Correction and Addendum for “Consistent Optical and Electrical Noise Figure”

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Optical and Unified Noise Figure, and Homodyne Noise Figure

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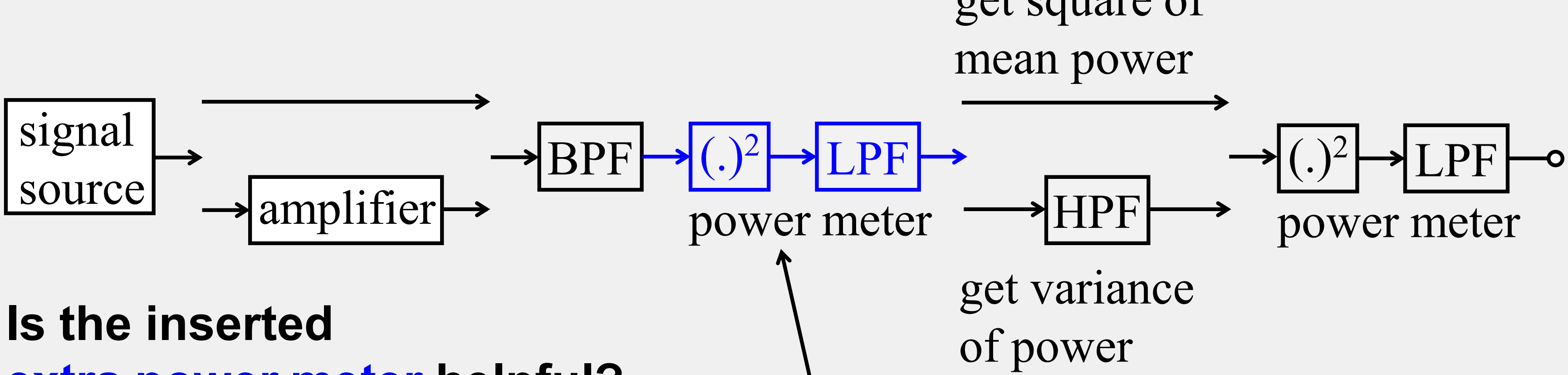
Overview

- Motivation
- Noise figures in linear/coherent optical receivers
  - In-phase and quadrature noise figure = the noise figure
  - In-phase (homodyne) noise figure = special case
- Comparison of noise figures
- Consistent unified noise figure
- Summary

How to determine noise and gain properties of amplifier

Standard electrical measurement



Is the inserted
extra power meter helpful?

Probably not. ;-) Now this is a photodiode
and we are talking about optical signals!

$G = e^{(a-b)t} = e^{(a-b)z/v_g}$ gain

$\mu = n_{sp}(G-1)$ mean number of detectable output noise photons per mode

$n_{sp} = \dfrac{a}{a-b}$ spontaneous emission factor

$P_{n,out}\tau = \mu hf = G\tilde{\mu}hf$ mean output noise energy per mode

(traditional optical NF)

F_e, gain, loss, power must be redefined if F_{pnf} is valid NF!

$$F=\frac{SNR_{in}}{SNR_{out}}=\frac{P_{s,in}P_{n,out}}{P_{n,in}P_{s,out}}=\frac{\text{noise gain}}{\boxed{\text{signal gain}}}$$

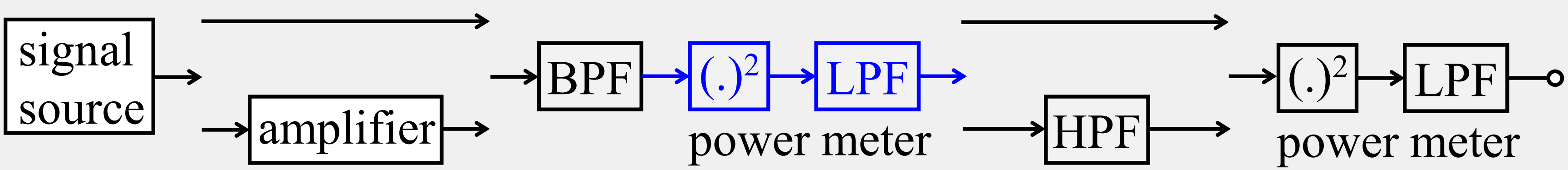
Science is **systematic** and **exact** and **does not tolerate contradictions**! Unit definitions **must not** depend on measurement method or f! F_{pnf} implies:

$$\frac{G^2\langle n\rangle^2}{\langle n\rangle^2}=G^2$$ **is** the signal „gain“! ⇔ No more (optical) dB are allowed; „gain“ must be given in „electrical“ dB! ⇔ Fiber loss @ 1550 nm is no longer 0.2 dB/km; „loss“ must be stated as 0.4 dB/km! ↯ ⇔

Thermal power meter can replace photodiode and allows going to low f.

⇔ Any electrical or optical amplifier with 20 dB gain has 40 dB „gain“! ↯ ⇔ $F_{pnf}=F_e^2$ ↯ ⇔ All „powers“ (optical, electrical, thermal, mechanical) must be ~ squared powers, since all powers can be converted into thermal power and compared! ↯ ⇔ „power“ is not linear! ↯ ⇔ work = sqrt(„power“)·time ↯

$$SNR_{pnf,in}=\frac{\langle n\rangle^2}{\langle n\rangle}\qquad SNR_{pnf,out}=\frac{G^2\langle n\rangle^2}{G\langle n\rangle+n_{sp}\left(2(G-1)G\langle n\rangle+\dots\right)}$$



Problem introduction

- **Electrical** noise figure (NF) is standardized since many decades.
- Traditional **optical** noise figure F_{pnf} was defined in 1990ies, for optical direct detection receivers (DD RX). Problematic aspects, in **conflict** with electrical NF:
  - Optical signals have in-phase and quadrature components, like electrical signals. But an optical DD RX **suppresses** phase information.
  - „Power“ in signal-to-noise (SNR) ratio calculation is ~ square of photocurrent in optical DD RX. Photocurrent is ~ optical power ~ **square** of field amplitude. SNR „power“ is ~ 4th power of field amplitude ~ **square** of power. ϟ
    Conflict with ~150 years of science: $P = U^2/R$, not $P \sim U^4$.

  $\Rightarrow$ NF = 2 for ideal optical amplifier, whereas NF = 1 for ideal electrical amplifier.
  - **Noise happens on a field basis. Looking at the power is insufficient!** ϟ
  - Ideal DD RX for intensity modulation without / with ideal optical amplifier needs 10 / 38 photoelectrons/bit for bit error ratio = 10^{-9}. Ideal DD RX for differential phase shift keying: 20 / 20 photoelectrons/bit. **Where is NF = 2?**
  - Optical: **Non**linear DD RX; **non**-Gaussian noise; amplifier NF depends on power and bandwidths. <u>Electrical: Linear RX; Gaussian noise; constant NF.</u>
  - Unification of all prior optical NF with electrical NF is inconsistent, contradictory.

Fields in coherent optical I&Q receiver

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\mathbf{e}_1 e^{j\omega t}$$

$P =: |\mathbf{E}|^2$ Power (for simplicity)

