the intersecting fields model.

4 Tomita-Takesaki theory

Considering the rotational fields system as a specific case of the Jacobian conjecture, it is also possible to conceptually infer some relations to Tomita-Takesaki (TT) modular theory [3].

In TT theory two intersecting algebras form two shared “modular inclusions” (with $+$ and $-$ half sided subalgebras) and a “modular intersection” (with an integer sided subalgebra).

The left and right half handed subalgebras will be images of each other, when they are commutative, or they will not be their mirror image when they are noncommutative.

Mapping the modular inclusion to its reflection image, the left and right subalgebras will be the opposite image of each other (reverting their initial signs) if they are commutative; if they are noncommutative, the initial left sided subalgebra will be the image of the right sided mapped subalgebra, and the initial right-handed subalgebra will be the image of the left sided mapped subalgebra.

TT theory decomposes a linear transformation into its modular building blocks, revealing its automorphisms.

Decomposing the bounded operator, it obtains the modular operator and the modular conjugation (or modular involution) which is a transformation that reverses the orientation, preserving distances and angles.

Translating the abstract algebraic terms to the fields model, two intersecting algebras would represent the two intersecting fields fluctuating with the same or opposite phase.

The half handed subalgebras (or “modular inclusions”) will be the transversal subfields of the nucleus shared by the intersecting fields, while the integer handed subalgebra (or “intersection inclusion”) will be our vertical subfields. In this context, we iden-

tify commutativity and noncommutativity with mirror symmetry and mirror antisymmetry, respectively.

The bounded operator that is decomposed will be the 90-degrees rotational matrix; The modular building blocks are the set of matrices that are obtained when applying the operator.

The modular operator will be the $\frac{1}{2}$ partial conjugate $A2$ matrix; And the modular conjugation will be the conjugate matrix $A4$, which forms the whole conjugation by adding the partial conjugations ($\frac{1}{2} + \frac{1}{2}$).

Therefore, by separating the conjugate matrix from the complex one the automorphism of the antisymmetric conjugate system is found.

The half sided algebras that form a modular inclusion are noncommutative, it means we are in the antisymmetric system where the left intersecting field contracts while the right one contracts and vice versa; in that system, the left transversal subfield will be the mirror symmetric image (it will be the mapped image) of the right transversal subfield when, later, the left intersecting field expands and the right one contracts.

In that sense, a past half handed subalgebra is being mapped with its future image. A time delay will exist between both subalgebras.

Considering Δ as the modular operator $A2$, J the modular conjugation $A4$, and M the intersection of two Von Neumann algebras, $\Delta^{-Yt} M \Delta^{it}$ will represent the positive and negative $\frac{1}{2}$ sided modular inclusions of the modular operator, being t a real time dimension and it an imaginary time dimension given by the partial conjugation of $A1$ or $A3$.

It is this different time dimension what makes non-commutative, as non-interchangeable, the modular $+$ and $-$ inclusions related to Δ in the antisymmetric system.

Applying the modular involution, yields $J^{yt} M' J^{-it}$.

Δ^{-yt} is transformed into J^{yt} and Δ^{it} is transformed into J^{-it} , being $J^{yt} M' J^{-it}$ the involutive automorphism of $\Delta^{-yt} M \Delta^{it}$.

The noncommutative, as non-interchangeable, Δ^{-yt} and Δ^{it} become commutative or interchangeable through time at $J^{yt} M' J^{-it}$, fixing their antisymmetry (restoring the lost mirror symmetry) in that way.

5 Reflection positivity

Related to the delay in time in the antisymmetric system, it can also be mentioned a property that all unitary quantum field theories are expected to hold: “reflection positivity” (RP). [4]

The positive increasing energy that appears in one side of the mirror system should also be reflected in the other side. However, in the context of the antisymmetric system, the positive or increasing energy of the contracting right transverse subfield does not mirror simultaneously in the expanding left transverse subfield, which exhibits negative or decreasing energy.

