Correction and Addendum for "Consistent Optical and Electrical Noise Figure"

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Abstract—**The minimum noise figure of an electrical amplifier is F_e=1. E. Desurvire's traditional optical noise F_{pnf} of an optical amplifier has the minimum value F_{pnf}=2. If F_{pnf} is a noise figure then power, gain and F_e need to be redefined. F_{pnf} is in conflict with physics and F_e. The correct optical noise figure $F_{o,IQ}$, observable in coherent I&Q receivers, has the minimum $F_{o,IQ}$=1 and is compatible with F_e. In the derivation of the consistent unified noise figure F_{IQ} for all frequencies, from F_e and $F_{o,IQ}$, thermal noise energy is needed. Its usual simplified expression kT is now replaced by Nyquist's correct result. This holds also in a unified homodyne noise figure F_I, against which H. Haus' unified noise figure F_{fas} is discussed.**

Index Terms—**Noise figure, Optical amplifiers, Optical fiber communication**

I. INTRODUCTION

EQUATION, figure and reference numbering of the original paper [17] is continued here. The correct optical I&Q noise figure $F_{o,IQ}$ as the 1:1 equivalent of the electrical noise figure F_e has been derived in [17]. Nothing needs to be changed there regarding optical noise figure (NF). The same is true for the optical homodyne NF $F_{o,I}$.
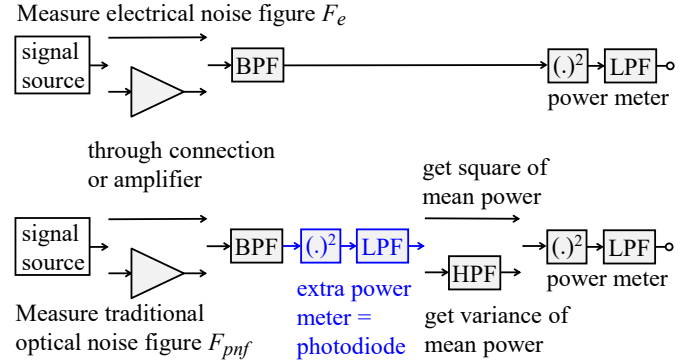
This paper adds to the description of the unified NF F_{IQ} for all frequencies. Thermal noise energy must be corrected from its usual simplified expression kT to Nyquist's accurate expression, in order to "avoid the UV catastrophe" (Section III). For sake of completeness, the unified NF F_{fas} [6] of pioneer H. Haus is compared against the corrected unified homodyne NF F_I (Section IV). But we start with a discussion:

II. DISCUSSION OF NOISE FIGURE ISSUES

A caution about lab jargon: When NF is given as a factor and not in dB then in reality the noise factor (= SNR quotient) is meant. "noise figure" = (10 dB)·log10("noise factor").

The reason for deriving $F_{o,IQ}$ [17] is illustrated in Fig. 5. The insertion of a photodiode into the signal path [3] as a kind of extra power meter defies NF definition (linear channel,

2 available quadratures, minimum amplifier NF equals 1) [18]. The resulting traditional optical NF of E. Desurvire [3], called F_{pnf} in [6], is in conflict with ~150 years of science:

Measure electrical noise figure F_e

Fig. 5: Measurement of electrical and of traditional optical noise figure. LPF/BPF/HPF = lowpass/bandpass/highpass filter

High-frequency engineers would reject the idea of inserting an extra squaring power meter into the linear signal path. And so should optical engineers. But the photodiode needed for F_{pnf} definition acts as a squarer and power meter.

Subsequently the needed "power"="P"$\sim I^2 \sim P^2$, i.e. electrical power of a photocurrent I flowing through a load resistor, is proportional to the square of the optical power P. It holds $P \sim |\mathbf{E}|^2$ where $\mathbf{E}$ is the optical field. This means "P"$\sim |\mathbf{E}|^4$!

By definition,

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{signal,in}P_{noise,out}}{P_{noise,in}P_{signal,out}} = \frac{G_{noise}}{G_{signal}} \qquad (65)$$

where "out" means "with device" and "in" means "without device". If we set $F = F_{pnf}$ and derive this NF using "P"$\sim I^2$ then we find that any amplifier with gain $G \equiv G_{signal}$ has a supposed gain $G_{signal} = G^2$!