$I = RP = e/(hf)\cdot P$ Photocurrent

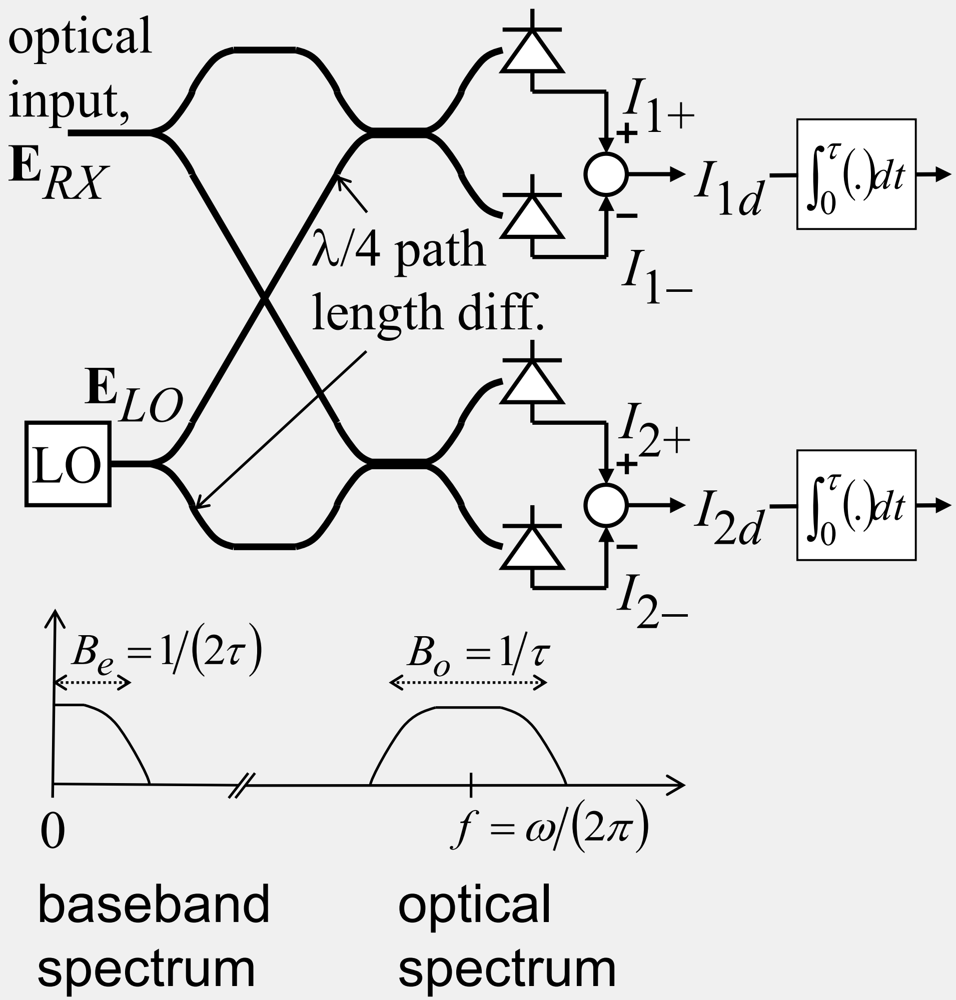
v_1, v_2 independent zero-mean Gaussian

$$\langle v_1^2 \rangle = \langle v_2^2 \rangle = \sigma^2 = 1$$

$\mathbf{e}_1$ normalized field (polarization) vector

Optical signal is linearly downconverted to baseband. Local oscillator (LO) is a strong unmodulated laser with (essentially) the same frequency as the received signal.

2 available quadratures



Baseband I&Q receiver is not mandatory!
Heterodyne receiver with image rejection filter gives the same results!

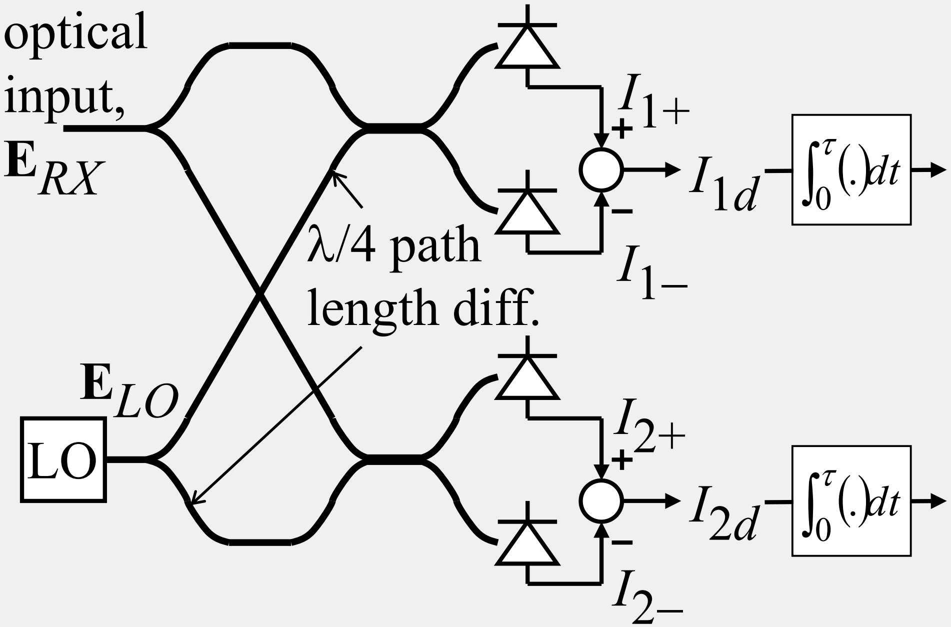
Photocurrents in coherent optical I&Q receiver ...

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\mathbf{e}_1 e^{j\omega t}$$

$$P =: |\mathbf{E}|^2 \qquad I = RP = e/(hf)\cdot P$$

(In practice, optical frequencies of signal and unmodulated local oscillator may differ a bit, causing the complex plane of I_{1d} and I_{2d} to rotate at the difference frequency.)



4 detected photocurrents

$$I_{1\pm} = R\left|\pm\mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + P_{LO}\right)$$

$$I_{2\pm} = R\left|\pm\mathbf{E}_{RX}/2 + j\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + P_{LO}\right)$$

...and their differences and sums

$$I_{1d} = I_{1+} - I_{1-} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

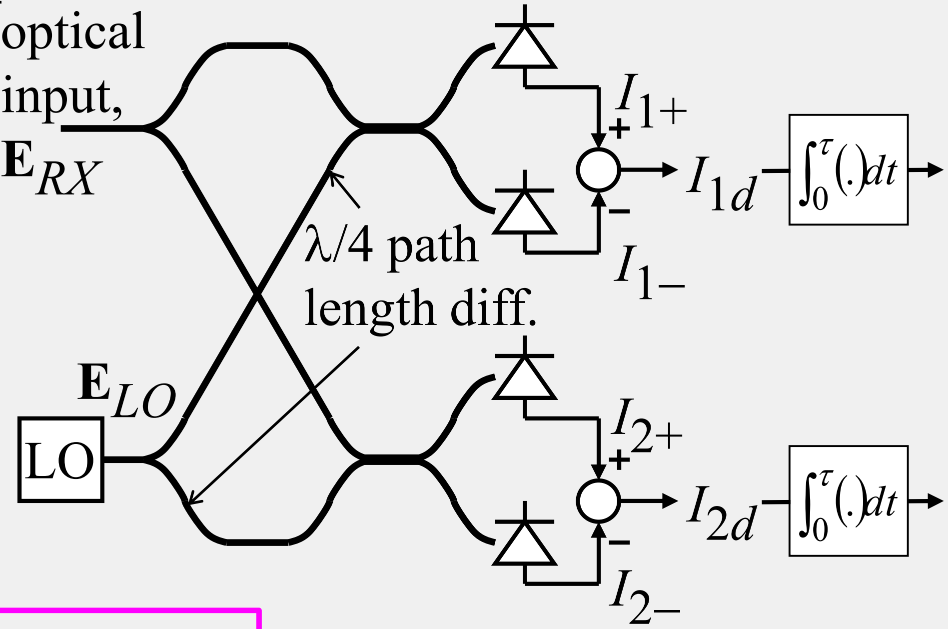
$$I_{2d} = I_{2+} - I_{2-} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = I_{1+} + I_{1-} = RP_{LO}/2$$

$$I_{2s} = I_{2+} + I_{2-} = RP_{LO}/2$$

Differences and sums of photocurrents

Neglect for $P_{LO} \to \infty$



Cancel in subtraction

4 detected photocurrents

$$I_{1\pm} = R\left|\pm \mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(\cancel{G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right)} \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + \cancel{P_{LO}}\right)$$

$$I_{2\pm} = R\left|\pm \mathbf{E}_{RX}/2 + j\,\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(\cancel{G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right)} \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + \cancel{P_{LO}}\right)$$

SNR in coherent optical I&Q receiver

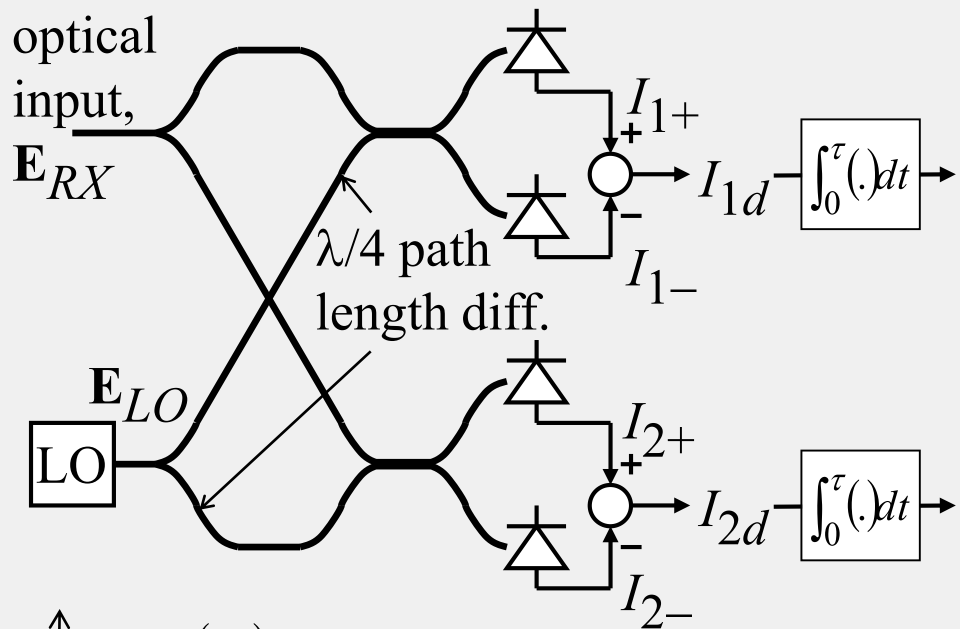
$$I_{1d} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