Therefore, to obtain a positive energy reflected at the left side, making the sides of the system virtually symmetric, a time reversal operation is needed.

To observe the positive energy reflected at the left side, it will be needed to go back in time to the moment where the left transversal subfield was contracting and had a positive energy. This operation is performed by a type of “Wick rotation”. [5] The main

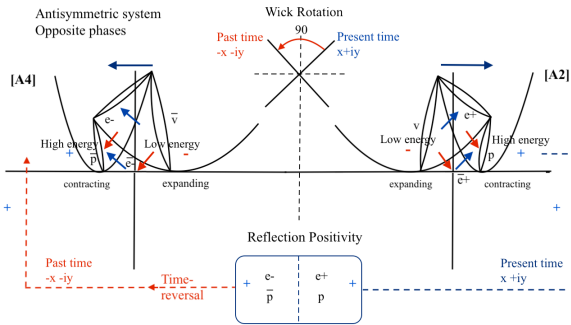


Figure 12: *Reflection positivity in the antisymmetric system.*

time phase of the symmetric system can be represented with the Y coordinate.

By performing a partial conjugation that involves a fractional derivative, the time coordinate Y undergoes a rotation into the purely imaginary dimension within the complex plane. At that moment, the mirror system becomes antisymmetric as one side of the

system keeps following the imaginary time of Y while the other side follows a harmonic phase. A positive or negative time lag has been introduced.

Reversion time on one side of the system serves as a symbolic tool to virtually restore symmetry to the time phases. To revert to the previous time, one could perform a reverse rotation of the complex time axis ($X + iY$) to achieve a full complex conjugation at $(-X - iY)$.

In the A matrix context, that time backward rotation represents an antiderivative of $-A$.

When the time reverse has been symbolically completed, in the left side of the mirror system the left subfield will be contracting, having an increased positive energy; this is a past reflection of the future positive energy that will emerge a moment later in the right side.

In the reverse past time, at the right side of the system the right subfield will be expanding, having a decreased negative energy.

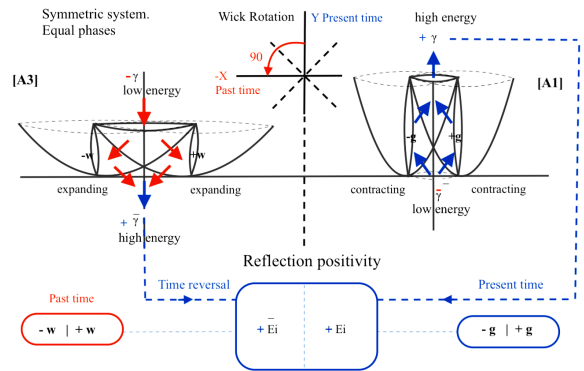


Figure 13: *Reflection positivity the symmetric system.*

In regard to the symmetric system, positivity is reflected between the right and left transverse subfields at the same time. In that sense, it's not necessary to use the Wick operation to reverse time.

Both left and right transversal subfields will be the mirror reflection of each other at the same time. However, in the case of the strong interaction in the symmetric system, when the contracting vertical subfield has an increased positive energy while ascending to

emit a pushing force, it will be necessary to virtually visit a past moment to look for a previous state where positivity could be reflected.

Going back in time, the vertical subfield will be losing its energy while expanding, moving downwards. Therefore, at that past moment, the vertical subfield will not display a positive energy.

Reflection positivity, however, can be found at that past moment in the convex side of the system of the two intersecting fields, where an inverted subfield with convex curvatures will be experiencing an increased energy.

That inverted subfield can mirror the vertical subfield which in a future state will be ascending in the concave side of the system through the Y axis.

The missing reflection positivity in the concave side of the system in the strong interaction can be related to a mass gap problem when it comes to the weak interaction.

