The reverse process of differentiation is known is **integration**.

To differentiate sums of terms of the form ax^n you multiply by the power then reduce the power by 1 to get nax^{n-1}

To **integrate** you do the exact opposite: you add 1 to the power then divide by the new power.

When you differentiate a constant, the result is zero. So when you perform an integration, you should add a constant, c, to allow for this. This is referred to as the **constant of integration**. The value of this constant can only be determined if further information is given.

When you perform an integration you can check your result by differentiating it – you should get back to what you started with.

Key point

Integrating x^n with respect to x is written as

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

When a function is multiplied by a constant, the constant can be moved outside the integral.

Key point

$\int a\mathrm{f}(x)\mathrm{d}x = a\int \mathrm{f}(x)\mathrm{d}x$ where a is a constant.

When integrating the sum of two functions, each function can be integrated separately.

You may see this rule referred to as the *sum rule*.

$$\int(\mathrm{f}(x)+\mathrm{g}(x))\mathrm{d}x = \int \mathrm{f}(x)\mathrm{d}x + \int \mathrm{g}(x)\mathrm{d}x$$

Given that displacement is a function of time $\mathrm{r}(t)$, then velocity $\mathrm{v}(t) = \mathrm{r}'(t)$. Reversing this, we get

Key point

$$\int \mathrm{v}(t)\mathrm{d}t = \mathrm{r}(t) + c$$

Given that velocity is a function of time $\mathrm{v}(t)$, then acceleration $\mathrm{a}(t) = \mathrm{v}'(t)$. Reversing this we get

Key point

$$\int \mathrm{a}(t)\mathrm{d}t = \mathrm{v}(t) + c$$

The Fundamental Theorem of Calculus shows how integrals and derivatives are linked to one another. The theorem states that, if $\mathrm{f}(x)$ is a continuous function on the interval $a \le x \le b$, then

Key point

$\int_a^b \mathrm{f}(x)\mathrm{d}x = \mathrm{F}(b) - \mathrm{F}(a)$ where $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{F}(x)) = \mathrm{f}(x)$