$$I_{2d} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = RP_{LO}/2$$

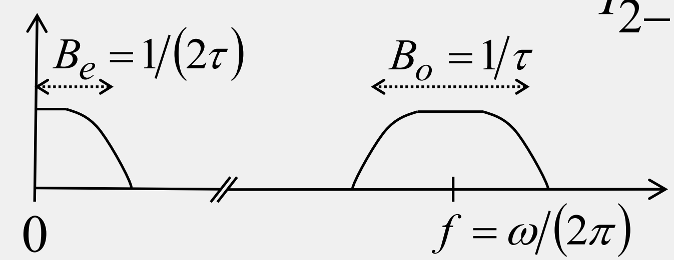
$$I_{2s} = RP_{LO}/2$$

Pure Gaussian PDFs of interference + field noises!



Shot noise PSD: $2eI_{1s}$, $2eI_{2s}$

Optical bandwidth: $B_o = 2B_e = 1/\tau$



Equivalent amplifier input noise PSD per mode: $\tilde{\mu}hf = P_n/B_o$

For SNR calculation take <u>either</u> noise in 1 mode <u>or</u> (like I do it) in 1 quadrature! (Factor 2 cancels in NF calculation.)

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO}\, G\tilde{\mu}hfB_o/2 + eRP_{LO}B_e} = \frac{P_S\tau}{\left(\tilde{\mu} + 1/G\right)hf/2}$$

Optical I&Q noise figure (or heterodyne with image rej.)

$$SNR_{o,IQ,out} = \frac{P_S \tau}{(\tilde{\mu} + 1/G) hf/2}$$

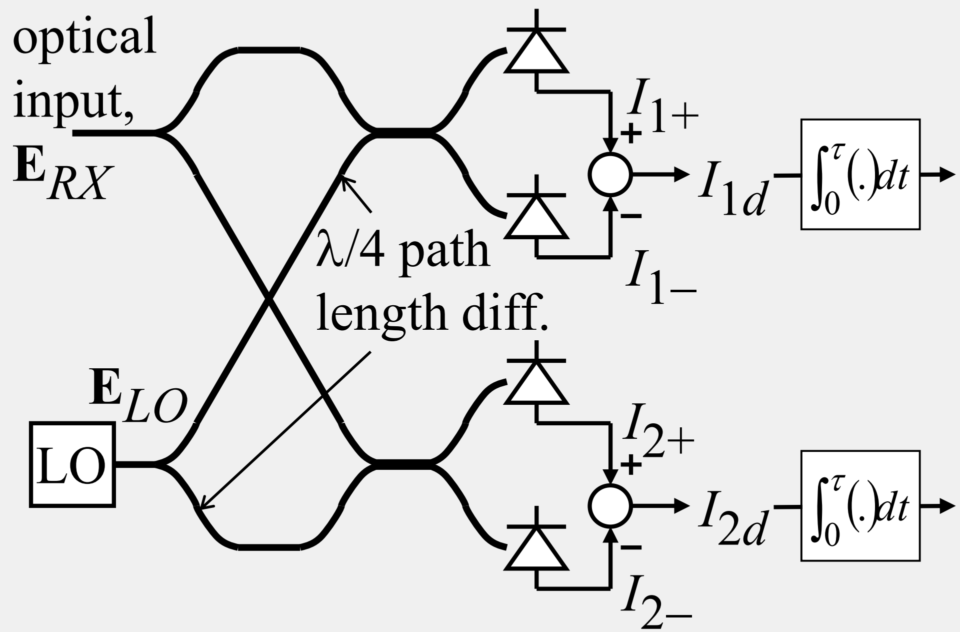
No amplifier, $G = 1, \quad \tilde{\mu} = 0$:

$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2}$$

$$\frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} =$$

$$\boxed{F_{o,IQ} = \tilde{\mu} + 1/G = n_{sp}(1 - 1/G) + 1/G}$$

$$= 1 + (n_{sp} - 1)(1 - 1/G) \geq 1$$



$F_{o,IQ}$ obeys the usual electrical NF definition, is SNR degradation factor; powers ~ squares of amplitudes; 2 available quadratures; linearity; ideal NF = 1; pure Gaussian noise!

$$F_{o,IQ} = (F_{pnf} - 1/G)/2 + 1/G$$

$$F_{pnf} = 2(F_{o,IQ} - 1/G) + 1/G$$

$$\boxed{F_{o,IQ} \approx F_{pnf}/2}$$

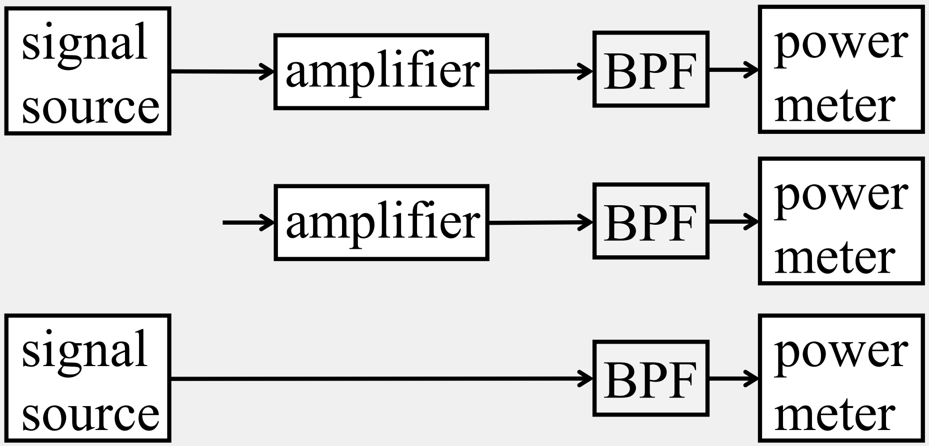
Conversion formulas

≈ 3 dB difference for large G

Measure optical I&Q noise figure with power meter

Optical amplifier must be loaded with extra optical signal power at other times/frequencies/polarization in order to keep G, $\tilde{\mu}$ constant.

Usually there are $p=2$ polarization modes. $p=1$ requires inserted polarizer.



$$P_1 = Gp\tilde{\mu}hfB_o + GP_S$$

$$P_2 = Gp\tilde{\mu}hfB_o$$

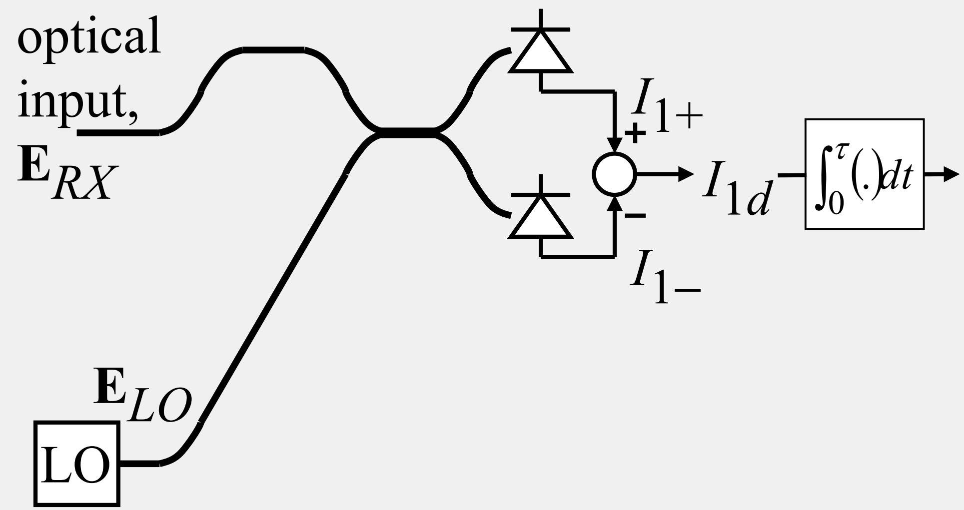
$$P_S$$

$$G = \frac{P_1 - P_2}{P_S} \qquad \tilde{\mu} = \frac{P_2}{pGhfB_o} \qquad \boxed{F_{o,IQ} = \tilde{\mu} + 1/G}$$

$F_{o,IQ}$ and all other optical NF can be determined from simple optical power measurements.