6 Mass gap problem

There will be a mass gap [6] in the system when the two intersecting fields simultaneously expand, and the vertical subfield experiences a decay of energy.

This case represents the ground state with the lowest possible energy of the vertical subfield, which is always greater than 0 because the highest rate of expansion of the intersecting fields prevents them from having zero curvature.

The zero point of the vacuum, where there should be no energy or mass, is placed at the point of intersection of the XY coordinates, and that point is never reached by the vertical subfield that descends through the Y axis while expanding during its decay.

An “upper” mass gap would refer to the highest possible mass of a particle in the strong interaction. Its limit would be given by the greatest rate of contraction of the intersecting spaces.

The zero point of the vertical subfield is marked in yellow on the above diagram, at the point of intersection of the left and right intersecting fields.

The gap is given by the distance from that point to the zero point where the X and Y coordinates

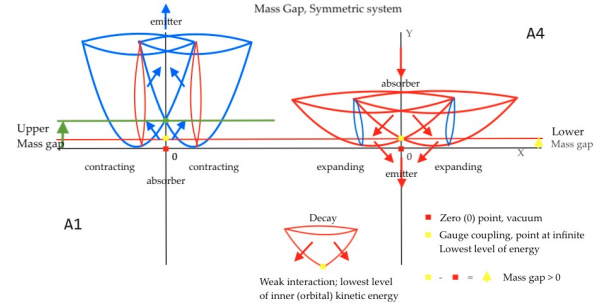


Figure 14: *Mass gap in the symmetric system; the upper gap occurs in the compressed photonic subfield when both intersecting fields contract, while the lower gap occurs in the decompressed subfield when both intersecting fields expand.*

intersect, represented by a red mark. An arrow shows the gap distance between those critical points.

However, in this model, the zero point does not represent a vacuum where neither energy nor mass exists.

When the mass and energy of the vertical subfield reach their weakest level in the concave side of the symmetric system, an equivalent amount of energy and mass arises in the convex side, where the zero point is located, as the result of the double pushing force caused by the displacement of the positive curvature of the expanding intersecting fields.

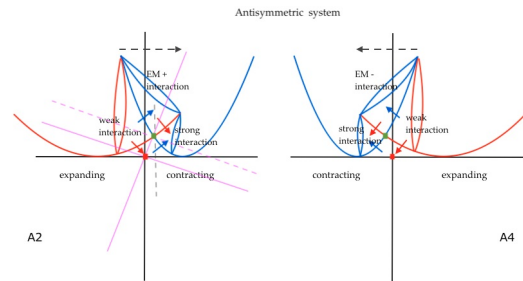


Figure 15: *Mass gap in the antisymmetric system, with the left and right displacements of the point of intersection.*

That mass and energy at this zero point will be considered dark from the point of view of the concave

side of the system.

In the antisymmetric system, the lowest energy level occurs when a transverse subfield experiences a double decompression due to the displacement of the concave curvature of the contracting intersecting field and the displacement of the positive curvature of the expanding intersecting field.

The corresponding double compression is then experienced by its mirror antisymmetric transverse subfield.

7 N=1 Supersymmetric atomic model with dual nucleus of matter and antimatter

The fields model emerges in the context of the heuristic development of a novel supersymmetric quantum model of an atom composed of two intersecting curved fields. These fields share a nucleus consisting of two transverse and two vertical subfields, symbolizing the mirror matter and antimatter of this dual structure. [7]

The following sections describe in a general way the dual nature of the nucleus, treating its symmetric and antisymmetric states as separate systems.

The atomic antisymmetric nucleus's composition varies based on the system's evolutionary stage. It could comprise a proton, a positron, and a antineutrino, or conversely, a neutrino, an electron, and an antiproton.

7.1 Fermionic antisymmetric system: the left intersecting field expands while the right one contracts (A2)

- The right contracting transversal subspace represents a proton.
- The left expanding transversal subspace represents a neutrino.
- The vertical subspace moving toward the right represents a positron.

