

Gravitational Fields and Gravitational Waves

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September 15, 2021

Abstract: The relative velocity between objects with finite velocity affects the reaction between them. This effect is known as general Doppler effect. The Laser Interferometer Gravitational-Wave Observatory (LIGO) discovered gravitational waves and found their speed to be equal to the speed of light c . Gravitational waves are generated following a disturbance in the gravitational field; they affect the gravitational force on an object. Just as light waves are subject to the Doppler effect, so are gravitational waves. This article explores the following research questions concerning gravitational waves: What is the spatial distribution of gravitational waves? Can the speed of a gravitational wave represent the speed of the gravitational field (the speed of the action of the gravitational field upon the object)? What is the speed of the gravitational field? Do gravitational waves caused by the revolution of the sun affect planetary precession? Can we modify Newton's gravitational equation through the influence of gravitational waves?

Keywords: law of gravitation^{[1][2]}; Doppler effect^[3]; gravitational wave^{[4][5]}; gravitational field^[6]; LIGO^[7]; gravitational constant^[8]; precession of the planets^[9]

1 Introduction

Newtonian gravity is a force that acts at a distance. No matter how fast an object travels, gravity acts upon the object instantaneously. Gravity is only related to the mass and distance of the object, equal to $\frac{G_0 M m}{r^2}$, of which the universal gravitational constant $G_0 = 6.67259 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$. G_0 is measured when two objects are relatively stationary. This can be regarded as a static gravitational constant. Newtonian gravity states that the speed of the gravitational field on an object is infinite, therefore, whether two objects are relatively stationary or moving, both can be considered unchanged, so there is no general Doppler effect. The Laser Interferometer Gravitational-Wave Observatory (LIGO) first discovered gravitational waves and measured their speed. This discovery thus leads us to consider whether the speed of a gravitational field is the same as that of the gravitational wave. If the gravitational field has a finite speed, there will be a general Doppler effect between the gravitational field and the object. To determine the speed of the gravitational field, we propose the following two hypotheses:

Hypothesis 1: The speed of the gravitational field is equal to the speed of light. For the convenience of analysis, we use X to represent the speed of the gravitational field.

Hypothesis 2: When the velocity of an object is equal to the velocity of the gravitational field, the gravitational field no longer acts upon the object.

Considering the above, how might the Newtonian equation of gravity change?

2 Derivation of the Relationship between Gravity and Velocity based on Newton's Gravity Equation

In a very short time slice dt , we can assume that m is stationary and the gravity received is constant. We can then accumulate the impulse generated by the gravity on each time slice and find the average relating to the entire time period to obtain effective constant gravitation and determine the relationship between the equivalent gravitation and velocity.

Consider the influence of velocity on gravity when the moving velocity of object m relative to M is not 0.

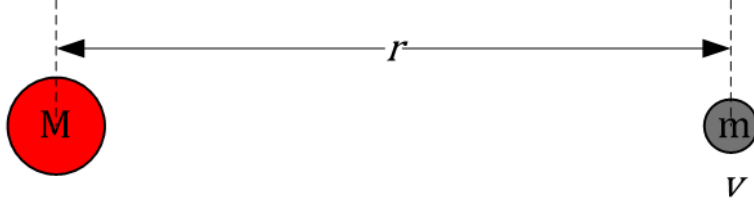


Figure 1: Gravity model

As shown in Figure 1, there are two objects with masses M and m , the distance between them is r , m has a moving velocity relative to M , the speed is v and the direction of the velocity is depicted by the straight line connecting them. $F(t) = \frac{G_0 M m}{(r + vt)^2}$ represents the gravity on m at time t . The Newtonian equation of gravity is used here. In any small time dt , m can be regarded as stationary. An accumulation of the impulse dp is obtained by multiplying the gravity and time in these small time slices. Then, the sum of the gravitational impulse received by m within a certain period can be obtained. Supposing that the gravitational impulse obtained by m is p after time T has passed, the gravity is integrated into the time domain:

