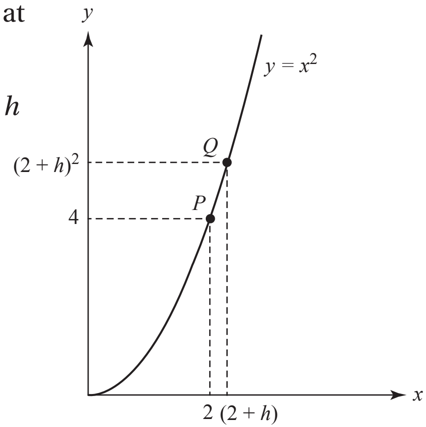
When looking at the graph of a function, the gradient of its curve at any given point tells you the rate of change. Differentiation from first principles is a method of calculating the gradient and, therefore, the rate of change.

For example, you can work out the gradient of the function $y = x^2$ at the point $P(2, 4)$ using differentiation from first principles.



Define a point Q that lies on the curve, a tiny horizontal distance h from P, so that Q has coordinates $(2 + h, (2 + h)^2)$

PQ is the chord that connects the points.

The gradient of the chord PQ is given by

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{(2+h)^2 - 4}{(2+h) - 2}$$
$$= \frac{4 + 4h + h^2 - 4}{(2 + h) - 2} = \frac{4h + h^2}{h}$$
$$= 4 + h$$

As the distance between P and Q becomes very small, h very small and m_{PQ} approaches 4

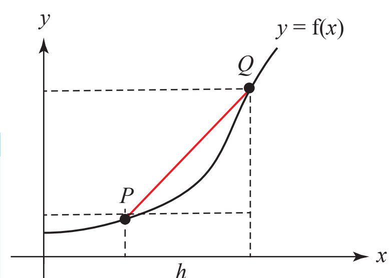
Gradient at $P = 4$

This method can be generalised for any function.

ICT Resource online

To investigate gradients of chords for a graph, click this link in the digital book.

Consider the graph $y = f(x)$
Let the point P lie on the curve and have x-coordinate x
Its y-coordinate is then $f(x)$
Let the point Q also lie on the curve, h units to the right of P
Its coordinates are therefore $(x + h, f(x + h))$



Key point

The **gradient** of PQ is given by

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

As h approaches 0, the point Q approaches P and the gradient of the chord PQ gets closer to the gradient of the curve at P

The gradient of the curve at P is defined as the **limiting value** of the gradient of PQ as h approaches 0. This limit is denoted by $f'(x)$ and is called the **derived function** or **derivative** of $f(x)$. See p.300 for a list of mathematical notation.

Key point

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation with this method is referred to as finding the derivative from **first principles**.

A limiting value, or **limit**, is a specific value that a function approaches or tends towards. "$\lim\limits_{h \to 0}$" followed by a function means the limit of the function as h tends to zero.