7.2 Antisymmetric system, the left intersecting field contracts while the right one expands (A4)

- The right contracting proton expands, becoming a right expanding antineutrino.
- The left expanding neutrino contracts, becoming a left-handed contracting antiproton.
- The vertical positron moves toward the left, becoming an electron.

The right transverse proton at moment A2, and the left transverse antiproton at moment A4 are contracting subspaces that experience a dual compression force, originating from the positive curvature of the expanding field and the negative curvature of the encompassing contracting field.

When the expanding field contracts and the contracting field expands, the side of the curvature that determines the force of pressure of the field changes, inverting the dynamics of the forces and the direction of the energies of the system.

It implies that when the right contracting proton of A2 is transformed into a right expanding neutrino at A4, and the left contracting antiproton of A4 is transformed into a left expanding antineutrino at A2, the neutrino and antineutrino will experience a double decompression force.

The topological transformation of the contracting proton in an expanding neutrino (and also of the expanding antineutrino in a contracting antiproton) can be described as an automorphic map, as the transverse subfield maps to itself through its periodic contraction and expansion, preserving the structure of its double curvature even though its size and physical properties change.

The curvature of each transverse subfield is half positive and half negative, and this irregular singularity is preserved during the whole evolution of the antisymmetric and symmetric systems.

The A1 Proton and the A3 antiproton exhibit chiral mirror reflection symmetry at different moments. Being identical with an inverse sign, they can be considered as the homeomorphic map of each other. The same applies to neutrino and antineutrino.

In this context, proton and antiproton, (and neutrino and antineutrino), are considered to be Dirac antiparticles at different times.

When the right intersecting field contracts and the left expands, the top vertical subfield will move rightward, acting as a positron at *A2*.

When the left intersecting field contracts and the right expands, the top vertical subfield will move leftward, acting as a positron at *A4*.

In this dual fields model, the electron and positron are the same automorphic subfield that maps onto itself at different moments, while oscillating left or right in a pendulum-like motion. Being their own antiparticles, they are modeled as Majorana antimatter.

When the vertical subfield moves toward the left acting as electron, it can be considered that it simultaneously exists in a virtual way on the system's right side, in the Aristotelian sense that although it actually does not exist yet it has the potential of becoming existent a moment later, when the right intersecting field contracts and the right field expands.

The existence of an electron and a positron in the same atom, also known as *positronium*, was predicted by Dirac in 1928. However, the positronium was formulated as an exotic atom with no proton in its nucleus.

In a similar way, the coexistence of proton and antiproton in the same atom is currently accepted as an exotic structure called *protonium*, with no electrons or positrons.

In the dual model introduced in this article, matter and antimatter would coexist in any electromagnetic nucleus, being related to each other by means of their chiral mirror reflection symmetry or antisymmetry.

That mirror symmetry would operate at different times in the fermionic antisymmetric system *A2 A4*, or at the same time in the bosonic symmetric system *A1* and *A3*.

Fig. 16 visually represents the limit states of the evolution of the antisymmetric system; however, they do not reflect the moment when the curvatures of the intersecting fields, having an opposite phase, are coincident. At that moment, the top vertical subfield will be passing through the central *Y* axis, which is the reference zero center of symmetry of the system,

and the left and right transverse subfields will exhibit the same curvature and charge, canceling each other.