Optical I noise figure (true homodyne; special case)

In such cases, phase locking is required between signal and LO or detector!
No power splitting $\Rightarrow$ In equations multiply each of $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ by 2.



$$\boxed{F_{o,I} = 2\tilde{\mu} + 1/G} \qquad (= F_{fas} = F_{pnf})$$

$$= 1 + (2n_{sp} - 1)(1 - 1/G) \geq 1$$

$F_{o,I}$ is similar to $F_{o,IQ}$ and F_e, but only 1 quadrature is available.
Lowest $F_{o,I} \to 1$ for $G \to 1$. Ideal $F_{o,I} = 2$ at $G \to \infty$. Why?

<u>Optical amplifier is not special! RX is special: 1 quadrature & detection noise!</u>

Without optical amplifier, true homodyne RX is twice as sensitive as I&Q RX because P_{RX} is not split. But with optical amplifier having $G \to \propto$, output power splitting like in the I&Q RX cannot have an SNR effect. So, behind the amplifier the homodyne RX "must" have the worse sensitivity of the I&Q RX. Amplifier halves homodyne SNR!

Phase-sensitive degenerate parametric optical amplifier passes only 1 quadrature and has ideal $F_{o,I} = 1$ and $F_{o,IQ} = 1/2$ (converts I&Q into more sensitive homodyne).

Zero point fluctuations can explain/replace shot noise

Shot noise can be derived either way (but only 1 way at a time, not 2 ways at a time):

- Semiclassical theory: Poisson distribution of photoelectrons has one-sided photocurrent power spectral density (PSD) $\boxed{2eI}$.
- Zero point fluctuations interfere with signal and cause shot noise PSD $\boxed{2eI}$.

Let us define field such that power is $P := |\mathbf{E}|^2$. Observation time is $\tau = 1/B_o$.

Zero point fluctuations have mean energy $W = P\tau$ equal to $hf/2$ per mode:

$$\mathbf{E}_0 = (u_1 + ju_2)\underline{\mathbf{e}}_1 e^{j\omega t} \qquad \sigma_{u1}^2 = \sigma_{u2}^2 = hf/(4\tau) \qquad |\underline{\mathbf{e}}_1| = 1$$

Signal field: $\mathbf{E}_S = \sqrt{P_S}\,\mathbf{e}_1 e^{j\omega t}$ Total field: $\mathbf{E}_S + \mathbf{E}_0$

Expected number of photoelectrons: $n_{S+0} = |\mathbf{E}_S + \mathbf{E}_0|^2 \tau/(hf)$

$$= \left(|\mathbf{E}_S|^2 + 2\,\mathrm{Re}\left(\mathbf{E}_0^+\mathbf{E}_S\right) + |\mathbf{E}_0|^2\right)\tau/(hf) \approx \left(P_S + 2u_1\sqrt{P_S}\right)\tau/(hf)$$

Mean: $\langle n_{S+0}\rangle = \dfrac{P_S\tau}{hf}$ Variance: $\sigma_{nS+0}^2 = \dfrac{hf}{4\tau} P_S \dfrac{2^2\tau^2}{h^2 f^2} = \dfrac{P_S\tau}{hf} = \langle n_{S+0}\rangle$

$$I = RP = \boxed{\frac{e}{hf}P} \qquad \langle I_{S+0}\rangle = \langle n_{S+0}\rangle \frac{e}{\tau} \qquad \sigma_{IS+0}^2 = \sigma_{nS+0}^2 \frac{e^2}{\tau^2} = \boxed{2e \cdot \boxed{\frac{e}{hf}P_S}} \cdot \boxed{\frac{1}{2\tau}}$$

one-sided electrical bandwidth

I&Q NF derived with zero point fluctuations (1)

$$\mathbf{E}_{RX1} = \left[\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_{11} + ju_{12})\right]\mathbf{e}_1 e^{j\omega t}$$

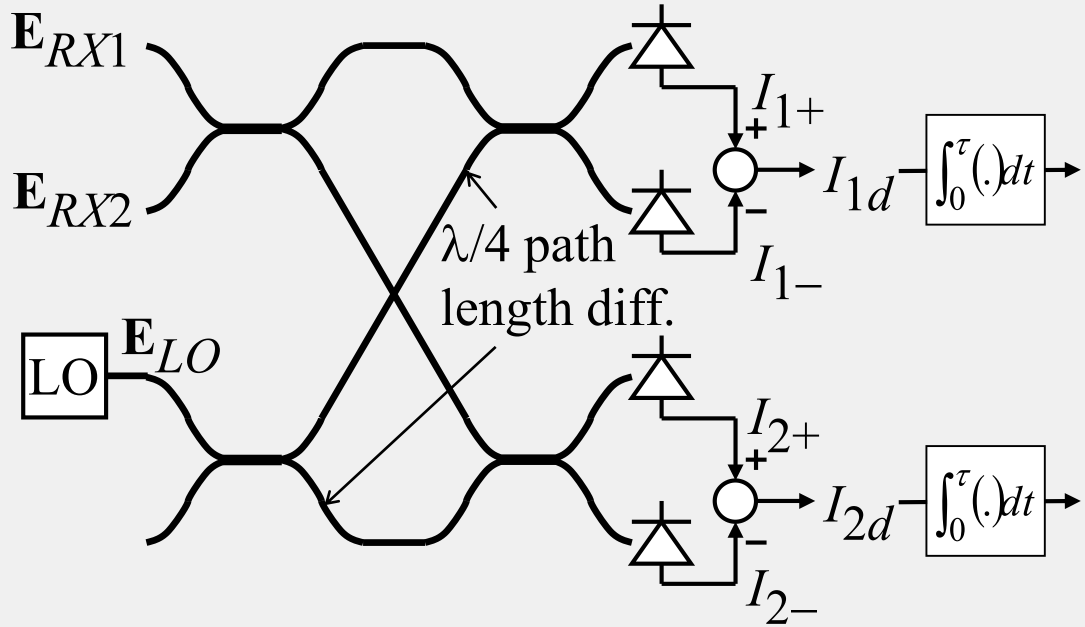
$$\mathbf{E}_{RX2} = (u_{21} + ju_{22})\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2$$

$$w_1 = u_{11} + u_{21} \qquad w_2 = u_{12} - u_{22}$$

$$2\langle u_{ij}^2\rangle = \langle w_k^2\rangle = \frac{hf}{2\tau} \qquad \langle v_k^2\rangle = 1$$



Zero point fluctuations occur at both signal ports. Mean power of zero point fluctuations is neglected for simplicity.

$$I_{1\pm} = R\left|\pm(\mathbf{E}_{RX1} + \mathbf{E}_{RX2})/2 + \mathbf{E}_{LO}/2\right|^2$$

$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

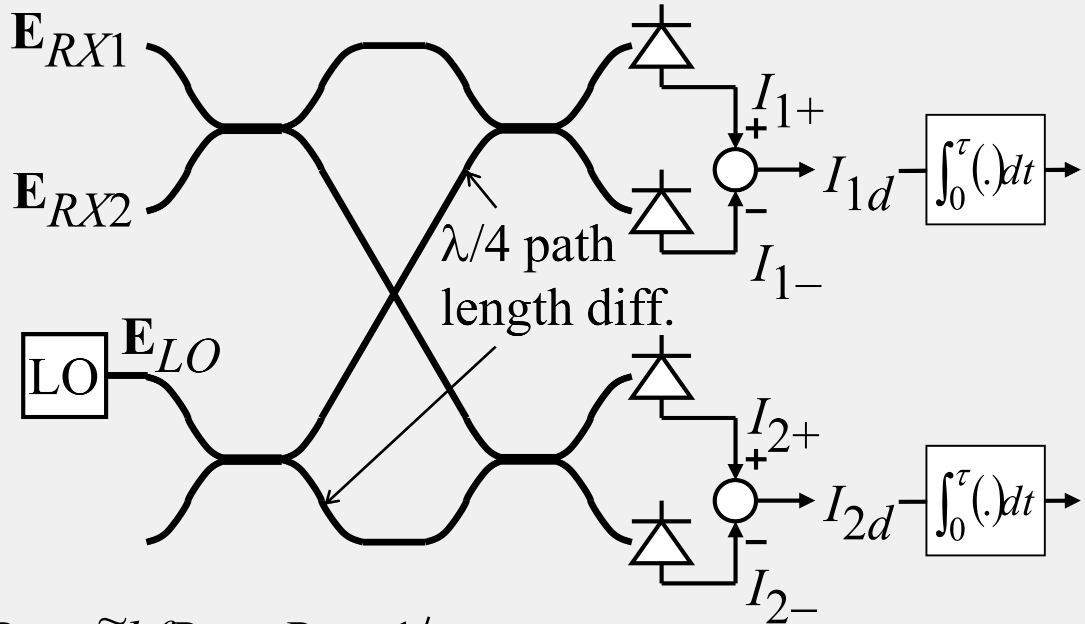
$$I_{2\pm} = R\left|\pm(\mathbf{E}_{RX1} - \mathbf{E}_{RX2})/2 + j\,\mathbf{E}_{LO}/2\right|^2$$