$$p = \int_0^T F(t) \times dt = \int_0^T \frac{G_0 M m}{(r + vt)^2} \times dt = \frac{G_0 M m}{r^2} \times \int_0^T \frac{1}{(1 + vt/r)^2} \times dt, \quad (1)$$

$$p = \frac{G_0 M m}{r^2} \times \left. \frac{-r/v}{1 + vt/r} \right|_0^T,$$

$$p = \frac{G_0 M m}{r^2} \times \frac{T}{1+vT/r}.$$

For an object m with a speed of v , the accumulated impulse p during time T can be expressed by an equivalent constant force multiplied by time T . For the convenience of description, we use $F(v)$ to express this equivalent force.

$$F(v) = p/T = \frac{G_0 M m}{r^2} / (1 + \frac{vT}{r}).$$

There is an inverse proportional relationship between the equivalent gravitational force and the speed v . The larger the v , the smaller the $F(v)$; the smaller the v , the larger the $F(v)$. When $v = 0$, it is Newtonian gravity. When v tends to infinity, $F(v) = 0$. The Newtonian gravitational equation is based on the premise that the gravitational field speed is infinite. Now, we may assume that the gravitational field has a finite speed X , therefore, the Newtonian gravitational equation is no longer applicable.

However, we also know that if there is relative velocity between any two objects, there will be a general Doppler effect between them. According to this general Doppler effect between the object and the gravitational field, two boundary conditions are introduced:

1. When an object's velocity relative to the source of gravity is 0, it is Newtonian gravity.
2. When an object's velocity relative to the gravitational field is 0, the gravitational force equation no longer applies.

As shown in the figure below, according to the chase effect, using boundary conditions $F(0) = \frac{G_0 M m}{r^2}$ and $F(X) = 0$, it can be easily calculated:

$$F(v) = F(0) + v \times \frac{F(X) - F(0)}{X} = F(0) \times \frac{X-v}{X} = \frac{G_0 M m}{r^2} \times \frac{X-v}{X}.$$

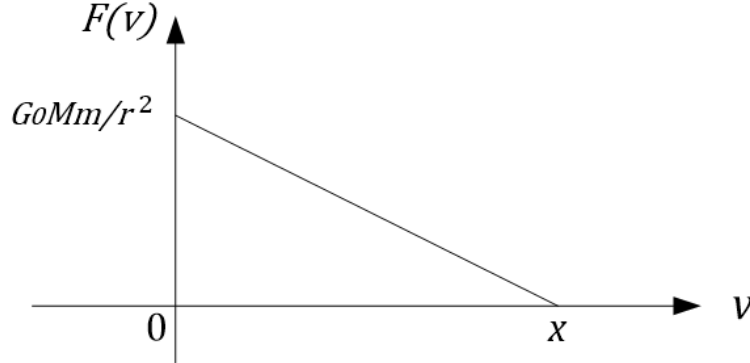


Figure 2: Linear relationship between gravity and speed

From the above analysis, the formula of universal gravitation with parameter v is as follows:

$$F(v) = \frac{G_0 M m}{r^2} \times f(v), f(v) = \frac{X-v}{X}, \quad (2)$$

If it is necessary to preserve the form of Newton's gravity equation, we may write it as follows:

$$F(v) = G(v) \times \frac{Mm}{r^2}, G(v) = G_0 \times \frac{X - v}{X}, \quad (3)$$

That is, the gravitational constant becomes a function of v , $G(v)$. Thus, we may understand that when the gravitational field has a different speed relative to m , the gravitational constant is also different. Next, we apply the new gravitational equation to the planetary orbit calculation to determine whether it is consistent with actual observations.