Let's see where F_{pnf} brings us. NF definition must not depend on detector type or frequency f. The photodiode can be

replaced by a thermal power detector (bolometer). Now "P" $\sim (\Delta\vartheta)^2$ is proportional to the square of the detected temperature difference $\Delta\vartheta$. Thermal power detectors can be built at all frequencies. With appropriate preamplifiers they can detect even smallest powers. We decrease f until we arrive in the electrical domain. Here we find "P" $\sim |U|^4$ where U is the input voltage, again $G \equiv G_{signal} = G^2$, and $F_{pnf} = F_e^2$! We have two competing NF for the same amplifier at the same f. F_{pnf} claims F_e to be wrong, and vice versa.

All this is direct consequence of calling F_{pnf} a NF. So, either F_{pnf} is a NF, or basic physics $P \sim |U|^2$ and G and F_e are correct. We know the latter holds.

All the same, F_{pnf} [3] has great historic merits in the development of optical communication.

$F_{pnf} = 2$ is found for an ideal optical amplifier. This irritates because the sensitivity of an ideal optical receiver with 2 available quadratures is not degraded by an ideal optical preamplifier.

While H. Haus [6] exposed F_{pnf} to violate NF definition, the proposed solutions F_{fas} (very rare case, optical homodyne NF for 1 quadrature, other than F_e for 2 quadratures), F_{ase} (no NF because it is not the SNR degradation factor in any optical receiver) also have a minimum value of 2 for an ideal amplifier, other than F_e. Usually the value 2 (instead of 1) is explained by claiming that optical amplifiers be special.

But standard optical amplifiers such as EDFAs are not special. They amplify 2 quadratures and add Gaussian amplitude (field) noise in the 2 quadratures [17], like standard electrical amplifiers. Also not special is optical detection noise (photon/particle aspect manifests) [19] as opposed to electrical source noise (thermal origin). Special in the light of detection noise are true homodyne detection and direct detection because these keep only 1 degree-of-freedom or quadrature and suppress the other. For an ideal optical amplifier one heuristically finds [18] the minimum

$$F_{opt,min} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}} \quad (66)$$

where F_{opt} is any optical NF. Even though optical direct detection is nonlinear, also F_{pnf} obeys (66) with 1 available quadrature in receiver. F_{ase} is not covered by (66), given that F_{ase} is not the NF in any known optical receiver. Clearly it makes sense to have the same number of available quadratures in amplifier and receiver, and to choose this number equal to 2 like in the electrical case F_e. This is confirmed by the correct NF $F_{o,IQ} \geq 1$ of a standard optical amplifier [17].

Instead of claiming $F_{o,min} = 2$ to be normal (F_{pnf},

$F_{fas} = F_{o,I}$, also F_{ase}, all with standard optical amplifier) one could with the same right claim $F_{opt,min} = 1/2$ to be normal (degenerate parametric optical preamplifier with $F_{o,IQ} = 1/2$ blocks one quadrature and increases I&Q receiver sensitivity to that of a true homodyne receiver). Both is mathematically correct, but none should be considered as the normal case.

I have been criticized for deriving $F_{o,IQ}$, $F_{o,I}$ [17] in semiclassical description. There, a photocurrent I has a one-sided noise power spectral density (PSD) $2eI$ due to a Poisson distribution of photoelectrons. Indeed one can alternately assume zero point fluctuations with energy $hf/2$ per mode. They give rise to the same noise PSD $2eI$ in a photocurrent I. Fig. 6 is similar to Fig. 1, but the splitters are replaced by 2×2 couplers and each of the 4 inputs gets zero point fluctuations. Interferences of zero point fluctuations from the LO coupler inputs with the LO signal eventually cancel upon photocurrent subtraction. Interferences of zero point fluctuations with the received signal are negligible since the LO is strong. Interferences of zero point fluctuations in $\mathbf{E}_{RX1,2}$ from the signal coupler inputs with the LO signal add upon photocurrent subtraction. Using zero point fluctuations, exactly the known $F_{o,IQ}$ and $F_{o,I}$ are obtained in the end. Likewise, F_{pnf} can be calculated with either semiclassical shot noise or zero point fluctuations.
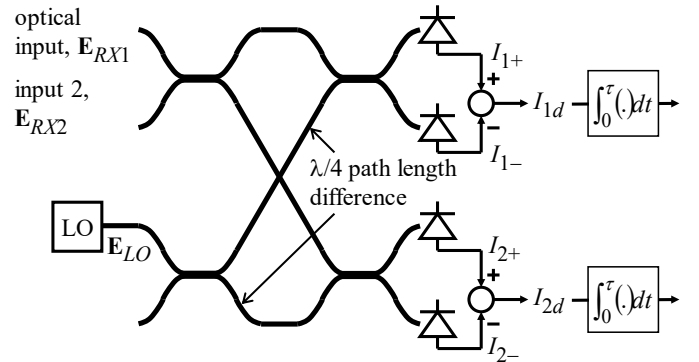