$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2\right) \pm 2\left(\sqrt{G}v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}} + P_{LO}\right)$$

I&Q NF derived with zero point fluctuations (2)

The 2 LO ports also carry zero point fluctuations. But these cancel upon subtraction of photocurrents.

$$I_{1d} = I_{1+} - I_{1-}$$
$$= R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}}$$
$$I_{2d} = I_{2+} - I_{2-}$$
$$= R\sqrt{G}\left(v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}}$$



$$\left\langle w_k^2 \right\rangle = \frac{hf}{2\tau} \qquad \left\langle v_k^2 \right\rangle = 1 \qquad P_n = \tilde{\mu} hf B_o \qquad B_o = 1/\tau$$

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO}\left(G P_n/2 + hf/(2\tau)\right)} = \frac{G P_S}{G\tilde{\mu} hf B_o/2 + hf B_o/2} = \frac{P_S \tau}{\left(\tilde{\mu} + 1/G\right) hf/2}$$

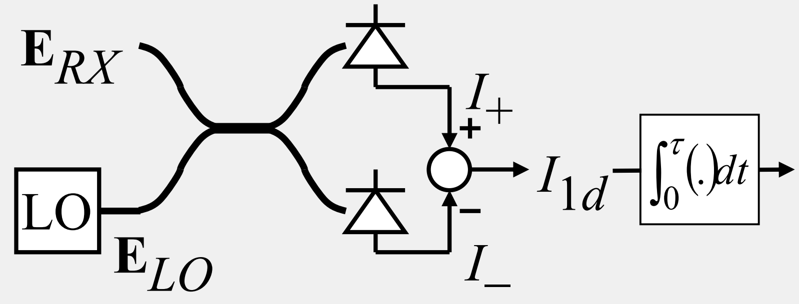
$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2} \qquad \frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} = F_{o,IQ} = \tilde{\mu} + 1/G$$

Same result as when derived with Poisson photoelectron distribution.

Homodyne NF derived with zero point fluctuations

$$\mathbf{E}_{RX} = \left(\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_1 + ju_2)\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$



$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2 \qquad \langle u_k^2 \rangle = \frac{hf}{4\tau} \qquad \langle v_k^2 \rangle = 1$$

$$I_\pm = R\left|\pm \mathbf{E}_{RX}/\sqrt{2} + \mathbf{E}_{LO}/\sqrt{2}\right|^2$$

$$\approx \frac{R}{2}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

$$I_d = I_+ - I_- = 2R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}}$$

$$SNR_{o,I,out} = \frac{\overline{I_d}^2}{\sigma_{I_d}^2} = \frac{4R^2 P_{LO} G P_S}{4R^2 P_{LO}\left(GP_n/2 + hf/(4\tau)\right)} = \frac{2GP_S}{2G\tilde{\mu} hfB_o/2 + hfB_o/2} = \frac{2P_S\tau}{\left(2\tilde{\mu} + 1/G\right)hf/2}$$

$$SNR_{o,I,in} = \frac{2P_S\tau}{hf/2} \qquad \frac{SNR_{o,I,in}}{SNR_{o,I,out}} = F_{o,I} = 2\tilde{\mu} + 1/G$$

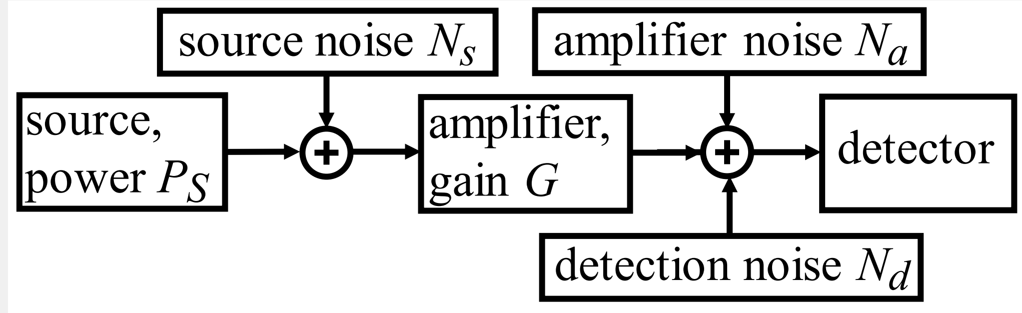
Same result as when derived with Poisson photoelectron distribution.

Structure of noise figure which fulfills Friis‘ formula

$$SNR_{in} = \frac{P_S}{N_s + N_d}$$

$N_s \sim kTB$?

$N_d \sim hfB$?



$$SNR_{out} = \frac{GP_S}{GN_s + N_d + N_a}$$

$$F = \frac{GN_s + N_d + N_a}{G(N_s + N_d)} = A + \frac{1-A}{G} + B$$

source noise fraction

$$A = \frac{N_s}{N_s + N_d}$$

added noise fraction

$$B = \frac{N_a}{G(N_s + N_d)}$$

Device cascade:

$$SNR_{out} = \frac{G_1 G_2 P_S}{G_1 G_2 N_s + N_d + G_2 N_{a1} + N_{a2}}$$

$$F = \frac{G_1 G_2 N_s + N_d + G_2 N_{a1} + N_{a2}}{G_1 G_2 (N_s + N_d)}$$

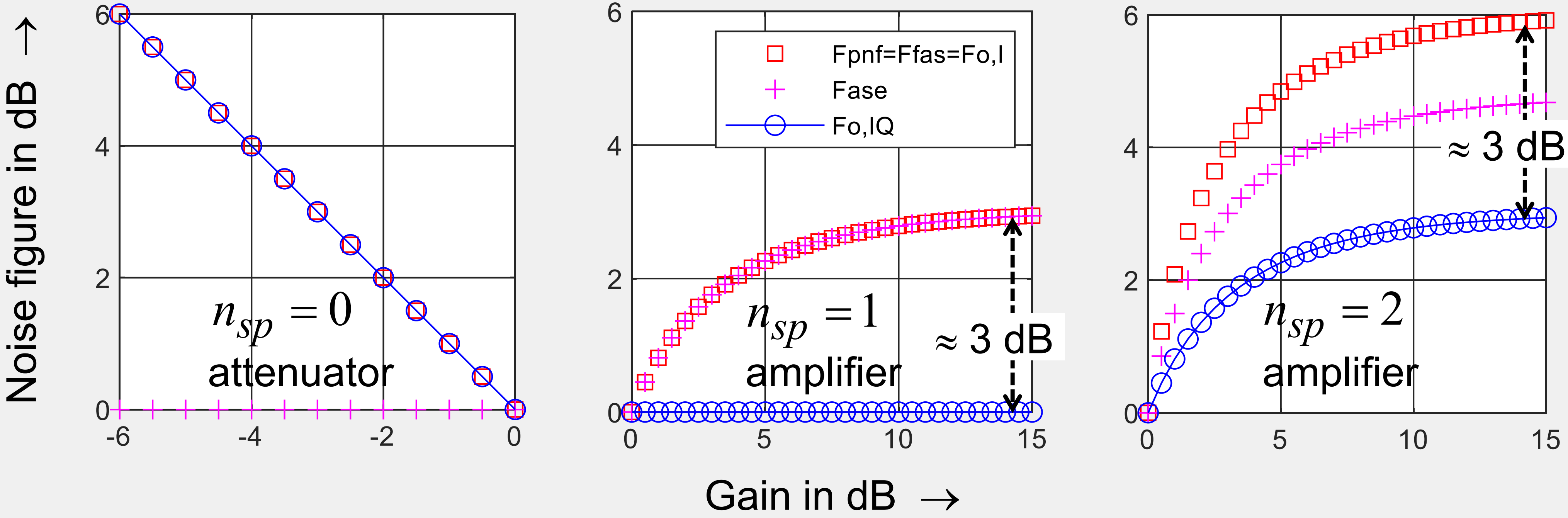
$$\Rightarrow \quad F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1}$$

Complete induction yields Friis’ formula:

$$F - 1 = \sum_{i=1}^{n} \frac{F_i - 1}{\prod_{k=1}^{i-1} G_k}$$

It holds for all noise figures which can be written like this, including $\underset{A=1}{F_e \,,\; F_{ase}}$, $\underset{A=0}{F_{pnf} = F_{fas} = F_{o,I} \,,\; F_{o,IQ}}$!