3 Calculation of the Influence of the New Gravitational Equation on Earth's Orbit

From the above derivation, we get the gravity formula with v as a parameter:

$$F(v) = \frac{G_0 Mm}{r^2} \times \frac{X-v}{X},$$

Considering that the velocity direction of the object m may have an angle with the gravitational field, we define v_r as the component of the speed in the direction of the gravitational field and then obtain a general formula:

$$F(v_r) = \frac{G_0 Mm}{r^2} \times \frac{X-v_r}{X},$$

The equation shows that when an object has a velocity component in the direction of the gravitational field, that is, there is a movement effect in the same direction between the gravitational field and the object, the gravitational force received decreases. When the object has a velocity component that is opposite to the direction of the gravitational field, that is, the two have the effect of moving towards each other, the gravitational force received increases. This leads us to thus consider what impact, under this general Doppler effect, it may have on the planet's orbit. Can planets maintain the conservation of mechanical energy in their orbits?

As shown in the figure above, under the new gravitational equation, as the planetary velocity has the same direction component v_r in the direction of the gravitational field in orbits A and B, the gravity decreases. Therefore, the planet gains extra force in the direction of the gravitational field. This force travels in the same direction as v_r . According to the power calculation formula $P = F \times v_r > 0$, the planetary mechanical energy increases.

Regarding regions C and D, as the planetary velocity has a reverse component v_r in the direction of the gravitational field, the gravitational force increases. Therefore, the extra force gained by the planet moves in the opposite direction of the gravitational field. This force is in the same direction as v_r . According to the power calculation formula $P = F \times v_r > 0$, the planetary mechanical energy increases.

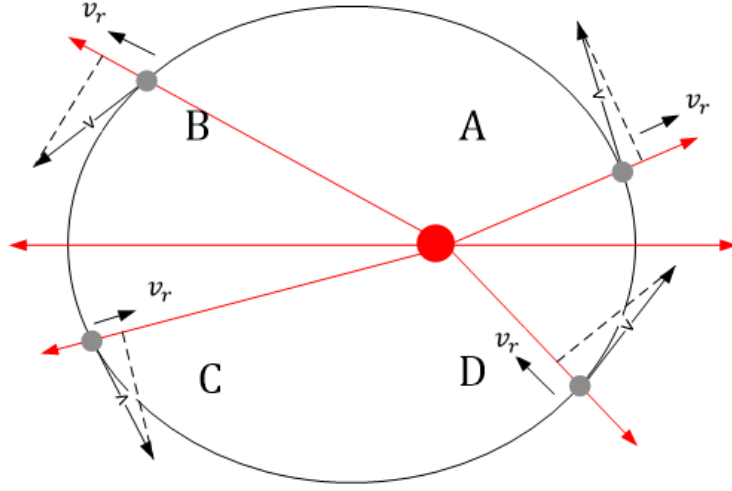


Figure 3: The velocity component of the planet's gravitational field direction

Therefore, under the new gravitational equation, the mechanical energy of the planet in the entire orbit continues to increase and the mechanical energy becomes larger and larger. This would cause the planet to gradually move away from the sun and eventually the solar system. Taking Earth as an example, using the new gravitational equation, after how many revolution cycles would Earth begin to move away from the solar system? Below we include our theoretical analysis and calculations.

3.1 Introduction of Polar Coordinates

Let the sun, mass M , lie at the origin. Consider a planet, mass m , in orbit around the sun. Let the planetary orbit lie in the $x - y$ plane. Let $\mathbf{r}(t)$ be the planet's position vector with respect to the sun. The planet's equation of motion is

$$m\ddot{\mathbf{r}} = -\frac{G_0 M m}{r^2} \times \frac{X - v}{X} \times e_r, \quad (4)$$

where $e_r = \mathbf{r}/r$ and $v_r = e_r \cdot \dot{\mathbf{r}}$. Let $r = |\mathbf{r}|$ and $\theta = \tan^{-1}(y/x)$ be plane polar coordinates. The radial and tangential components of (4) are

$$\ddot{r} - r\dot{\theta}^2 = -\frac{G_0 M}{r^2} \left(1 - \frac{\dot{r}}{X}\right) \quad (5)$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad (6)$$