Fig. 6: Coherent I&Q receiver with polarization matching

III. CORRECT THERMAL NOISE AT ALL FREQUENCIES

In the derivation of a unified NF F_{fas} [6] the mean value of detectable thermal photons per mode was given (in other, equivalent nomenclature) as $\langle n_\vartheta \rangle = kT/(hf)$; see eqn. (13) of [6]. Mean thermal noise energy or thermal PSD is hence $\langle n_\vartheta \rangle hf = kT$. In [17] I have adopted this and have derived the unified I&Q NF F_{IQ}, using the optical I&Q NF $F_{o,IQ}$. But in unified noise figures the term kT needs to be corrected at high frequencies! Total thermal power is finite. It is expedient to write

$$k'T = \frac{hf}{e^{hf/(kT)} - 1} \qquad \text{thermal power spectral density.} \quad (67)$$

The right hand side is eqn. (7) in [20], by Nyquist. Only for $hf \ll kT$ it approaches kT. The left hand side $k'T$ is defined such that where k was written in the derivation of F_{IQ} this is now <u>replaced</u> by k'. k' is a frequency- and temperature-dependent function which approaches the Boltzmann constant k in the case $hf \ll kT$, and 0 in the case $hf \gg kT$.

In agreement with $k'T$ (67), total noise energy or spectral density per mode is $hf/2 + k'T$. Here $hf/2$ stands for zero point fluctuations which we can alternately express by shot noise.

Now consider electrical NF measurement. Since a real power detector is thermally noisy the SNR degradation factor underestimates the true F_e. In particular, a decent amplifier with gain G in front of a very, very noisy detector will even improve the SNR. To get rid of thermal detector noise one puts the source at two different temperatures and measures noises with the power detector. Linear extrapolation of the measured noises to $T = 0$ K yields the own thermal noise of the power detector. It is subtracted in all NF calculations. This way the calculated SNRs and NF become higher. In practice, $T = 0$ K cannot be reached, and maybe the power detector wouldn't even work at $T = 0$ K. But this does not matter. The NF is simply the quotient of SNRs that one would achieve if the power detector had no thermal noise.

The same principle must be applied for the unified F_{IQ}. In practice, thermal noise occurs also at the unused input 2 (Fig. 6). It must not enter into the NF equations. To this purpose we assume an ideal cooled absorber with temperature 0 K at input 2. Thermal noise in the electronics behind the photodiodes is likewise eliminated because we have assumed $P_{LO} \to \infty$. A few control measurements allow isolating and removing these thermal noises in a practical receiver.

There is yet another idealization: In the responsivity $R = \dfrac{\eta e}{hf}$ the efficiency η has been set as $\eta = 1$. This maximizes the SNR. The same $\eta = 1$ is also used in the traditional F_{pnf}.

The <u>corrected</u> equations (k' instead of k) are:

$$SNR_{IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2}$$

$$= \frac{R^2 GP_S P_{LO}}{R^2 P_{LO} GF_e k'TB_o/2 + R^2 P_{LO}\, \tilde{\mu} GhfB_o/2 + eRP_{LO}B_e} \quad (47)$$

$$= \frac{GP_S}{GF_e k'TB_o/2 + \tilde{\mu} GhfB_o/2 + hfB_e}$$

$$= \frac{P_S\tau}{F_e k'T/2 + F_{o,IQ} hf/2} = \frac{P_S\tau}{k'(T+T_{ex})/2 + (\tilde{\mu}+1/G)hf/2}$$

$$SNR_{IQ,in} = \frac{P_S\tau}{k'T/2 + hf/2}, \quad (48)$$

$$F_{IQ} = \frac{SNR_{IQ,in}}{SNR_{IQ,out}} = \frac{F_e k'T + F_{o,IQ} hf}{k'T + hf}$$

$$= \frac{k'(T+T_{ex}) + (\tilde{\mu}+1/G)hf}{k'T + hf} \qquad (A = \frac{k'T}{k'T + hf}). \quad (49)$$

$$= A + (1-A)/G + (AT_{ex}/T + (1-A)\tilde{\mu})$$

The change k' instead of k affects also Table I, Figs. 2, 3 and other places in [17]. The same change is needed in [18]. We recognize thermal source noise $k'T$, thermal noise $k'T_{ex}$ added in amplifier, spontaneous emission field noise $\tilde{\mu} hf$ added in amplifier and shot noise hf/G in detector, all input-referred and per mode. For an amplifier it is not important to know the individual contributions of F_e, $F_{o,IQ}$; only the resulting F_{IQ} counts.

