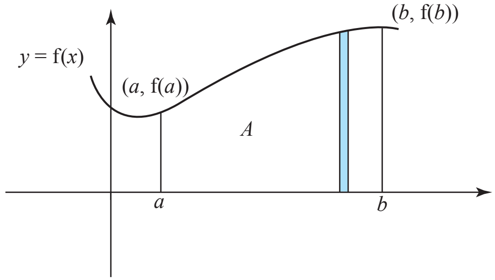
You can use integration to find the area between a curve and the x-axis. To do this, you perform a calculation using a **definite integral**.

Key point

A definite integral is denoted by $\int_a^b f(x)\,dx$

b is called the **upper limit**, and a the **lower limit**.



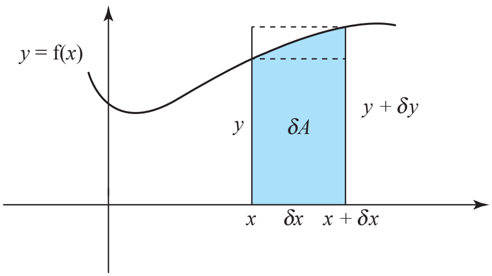
Consider a continuous function $y = f(x)$ over some interval and where all points on the curve in that interval lie on the same side of the x-axis. The area, A, is bound by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ where $a < b$

Consider a small change in x and the change in area, δA, that results from this change.

Use Leibniz notation where δx represents a small change in x and δy represents the corresponding change in y

ICT Resource online

To experiment with numerical integration using rectangles, click this link in the digital book.



Zooming in you can see a small section of the area, trapped between vertical lines at x and at $x + \delta x$

The small area between the vertical lines at x and $x + \delta x$ (shaded) is denoted by δA

$$y\delta x \le \delta A \le (y + \delta y)\delta x$$

Dividing by δx: $$y \le \frac{\delta A}{\delta x} \le (y + \delta y)$$

As you let δx tend to zero:

$$y \le \lim_{\delta x \to 0} \frac{\delta A}{\delta x} \le \lim_{\delta x \to 0}(y + \delta y) \Rightarrow y \le \frac{dA}{dx} \le y$$

Remember

$\lim_{\delta x \to 0} \delta y = 0$ and $\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$

So $$y = \frac{dA}{dx}$$

Integrating this will give you a formula for the area from the origin up to the upper bound, namely $A = \int y\,dx$. Note that, when calculating this small area, the lines $x = a$ and $x = b$ were not used so this has given us a *general* formula for calculating the area. A further calculation is needed to obtain the area between $x = a$ and $x = b$

If you wish to calculate the area between two vertical lines, $x = a$ and $x = b$, then you need only integrate to get the formula for area and substitute a and b for x. The difference between your results is the required area.

If $\int \mathrm{f}(x)\,\mathrm{d}x = \mathrm{F}(x) + c$ then $(\mathrm{F}(b) + c) - (\mathrm{F}(a) + c) = \mathrm{F}(b) - \mathrm{F}(a)$

Key point

The area under a curve between the x-axis, $x = a$, $x = b$ and $y = \mathrm{f}(x)$, is given by $A = \int_a^b \mathrm{f}(x)\,\mathrm{d}x = \mathrm{F}(b) - \mathrm{F}(a)$

Working with areas below the x-axis will produce negative results. As area is a positive quantity you should use the magnitude of the answer only (ignore the negative sign).

The constants of integration sum to zero. This means you don't need to worry about the constant of integration when calculating a definite integral.