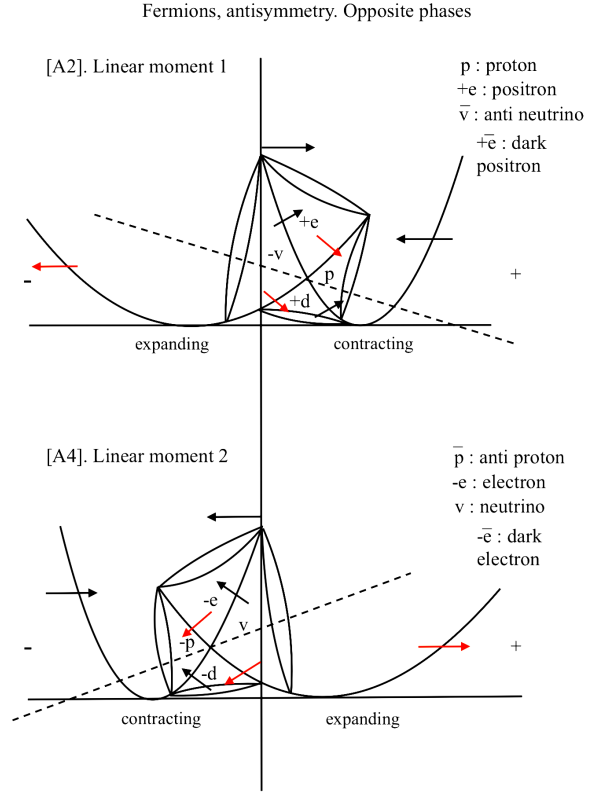


Figure 16: *Fermionic antisymmetric system, lineal evolution.*

This state of neutrality can be considered analogous to a balance scale, where the equal weights on each side cancel each other. It is at this moment that the neutron concept would emerge.

All the subfields in the antisymmetric system are considered fermions with noninteger $\frac{1}{2}$ spin.

Each field is determined by two forces of pressure or decompression, each one being represented by a vector in the fields model. The spin of each subfield is determined by the force of pressure of only one of the intersecting fields, which is represented only by the vector that changes its sign.

These fermionic subfields are governed by the Pauli exclusion principle: when at the right side of the system the transverse subfield contracts acting as a proton, the left transverse subfield cannot be contracting acting as an antiproton, it must be expanding, acting as an antineutrino, as a consequence of the opposite phases that govern the antisymmetric system.

Similarly, when the vertical subfield moves toward the left, acting as an electron, it cannot simultaneously move toward the right, acting as a positron.

Although this unconventional fields model is causal and deterministic, it should be possible to be described in probabilistic terms. In that case, these fermionic fields should follow Fermi-Dirac statistics.

In that same context, considering Schrödinger's cat in the context of the dual atom model, it could be figuratively said that the right alive contracting cat will be the delayed reflection of the left dead expanding cat, and vice versa.

Here, there will not be a single alive and dead cat, but two identical cats with opposite states and positions that are mirror reflections of each other.

Considering there is a unique cat, the simultaneous states of being "alive" and "dead" will be considered "superposed". But in the context of two mirroring cats, their opposite states will be considered entangled with an opposite phase.

7.3 Bosonic symmetric system, when the left and right intersecting fields contract (A1)

- The right and left expanding transversal subspaces represent a right-handed positive and a left-handed negative gluon.
- The top vertical ascending subspace that contracts receiving a double force of compression will be the electromagnetic subfield that emits a photon while pushing upward.
- The inverted bottom vertical subspace at the convex side of the system represents the dark decay of a previous dark antiphoton.

7.4 Symmetric system, when the left and right intersecting fields expand (A3)

- The right and left expanding transverse subspaces may represent $-W$ and $+W$ bosons.
- The top vertical descending subspace will be the electromagnetic subfield losing its previous energy, after having emitted a photon.
- The inverted bottom vertical subspace at the convex side of the system is the dark anti electromagnetic subfield that emits a dark antiphoton.