Optical noise figures [dB] vs. gain [dB]



Only $F_{o,IQ}$ behaves like F_e (1 for amplifier with $n_{sp}=1$; $1/G$ for attenuator).

2 quadratures, like F_e ☺

$$\boxed{F_{o,IQ}=n_{sp}\left(1-1/G\right)+1/G}$$

1 quadrature ϟ

$$\boxed{F_{pnf}=2n_{sp}\left(1-1/G\right)+1/G=F_{fas}=F_{o,I}}$$

Is not the SNR degradation factor in any optical receiver!

$$\boxed{F_{ase}=n_{sp}\left(1-1/G\right)+1}$$

Assumes source noise ϟ

Properties of noise figures

Type of noise figure F	SNR degradation factor	Linear	Available quadratures	F of ideal ampl. $G \to \infty$	F of atten., $G<1$	M of ampl.	Input-referred energy per mode, kT_{ex} or $\tilde{\mu}hf$
F_e	yes	yes	2	1	$1/G$	≥ 0	$kT(F-1)$
$F_{o,IQ} = n_{sp}(1-1/G)+1/G$	yes	yes	2	1	$1/G$	$n_{sp}-1 \geq 0$	$hf(F-1/G)$
$F_{pnf} = F_{fas} = F_{o,I}$ $= 2n_{sp}(1-1/G)+1/G$	yes	not F_{pnf}	1	2	$1/G$	$2n_{sp}-1$ ≥ 1	$hf(F-1/G)/2$
$F_{ase} = 1+n_{sp}(1-1/G)$	no	yes	2	2	1	$n_{sp} \geq 1$	$hf(F-1)$

<u>Only</u> $F_{o,IQ}$ matches conceptually with F_e!

Note: NF is lab jargon. Precisely, F is the noise factor and $(10\text{ dB})\cdot\log_{10}(F)$ is the noise figure.

For lowest NF of a cascade, order amplifiers according to ascending noise measure M.

$$M = \frac{F-1}{1-1/G}$$

Ideal optical amplifier noise figure at large gain is ... ?

$$\boxed{\text{ideal optical NF} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}}}$$

(according to the foregoing)

quadratures	1	2
optical amplifier	phase-sensitive	phase-insensitive
optical receiver	direct detection (or homodyne)	I&Q, or heterodyne with image rejection

Common answer since mid 1990ies:

$$F_{pnf} = 2 = F_{fas} = F_{o,I}$$

But with the same logic one could answer: $F_{o,IQ} = 1/2$

Other cases are considered as special.

It makes most sense to pair amplifiers and receivers with same number of available quadratures:

optical amplifier	phase-sensitive	phase-insensitive
optical receiver	homodyne (or direct detection)	I&Q, or heterodyne with image rejection
Nonlinear! Can it yield a NF?	$F_{o,I} = 1$	$F_{o,IQ} = 1$ (like $F_e = 1$)

By far most frequent optical + electrical scenario <u>today</u>!

User must provide phase reference! RX can also contain phase-sensitive amplifier!

2 unequal NF for 1 scenario? 1 NF for 2 unequal scenarios?

One cannot say one NF (F_e) is for electrical detectors and another (F_{pnf}) is for quantum detectors (photodiodes), because one might become able to build both detector types for the same f (low THz region?): This would oppose **unequal** NF for same usage of same amplifier at same f! NF must be detector-independent!
The term "noise figure" without additions suggests the properties of F_e, i.e. SNR degradation factor in linear system with 2 quadratures (and preferably Gaussian noise).

$\Rightarrow$ Term "optical noise figure" seems fit only for $F_{o,IQ} = \tilde{\mu} + 1/G$.

To avoid misinterpretation, $F_{pnf} = 2\tilde{\mu} + 1/G$ could be called "high-power optical χ^2 (chi-square) noise estimator", "photoelectron number fluctuation indicator", ...

Likewise, $F_{o,I}\ (= F_{fas}\ (= F_{pnf}))$ can be called "optical 1-quadrature NF" (= in-phase).

If SNR is defined with only in-phase noise then the electrical 1-quadrature NF $F_{e,I}$ equals F_e. I have combined $F_{e,I}$ with $F_{o,I}$ to form a 1-quadrature NF F_I.

Result is similar to a corrected F_{fas}. But number of quadratures in F_{fas} is not given and one is left to assume that in the electrical domain F_{fas} is for 2 quadratures. **1 ≠ 2**!

An interpretation difference is that in F_{fas} added thermal noise is considered not separately, but as caused by spontaneous emission (set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$). In a phase-sensitive amplifier, ideal $F_{o,I} = F_{fas} = 1$.

Removing avoidable receiver or power meter noise

In NF measurement the power meter or RX is always assumed to be free of avoidable noise. In practice it is not possible to cool a power meter or RX to $0\ \mathrm{K}$ in order to avoid its thermal noise. For this reason the intrinsic power meter noise is measured, and subtracted during NF measurement, thereby maximizing the resulting NF.

In the coherent RX we also must assume zero thermal noise. In the foregoing this has been achieved by letting $P_{LO} \to \infty$. Practically one must subtract RX thermal noise.

Shot noise of LO is unavoidable. But shot noise of received signal is avoidable by $P_{LO} \to \infty$. Practically one must subtract shot noises caused by GP_S and P_S .

Nonideal quantum efficiency η also reduces and falsifies measured NF. Hence we have assumed $\eta = 1$ in the responsivity $R = \eta e/(hf)$. Practically one must correct measurements such that they represent the case $\eta = 1$.

In the coherent I&Q RX the signal splitter can be viewed as a 2×2 coupler. When considering all frequencies, thermal noise enters also at the 2nd, unused coupler input. That can be avoided by cooling the termination of the 2nd coupler input to $0\ \mathrm{K}$. Practically, thermal noise due to the 2nd coupler input must be subtracted.

These corrections have been implemented. They minimize RX noise and maximize measured NF. – Direct power measurements (p. 27) achieve the same and are easier!

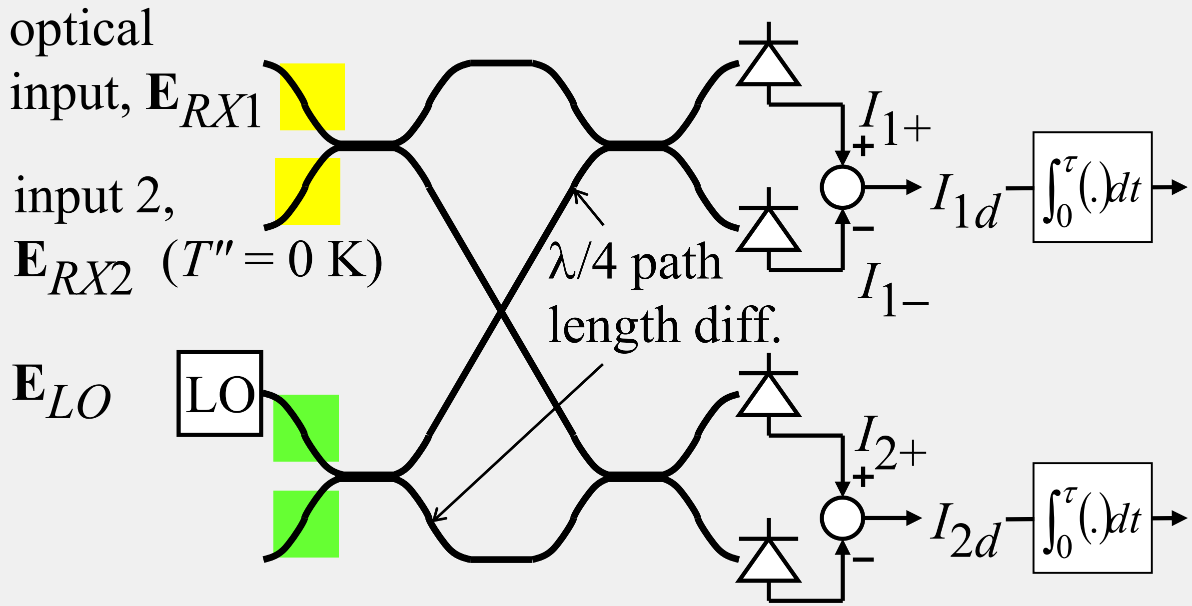
Thermal and optical noises

Shot noise is represented either semiclassically by a Poisson distribution of photoelectrons or by zero point fluctuations at all inputs.

Thermal noise energy per mode approaches kT only at low frequencies f. For all f the correct expression is:

$$\frac{hf}{e^{hf/(kT)}-1}$$

For NF measurement, always an ideal RX or power meter is assumed!