The crossover condition $hf = k'T$ of equally strong thermal and quantum noises yields $hf = kT \ln 2$. This requires $f = 194$ THz / 28 THz / 4.3 THz / 1.1 THz / 58 GHz at $T = 13400$ K / 1940 K / 300 K / 77 K / 4 K, respectively. In [21], a 66 GHz electronic circuit operates at 4 K. Quantum noise plays a role here. Cryo and space electronics in the mm wave range and possible future THz applications need F_{IQ}. The same would hold for an extremely hot attenuator or amplifier at the CO_2 laser frequency 28 THz.

IV. UNIFIED HOMODYNE NOISE FIGURE

This is investigated in order to complete the picture in the context of F_{fas} in [6]. According to eqn. (35) of [17], the signal power P_S in a true homodyne receiver appears multiplied by $4R^2 GP_{LO}$. The same holds for the in-phase part $F_e k'TB_o/2$ of the total received thermal noise power $F_e k'TB_o$. We add the product to the denominator of (35) and obtain

$$SNR_{I,out}$$

$$= \frac{4R^2 GP_S P_{LO}}{4R^2 P_{LO} GF_e k'TB_o/2 + 4R^2 P_{LO}\,\tilde{\mu} GhfB_o/2 + 2eRP_{LO}B_e}$$

$$= \frac{2GP_S}{2GF_e k'TB_o/2 + 2\tilde{\mu} GhfB_o/2 + hfB_e} \quad (68)$$

$$= \frac{2P_S\tau}{F_e k'T + F_{o,I} hf/2} = \frac{2P_S\tau}{k'(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}$$

In the absence of noise and gain this becomes

$$SNR_{I,in} = \frac{2P_S\tau}{k'T + hf/2}. \quad (69)$$

We get the homodyne / in-phase / single-quadrature NF

$$F_I = \frac{SNR_{I,in}}{SNR_{I,out}} = \frac{F_e k'T + F_{o,I}\, hf/2}{k'T + hf/2}$$

$$= \frac{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}{k'T + hf/2} \qquad (A_I = \frac{k'T}{k'T + hf/2})$$

$$= A_I + (1 - A_I)/G + (A_I\, T_{ex}/T + (1 - A_I)2\tilde{\mu}). \qquad (70)$$

Note that in the electrical domain, single-quadrature or homodyne analysis simply means that the other quadrature is suppressed. This can be done by downconversion to baseband in a multiplier/mixer, or by degenerate parametric amplification. The electrical homodyne NF $F_{e,I}$ equals the electrical I&Q NF, $F_{e,I} = F_e \equiv F_{e,IQ}$ because there is (thermal) source noise. In (68)-(70) one could write $F_{e,I}$ instead of F_e.

F_I will probably not be needed in the electrical domain (mm waves at low temperatures) because single-quadrature electric amplifiers have no noise advantage over standard electrical amplifiers. F_I could be applied for an extremely hot device and a homodyne receiver at the CO_2 laser frequency.

For $hf \gg kT$, H. Haus' unified NF F_{fas}, eqn. (18) in [6], becomes $F_{o,I}$. This means F_{fas} is a homodyne NF at optical frequencies. F_{fas} is very similar to F_I. Differences are:

- F_{fas} contains kT in $\langle n_g \rangle$ instead of the correct $k'T$.

- No clear number of quadratures is defined for F_{fas}. Since F_{fas} should generalize the familiar F_e one is left to assume that in F_{fas} there is 1 quadrature at optical f, 1...2 quadratures at intermediate/thermal f and the usual 2 quadratures at electrical f. Such transition is of course not possible.

- In F_{fas} it is defined $T_{ex} = 0$ (which is allowed), and all added thermal noise is assumed to be contained in a sufficiently large spontaneous emission factor n_{sp}. For a pure attenuator one must guess and set $n_{sp} = -k'T/(hf)$. Otherwise the needed $F_{fas} = 1/G$ is not reached. For comparison, $F_I = 1/G$ is easily derived from the known $F_{e,I} = F_e = F_{o,I} = 1/G$ and $n_{sp} = 0$.

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