Correcting dendrometer measurements for stem curvature

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The dendrometer consists in a band made out of metal or plastic applied around a stem, usually at breast height (~1.3 m), to measure tree diameter growth.

The measurements are made using a caliper between two marked points on the band. Hence, the caliper measures the length of the chord (see fig1).

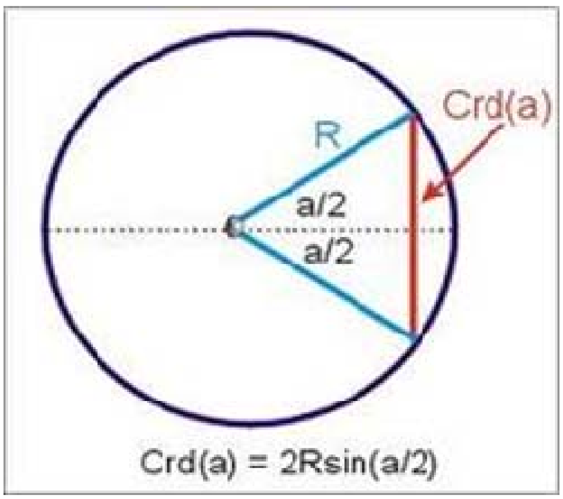

Fig 1.

The relation between the diameter and the chord is a function of the angle *a* subtended by the chord. In order to correctly compute the diameter of successive measurements (denoted as 1 and 2) we need to solve a system of three equations:

$$C_1 = D_1 \sin\frac{a_1}{2} \tag{1}$$

$$C_2 = D_2 \sin\frac{a_2}{2} \tag{2}$$

$$D_1\left(\pi - \frac{a_1}{2}\right) = D_2\left(\pi - \frac{a_2}{2}\right) \tag{3}$$

where the last equation is derived considering that the length of the arc of the band behind the chord remains constant between measurements.

Unfortunately, the above system does not have an explicit analytical solution for the unknown diameter D_2 when D_1, C_1 and C_2 are measured. However it can be easily solved numerically. Substituting (1) and (2) into (3) we obtain:

$$D_2\left(\pi - \sin^{-1}\left(\frac{C_2}{D_2}\right)\right) = D_1\left(\pi - \sin^{-1}\left(\frac{C_1}{D_1}\right)\right) \qquad (4)$$

The solution can be found by iteratively increasing the value of D_2 starting from D_2=D_1 until both side of (4) are equal.

Example:

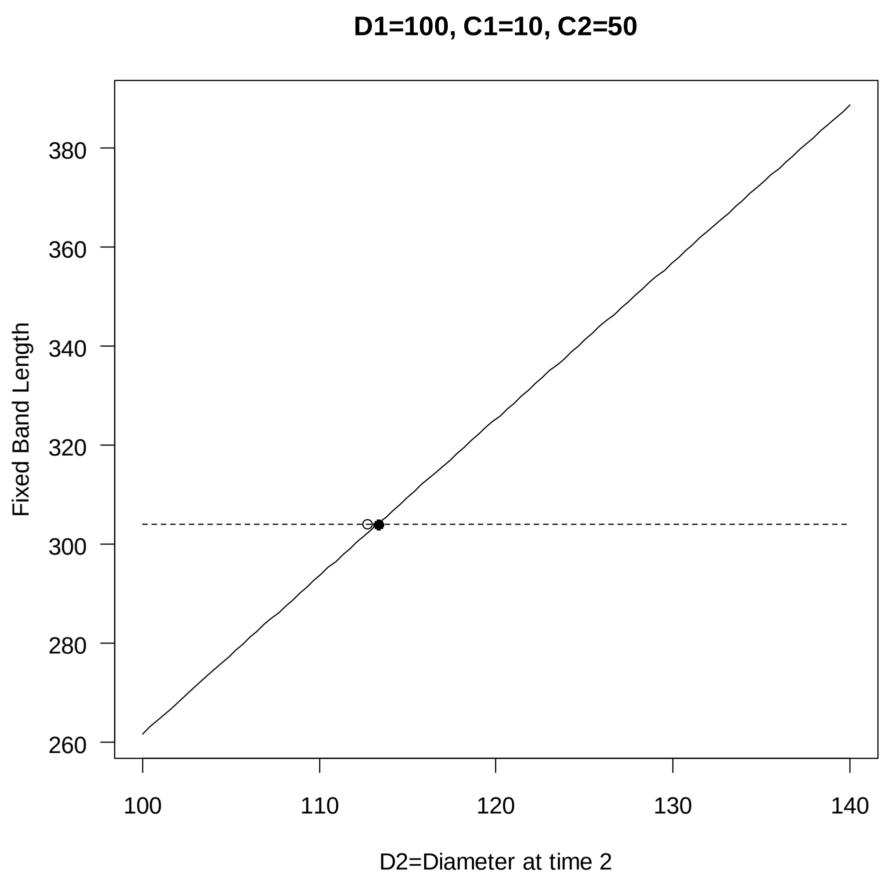

Fig 2. The solid and dashed lines represent the left and right sides of equation 4, respectively. The open circle shows the approximate solution of 112.73, obtained as $D_2 \approx D_1 + \frac{C_2 - C_1}{\pi}$. The solid circle shows the exact solution of 113.30, obtained numerically.

<u>R code for obtaining the true d2:</u>

```
# adjusting for curvature in dendrometer measurements

#d1=diameter at time 1 (measured with diameter tape)
#c1=length of the chord of the opening on the dendrometer at time 1
#c2=length of the chord at time 2
#d2=diameter at time 2 (to be determined using the function below)

gettrued2=function(c1,c2,d1) {
 rhs=d1*(pi-asin(c1/d1))
 d2=optimize(difdendro,c(0,d1+c2),c2=c2,rhs=rhs)
 return(d2$minimum)
} # end gettrued2

difdendro=function(d2,c2,rhs) {
 lhs=d2*(pi-asin(c2/d2))
 return(abs(lhs-rhs))
}


# graphical illustration of the problem
graphtrued2=function(c1,c2,d1) {
 rhs=d1*(pi-asin(c1/d1))
 d2s=seq(d1,d1+c2-c1,length=100)
 lhs=d2s*(pi-asin(c2/d2s))
 par(las=1)
 plot(d2s,lhs,type="l",xlab="D2=Diameter at time 2",ylab="Fixed Band Length",
   main=paste("D1=",d1,", C1=",c1,", C2=",c2,sep=""))
 lines(d2s,rep(rhs,100),lty=2)
 approxd2=d1+(c2-c1)/pi
 points(approxd2,rhs)
 goodd2=gettrued2(c1,c2,d1)
 points(goodd2,rhs,pch=16)
} # end graphtrued2
```