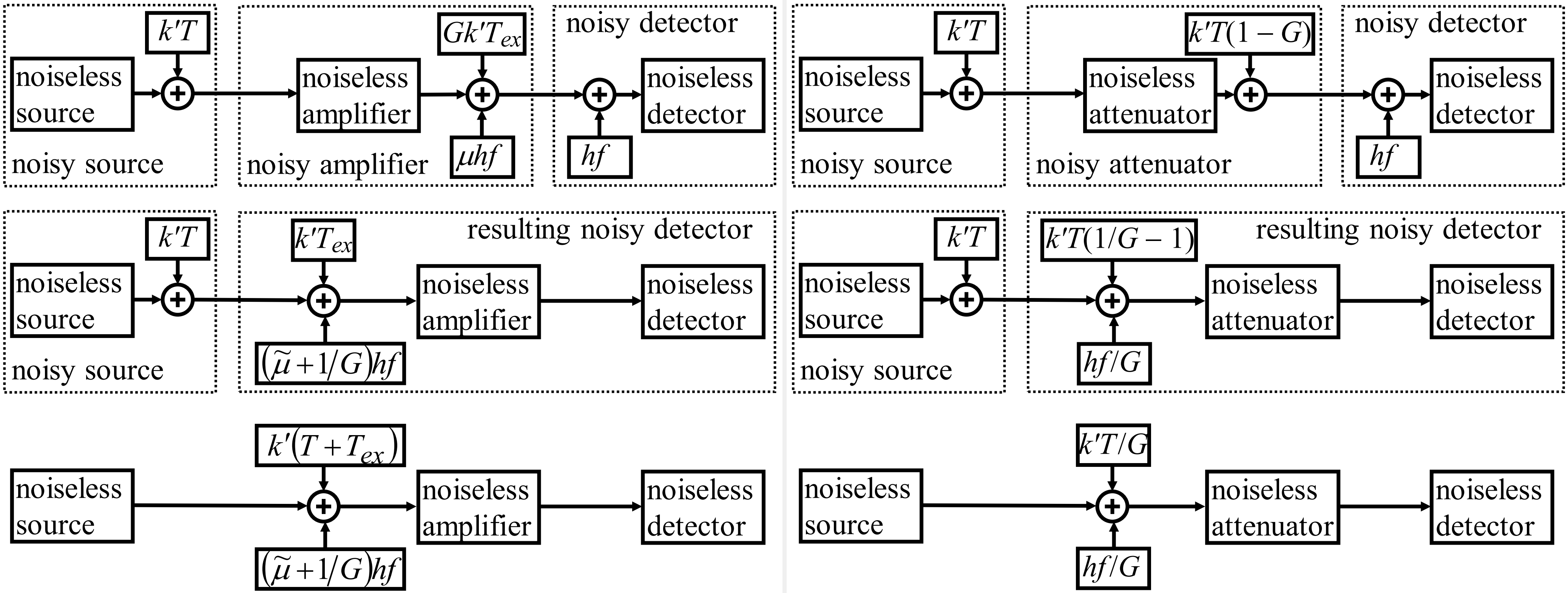
For the source with temperature T we define: $$k'T = \frac{hf}{e^{hf/(kT)}-1}$$

Signal input 2 shall be terminated by an absorber having $T''=0\text{ K}$. No thermal noise enters there.

All noises interfere essentially with the strong LO fields ($P_{LO}\to\infty$).

LO interferences of noises from the signal inputs add / LO inputs cancel upon photocurrent subtractions.

Block diagrams with thermal and optical noises



Source, amplifier (left) or attenuator (right) and detector (electrical or coherent optical), **all I&Q**, noisy or noiseless with equivalent added noise energies per mode. Individual devices (top) and equivalent interpretations (middle, bottom). Detector is for 2 available quadratures. If detector were for 1 quadrature, hf would be replaced by $hf/2$.

Upconversion e-o is possible with an I&Q modulator (or DSB modulator + SSB filter), downconversion o-e with an I&Q RX (or heterodyne + image rejection filter).

SNR in the presence of thermal and optical noises

To derive a consistent unified NF (**I&Q** !) we add noises of F_e and $F_{o,IQ}$ for all f.

Optical and electrical gains G are identical because they manifest at same f.

Thermal noise in bandwidth $B_o = 1/\tau = 2B_e$ is $GF_e k'TB_o$ at amplifier output.

Half of this is in phase with signal. In coherent I&Q RX it appears multiplied with $R^2 P_{LO}$, like amplified signal power GP_S. Corresponding variance σ_e^2 is added.

$$SNR_{IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2}$$

Detector type does not matter, as long as it is usable in linear I&Q receiver:

$$= \frac{R^2 P_{LO} G P_S}{R^2 P_{LO} GF_e k'TB_o/2 + R^2 P_{LO}\tilde{\mu} GhfB_o/2 + eRP_{LO}B_e}$$

Powers in I&Q receiver with quantum detectors

$$= \frac{GP_S}{GF_e k'TB_o/2 + \tilde{\mu} GhfB_o/2 + hfB_e}$$

Powers in electrical I&Q receiver

$$= \frac{P_S\tau}{F_e k'T/2 + F_{o,IQ}hf/2} = \frac{P_S\tau}{k'(T + T_{ex})/2 + (\tilde{\mu} + 1/G)hf/2}$$

Thermal source noise
Thermal amplifier noise
Spontaneous emission field noise in amplifier
Shot noise in detector

I&Q noise figure from electrical to optical frequencies

$$SNR_{IQ,out} = \frac{P_S\tau}{F_e k'T/2 + F_{o,IQ} hf/2} = \frac{P_S\tau}{k'(T+T_{ex})/2 + (\tilde{\mu}+1/G)hf/2}$$

Linear!

Pure Gaussian noises!

$$SNR_{IQ,in} = \frac{P_S\tau}{k'T/2 + hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{IQ,in}}{SNR_{IQ,out}} = \boxed{F_{IQ} = \frac{F_e k'T + F_{o,IQ} hf}{k'T+hf} = \frac{k'(T+T_{ex}) + (\tilde{\mu}+1/G)hf}{k'T+hf}}$$

2 available quadratures!

$$= A + (1-A)/G + \left(A T_{ex}/T + (1-A)\tilde{\mu}\right) \qquad \boxed{A = k'T/(k'T+hf)}$$

Fulfills Friis' formula!

Measured F_{IQ} is just observed SNR degradation in linear system with 2 quadratures.

In amplifier, F_e, $F_{o,IQ}$ may not be known. Anyway, $k'T_{ex} + \tilde{\mu}hf$ is total added noise.

In attenuator, clear separation yields the correct result: $G<1$, $T_{ex} = T(1/G-1)$,

$n_{sp}=0$, $\tilde{\mu}=0 \quad \Rightarrow \quad F_{IQ} = 1/G = F_e = F_{o,IQ}$

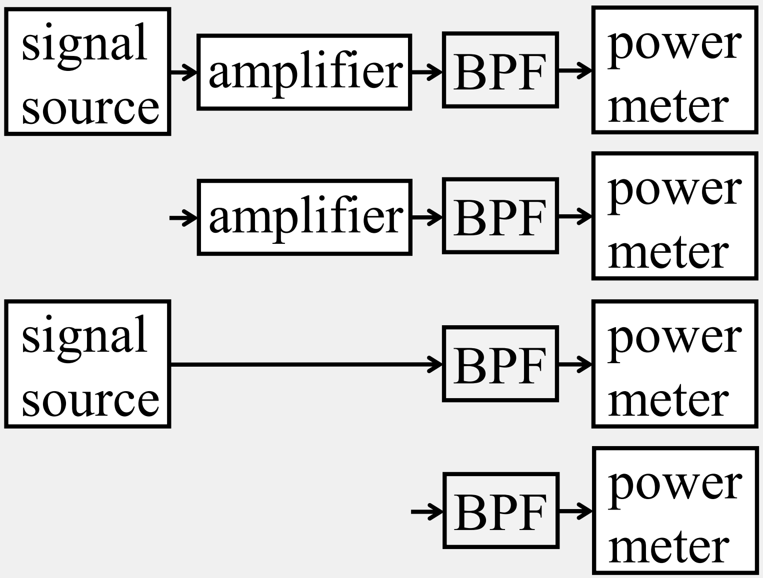
At low f: $F_{IQ} \to F_e$. At high f: $F_{IQ} \to F_{o,IQ}$.

https://ieeexplore.ieee.org/document/9783564 = 66 GHz @ 4 K

At 13400 / 1940 / 300 / 77 / 4 K, equal $k'T = hf$ is at f = 194 / 28 / 4.3 / 1.1 / 0.06 THz.