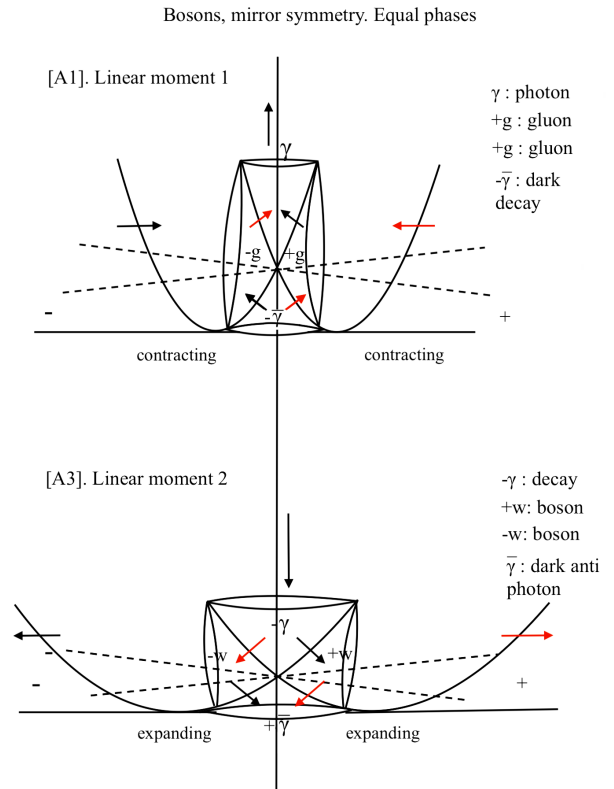


Figure 17: *Bosonic Symmetric system, lineal evolution.*

The identity of the symmetric transversal subfields, labeled before as “W bosons” and “gluons”, requires further clarification in this model.

The left and right transversal subspaces will be mirror symmetric antimatters at the same time. As both experience simultaneously the same state of being contracting (or later being expanding), they are considered to be bosons not ruled by the Pauli exclusion principle. They should then obey the Fermi-Dirac statistics.

However, photons and antiphotons cannot exist simultaneously in the same state. When a photon emerges in the concave side at A1, an antiphoton decays in the convex side. Conversely, when a photon decays at A3, an antiphoton emerges. From here, it is inferred that photon and antiphoton are governed by the Pauli exclusion principle.

This occurs because the curvatures of both the left and right transverse bosonic subfields follow the same phase of variation, which is opposite to the phase of the intersecting fields that encompass them.

7.5 Supersymmetric system

While the Standard Model describes fermions and bosons as fundamentally distinct particles, string theory suggests a connection between them through additional supersymmetric particles that would act as superpartners between them.

In the two intersecting fields model proposed in this article, supersymmetry emerges from the dynamic evolution of intersecting fields. As the system rotates, the same four subfields manifest as fermions or bosons according to the phase difference between the intersecting fields. This phase difference determines the fermionic or bosonic topological states that the subfields manifest at different stages of the rotational evolution.

This rotational framework seamlessly unifies fermionic and bosonic systems through achieving a N=1 supersymmetry.

Although in this model the fermionic partners of the photonic subfield are the electron and positron subfields, it still can be considered as an N=1 correspondence. This is because in this model positron and electron are the same subfield placed at opposite

sides at different moments. In that sense, the $\frac{1}{2}$ top spin of the positron at A2 and the $-\frac{1}{2}$ top spin of the electron at A4 simultaneously converge at A1 in the vertical photonic field.

Supersymmetry could also be explained in terms of the periodic synchronization and desynchronization of the intersecting fields. However, the interpolation caused by the rotational dynamics still seems necessary to introduce a quantization of the space time. When considered independently, the bosonic symmetric and the fermionic antisymmetric systems exhibit the classical linear continuity of longitudinal waves. However, when interpolated within the rotational context, an apparent quantum discontinuity arises, periodically breaking and restoring the symmetry of the rotational system. This smooth quantum phenomenon can be interpreted as a non-linear rotational continuity.

However, the nonlinear interpolation introduced by rotational dynamics seems to play a crucial role in introducing quantization to spacetime. When considered independently, the bosonic symmetric and fermionic antisymmetric systems exhibit a classical continuity. However, within the rotational context, a quantum discontinuity arises, periodically breaking and restoring the symmetry of the system. This smooth quantum phenomenon can be interpreted as a non-linear rotational continuity.