(6) can be integrated to give

$$r^2\dot{\theta} = h \quad (7)$$

where h is the conserved angular momentum per unit mass. (5),(7) can be combined to give

$$\ddot{r} - \frac{h^2}{r^3} = -\frac{G_0 M}{r^2} \left(1 - \frac{\dot{r}}{X}\right) \quad (8)$$

3.2 Energy Conservation

Multiply (8) by \dot{r} . We obtain

$$\frac{d}{dt} \left(\frac{\dot{r}^2}{2} + \frac{h^2}{2r^2} - \frac{G_0 M}{r} \right) = \frac{G_0 M \dot{r}^2}{r^2 X} \quad (9)$$

or

$$\frac{d\epsilon}{dt} = \frac{G_0 M \dot{r}^2}{r^2 X} \geq 0 \quad (10)$$

where

$$\epsilon = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{G_0 M}{r} \quad (11)$$

is the energy per unit mass. (10) demonstrates that the Doppler shift correction to the law of force causes the system to cease conserving energy. The orbital energy grows without limit. This means that the planet will eventually escape the sun's gravitational pull (when its orbital energy becomes positive).

3.3 Solution of Equations of Motion

Let $1/r = u[\theta(t)]$. It follows that

$$\dot{r} = -h \frac{du}{d\theta}, \quad (12)$$

$$\ddot{r} = -u^2 h^2 \frac{d^2 u}{d\theta^2}, \quad (13)$$

Thus, Eq. (8) becomes

$$\frac{d^2 u}{d\theta^2} - \gamma \frac{du}{d\theta} + u = \frac{G_0 M}{h^2}, \quad (14)$$

where

$$\gamma = \frac{G_0 M}{hX}, \quad (15)$$

is a small dimensionless constant. To first order in γ , an appropriate solution of (14) is

$$u \approx \frac{G_0 M}{h^2} (1 + e \exp(\gamma \theta) \cos \theta), \quad (16)$$

where e is the initial eccentricity of the orbit. Thus

$$r(\theta) = \frac{r_c}{1 + e \exp(\gamma \theta) \cos \theta}, \quad (17)$$

where

$$r_c = \frac{h^2}{G_0 M}, \quad (18)$$

It can be observed that the orbital eccentricity grows without limit as the planet orbits the sun. Eventually, when the eccentricity becomes unity, the planet will escape the sun.

3.4 Estimation of Escape Time

The planet escapes when its orbital eccentricity becomes unity. The number of orbital revolutions, n , required for this to happen is

$$e \exp(\gamma n 2\pi) = 1, \quad (19)$$

where e is the initial eccentricity. Thus, $n = \frac{1}{2\pi\gamma} \ln(\frac{1}{e})$,

but,

$$\gamma = \frac{2\pi a}{TX(1 - e^2)^{\frac{1}{2}}}, \quad (20)$$

where a is the initial orbital major radius and T is the initial period. Hence,

$$n = \frac{TX(1 - e^2)^{\frac{1}{2}} \ln(\frac{1}{e})}{4\pi^2 a}. \quad (21)$$

For Earth, $T = 3.156 \times 10^7 s$, $X = c = 2.998 \times 10^8 m/s$, $a = 1.496 \times 10^{11} m$, and $e = 0.0167$. Hence, Earth would escape from the sun's gravitational influence

after

$$n = \frac{(3.156 \times 10^7)(2.998 \times 10^8 m/s)(1 - 0.0167^2)^{\frac{1}{2}} \ln(\frac{1}{0.0167})}{4\pi^2(1.496 \times 10^{11})} \approx 6.6 \times 10^3, \quad (22)$$

revolutions. If each revolution takes approximately 1 year, then the escape time is a few thousand years. However, the age of the solar system is 4.6×10^9 years. The escape time is smaller than this by a factor of approximately one million. Therefore, the speed of gravitational waves cannot represent the speed of the gravitational field. From equation (22), the speed of a gravitational field X must be much greater than the speed of light c ; this is more in line with Newton's argument that the force of gravity acts at a distance.