Measure I&Q noise figure with power meter

Usually there are $p=2$ polarization modes. $p=1$ needs a polarizer to be inserted.
P_0' is the power readout offset caused by noise generated inside the power meter.



$$P_1 = \left(k'G\left(T+T_{ex}\right)+\tilde{\mu}Ghf\right)pB_o + P_0' + GP_S$$

$$P_2 = \left(k'G\left(T+T_{ex}\right)+\tilde{\mu}Ghf\right)pB_o + P_0'$$

$$P_3 = k'TpB_o + P_0' + P_S$$

$$P_4 = k'TpB_o + P_0'$$

$$\left(k'T = \frac{hf}{e^{hf/(kT)}-1}\right)$$

$$G = \frac{P_1-P_2}{P_3-P_4}$$ Gain

$$k'T_{ex} + \tilde{\mu}hf = \frac{1}{G}\left(\frac{P_2-P_4}{pB_o} - k'(G-1)T\right)$$ Added noise

It doesn’t matter, and needn’t be known, in how far added noise is of thermal or quantum origin. F_{IQ} and all other NF can be determined from simple static power measurements.

$$F_{IQ} = \frac{k'T + \left(k'T_{ex} + \tilde{\mu}hf\right) + hf/G}{k'T + hf}$$

SNR with 1-quadrature noises and homodyne receiver

No power splitting $\Rightarrow$ $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ must be multiplied by **2** compared to $F_{o,IQ}$ calculation. Only 1 RX input! Total thermal noise in bandwidth B_o at amplifier output is $GF_e k'TB_o$. Half of this is in phase with the signal. In the coherent 1-quadrature (homodyne) RX it appears multiplied with $4R^2 P_{LO}$, like the amplified signal power GP_S . RX for 1 quadrature is a special case!

$$SNR_{I,out} = \frac{4\overline{I_{1d}}^2}{4\sigma_e^2 + 4\sigma_{I_{1d}}^2 + 2\sigma_{I_{1s}}^2}$$

(Quantities found in I&Q RX are multiplied here by **2**·**2** or **2**.)

$$\boxed{F_{e,I} = F_e}$$

$$= \frac{4R^2 P_{LO} G P_S}{4R^2 P_{LO} G F_e k'TB_o/2 + 4R^2 P_{LO} \tilde{\mu} G hf B_o/2 + 2eRP_{LO}B_e}$$

Powers in homodyne receiver with quantum detectors

$$= \frac{2GP_S}{2GF_e k'TB_o/2 + 2\tilde{\mu} G hf B_o/2 + hf B_e}$$

$$= \frac{2P_S\tau}{F_e k'T + F_{o,I} hf/2} = \frac{2P_S\tau}{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}$$

Thermal source noise

Thermal amplifier noise

Spontaneous emission field noise in amplifier

Shot noise in detector

1-quadrature / homodyne unified noise figure

$$SNR_{I,out} = \frac{2P_S\tau}{F_e k'T + F_{o,I} hf/2} = \frac{2P_S\tau}{k'(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}$$

$$SNR_{I,in} = \frac{2P_S\tau}{k'T + hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{I,in}}{SNR_{I,out}} = F_I = \frac{F_e k'T + F_{o,I}\, hf/2}{k'T + hf/2} = \frac{k'(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}{k'T + hf/2}$$

$$= A_I + (1-A_I)/G + \left(A_I T_{ex}/T + (1-A_I)2\tilde{\mu}\right) \qquad \boxed{A_I = k'T/(k'T + hf/2)} \neq A$$

$F_{o,I} \neq F_{o,IQ}$ because there is detection noise!

$F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!

(set $T_{ex}=0$ and take a high $\tilde{\mu}$, with $\tilde{\mu}\to\infty$ for $f\to 0$)

1-quadrature / homodyne F_I is close to F_{fas} (except k' and interpretation difference)!

In definition of F_{fas}, number of quadratures was not discussed. F_{fas} is intended to be identical with the normal electrical F_e, which is understood to be for 2 available quadratures. So, one is left to assume that F_{fas} has 2 quadratures in the electrical and 1 quadrature in the optical domain. But that is contradictory, impossible!

1-quadrature / homodyne unified noise figure

$$SNR_{I,out}=\frac{2P_S\tau}{F_e k'T+F_{o,I}hf/2}=\frac{2P_S\tau}{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}$$

$$SNR_{I,in}=\frac{2P_S\tau}{k'T+hf/2}$$ (obtained with $T_{ex}=0,\ \tilde{\mu}=0,\ G=1$)

$$\frac{SNR_{I,in}}{SNR_{I,out}}=F_I=\frac{F_e k'T+F_{o,I}\,hf/2}{k'T+hf/2}=\frac{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}{k'T+hf/2}$$

$$=A_I+(1-A_I)/G+(A_I\,T_{ex}/T+(1-A_I)2\tilde{\mu})$$

$\boxed{A_I=k'T/(k'T+hf/2)}\neq A$

$F_{o,I}\neq F_{o,IQ}$ because there is detection noise!

$F_{e,I}=F_{e,IQ}\equiv F_e$ because there is source noise!

(set $T_{ex}=0$ and take a high $\tilde{\mu}$, with $\tilde{\mu}\to\infty$ for $f\to 0$)

1-quadrature / homodyne F_I is close to F_{fas} (except k' and interpretation difference)!

Attenuator: I simply say

$T_{ex}=T(1/G-1),\ n_{sp}=0=\tilde{\mu}$,

$F_I=1/G=F_{o,I}=F_e\quad(=F_{e,I})$.

Attenuator: To get $F_{fas}=1/G\ (=F_I)$ I find I must set $T_{ex}=0,\ n_{sp}=-k'T/(hf),\ \tilde{\mu}=n_{sp}(1-1/G)$.

$f\to\{\infty,0\}\ \Rightarrow\ n_{sp}\to\{0,-\infty\},\ \tilde{\mu}\to\{0,\infty\}$!

Noise figures

RX or power detector noise would cause F_e to be underestimated. Therefore RX noise is always subtracted, using reference measurements. The same way, avoidable optical RX noise can and must be subtracted at high f. Only LO shot noise is fundamental and is kept. Photodiode efficiency must be set equal to 1. We get:

Optical $kT << hf$	Unified/generalized	Electrical $kT >> hf$
$\tilde{\mu} = n_{sp}(1-1/G)$	$\tilde{\mu} = n_{sp}(1-1/G)$ $\quad k'T = \dfrac{hf}{e^{hf/(kT)}-1}$	kT
RX with 2 available quadratures (I&Q); <u>the</u> noise figure:		
$F_{o,IQ} = \tilde{\mu} + 1/G$	$F_{IQ} = \dfrac{k'(T+T_{ex}) + (\tilde{\mu}+1/G)hf}{k'T+hf}$	$F_e = 1 + \dfrac{T_{ex}}{T}$
RX with 1 available quadrature; important only in true optical homodyne systems:		
$F_{o,I} = 2\tilde{\mu} + 1/G$	$F_I = \dfrac{k'(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}{k'T+hf/2}$	$F_{e,I} = 1 + \dfrac{T_{ex}}{T}$

Shot noise occurs upon <u>detection</u>.
$\Rightarrow$ <u>Optical homodyne</u> is different.

In all cases, NF of pure attenuator is $1/G$.

Summary

- All prior optical and unified NF F_{pnf}, F_{fas}, F_{ase} are in conflict with electrical NF F_e.
- A „noise figure" without special name is expected to be the SNR degradation factor in a linear system with 2 available quadratures (and Gaussian noise?!), like F_e.
- The only optical NF which fulfills this is the optical I&Q NF $F_{o,IQ}$. It is ≥ 1, like F_e.
- Coherent I&Q receivers are linear field sensors. They linearize the quadratic field behavior of photodiodes. Heterodyne with image rejection is also fine.
- At high gain, $F_{o,IQ} \approx F_{pnf}/2$, i.e. ≈3 dB less when expressed in dB.
- Electrical and optical I&Q NF are limit cases of <u>the</u> NF F_{IQ}, unified for all f.

  Quantum noise / F_{IQ} plays a role in today's electronics at low $T = 4\ \mathrm{K}$.
- The in-phase equivalent of $F_{o,IQ}$ is $F_{o,I}$, a limit case of the unified F_{fas}. So, F_{fas} is a 1-quadrature NF and its other limit is F_e for 1 quadrature, not the expected 2.
- Information conveyed by the full F_{pnf} (including sp-sp) of a specific DD RX can be obtained, more accurately, from $F_{o,IQ}$ (pure Gaussian noise).
- Optical amplifier adds Gaussian I&Q field noise (wave aspect). Photodetection adds shot noise (particle aspect).

(after correction $k \rightarrow k'$)