In that context, a single complex function, or a separate complex conjugate function, cannot describe in a non probabilistic way the whole dynamics of this rotational system, as the fermionic states appear as the harmonic partial conjugation of the bosonic states, and vice versa, being the symmetric and the antisymmetric systems essentially intertwined.

In the context of this rotational system, a single complex function or its complex conjugate cannot capture the complete dynamics in a non-probabilistic manner. This is due to the harmonic partial conjugation of fermionic and bosonic states, which renders the symmetric and antisymmetric moments of the system inherently interdependent through time.

The bosonic and the fermionic states can be considered as the intertwined electric and magnetic moments of the system, respectively.

The next diagram represents the signs of the cur-

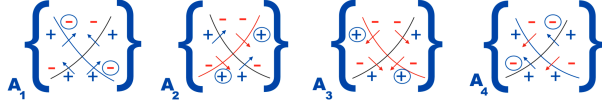


Figure 18: *Vector forces of pressure and signs of curvatures in the nuclear subfields of the interpolated symmetric and antisymmetric systems.*

vatures of the four nuclear subfields. A colored curvature indicates the intersecting field that changes its curvature, contracting or expanding, creating a force of pressure from the side of the curve represented by a circle on one of the curvature's sign.

7.6 Two-genus geometry

The geometry of intersecting fields model can also be described in terms of a two-genus torus or two related tori.

The outer positive and the inner negative curvatures of the torus can be seen as the simultaneous representation of the expanding or contracting moments of the oscillating fields when looking at them from above, in an orthographic projection.

The symmetric and antisymmetric subfields can be described as cobordant [8] subspaces. The vertical subspaces share borders with the left and right transversal subspaces, and they all share borders with the two intersecting spaces.

These borders can be thought of as one-dimensional lines described by the curvatures of the intersecting fields.

From that point of view, the fields model could be related to string theories.

7.7 T-duality and SYZ conjecture

In string theory, the SYZ conjecture [9] states that there exists a special type of Calabi-Yau manifold [10] that is related to another Calabi-Yau manifold by a T-duality [11] transformation.

T-duality is a concept that relates the spaces described by type IIA and type IIB string theories.

In Type IIA string theory, the strings can move freely in the Calabi-Yau transverse space with a larger radius, while in type IIB string theory, the strings are confined to the boundaries of the transverse space of smaller radius. This means that a string theory compactified on a large Calabi-Yau space is equivalent to a string theory compactified on a small Calabi-Yau space.

T-duality relates these two different types of larger and smaller transversal spaces by means of a type of inversion that exchanges the roles of the large and small radii transverse spaces.

In the context of the dual fields model, the Calabi-Yau spaces of smaller or larger radius can be considered equivalent to the transverse contracting or expanding subspaces that are mapped to each other by means of their topological transformation through time, as described before in the antisymmetric rotational system.

The elliptic orbits inside of the transversal subfields, caused by their periodic expansion and contraction, can be visually related to the notion of elliptic fibrations.

The inner orbits of the transverse subfields would be elliptic fibrations.

7.8 Unified interactions. Higgs fields

When one of the subfields contracts, its internal kinetic energy increases, and when it expands, its internal kinetic energy decreases, accelerating or decelerating its inner orbital motion. This increase or decrease in energy is considered here to be a strong or weak interaction, forming the stronger or weaker bond that unites the nuclear subfields and the whole dual system.

The electromagnetic interaction is represented by the vertical subfield, which moves left or right in the antisymmetric magnetic moment or upward or downward in the symmetric electric moment.

The gravitational interaction is represented by the fluctuating curvatures of the intersecting fields, which determine the mass of the subfields acting as a Higgs mechanism.

8 Apendix. Hodge cycles.

The supersymmetric dual nucleus model previously described may also be related to Hodge theory.

In that context, the transversal subfields of $A1$ would represent a projective algebraic variety, related to another projective algebraic variety represented by the transversal subfields of $A3$. Both varieties are linked in a cohomology group by an algebraic cycle.

The same can be said with respect to the cohomology group of the transversal subfields of $A2$ and $A3$, and their respective algebraic cycle.

The algebraic cycle within each cohomology group would establish a cyclic mapping between its respective varieties, making each variety the inverse of the other.

By combining the algebraic cycles of both groups, it can be constructed a type of super-cycle that relates in a smooth way all the associated algebraic varieties and the distinct symmetries they carry.

Following the rotational context described earlier, the algebraic cycles would be smoothly combined or interpolated through 90-degrees rotations of the complex plane, giving rise to the cycle $A1 + A2 + A3 + A4 + A1 + \dots$, where each algebraic cycle is the harmonic partial conjugation of the other.

The cycle formed by combining in this way those algebraic varieties would constitute a Hodge cycle, which links the symmetry of each cohomology group in a dual Hodge structure.

In this sense, the mechanics of the dual atomic model can represent a specific case of the Hodge conjecture, which states that "for particularly nice types of spaces called projective algebraic varieties, the pieces called Hodge cycles are actually rational linear combinations of geometric pieces called algebraic cycles". [12]

However, as was previously seen, the combination of systems in the rotational fields model, although smooth, cannot be considered linear. This suggests that the supersymmetric dual nucleus model may point towards a broader interpretation of the Hodge conjecture beyond the classical linearity.

Keywords: Jacobian, Gorenstein, supersymmetry, mirror symmetry, Tomita-Takesaki, modularity, mass

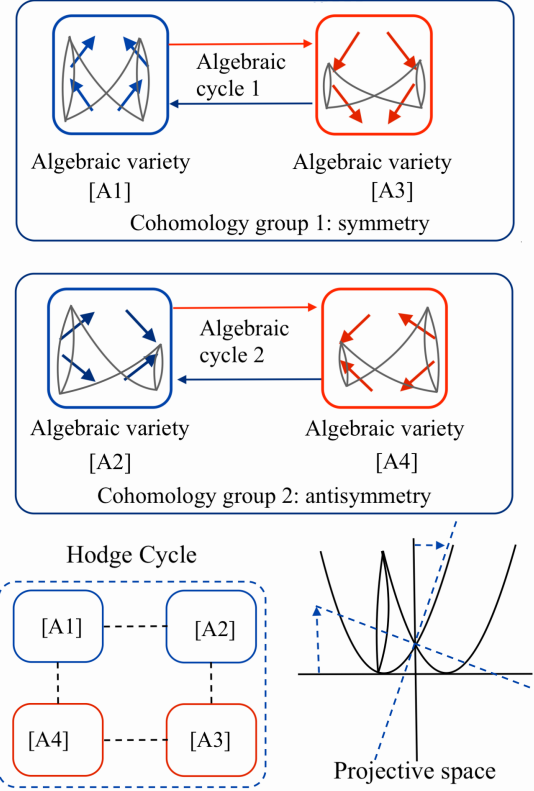


Figure 19: *Algebraic varieties and Hodge cycle. The left- and right-transverse subfields of the dual nucleus can be considered as algebraic varieties of projective spaces (the coordinates of the intersecting fields are projected onto the diagonal to constitute the extradimensional coordinates of the transverse subspaces). An algebraic variety and its inverse form a cohomology group of symmetry and are linked by a linear cycle of mapping or transformations that connect them through time. The nonlinear combination of the algebraic varieties of two groups of cohomology gives rise to a supergroup of cohomology whose cycle would represent a quantized Hodge cycle.*

gap, reflection positivity, quantum field theory, dual nucleus, antimatter, t-duality, SYZ conjecture, elliptic fibration, Calabi-yau, Higgs field, Hodge cycles.

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