We may consider the following analogy: we use a rope to pull a kite. When we shake it hard, the rope will fluctuate and pass to the kite at a certain wave speed, however, when we loosen the rope, the kite instantly loses control. It is inappropriate to use the wave speed of the rope to represent the speed of the force of the rope on the kite. With this considered, how do gravitational waves affect gravity? Since the revolution speed of the sun will cause gravitational waves, how are gravitational waves distributed around the sun?

4 The Influence of Gravitational Waves Produced by the Sun on the Surrounding Gravity

Gravitational waves caused by the movement of the sun are akin to water waves caused by ships. For the convenience of explanation, we have turned the three-dimensional space problem into a two-dimensional problem. The gravitational influence caused by gravitational waves is different in the direction of the sun's velocity and the vertical direction, as shown in Figure 4.

Assuming that, without considering the general Doppler effect, the ratio of the gravitational increase caused by gravitational waves to Newtonian gravitation is r_w , we introduce a gravitational wave influence factor of f_w and $f_w = 1.0$. The above figure shows that, due to the general Doppler effect of gravitational waves, the energy of gravitational waves is largest in the direction of the sun's velocity and the impact on gravity is the greatest. The planet's orbital surface is perpendicular to the direction of the sun's velocity and the gravitational wave is relatively small, as shown below:

4.1 Calculation of the Influence Factor of Gravitational Waves in the Direction of the Sun's Velocity

We know that the revolution speed of the sun is v_s . Assuming that the sun moves from position O to position O' after time T, the gravitational waves generated in the direction

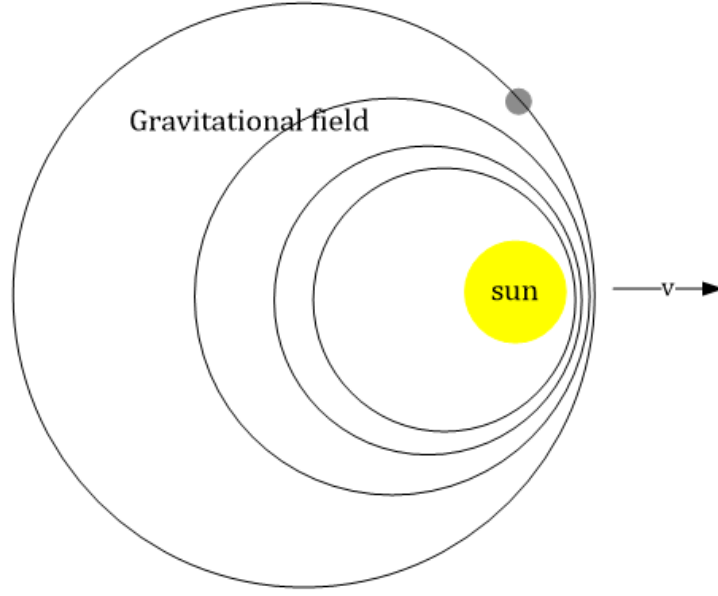


Figure 4: The gravitational wave model generated by the sun's movement.

of the sun's velocity during this period are all located between O'B. According to the general Doppler effect of gravitational waves, the influence factor of gravitational waves in this direction is as below:

$$f_w = \frac{c + v_s}{c} > 1.0. \quad (23)$$

4.2 Calculation of the Influence Factor of Gravitational Waves in the Vertical Direction of the Sun's Velocity

The gravitational waves in the direction perpendicular to the sun's velocity are located between O'A; it is only necessary to calculate the ratio between O'B and O'A to determine the gravitational wave density relationship in the two directions.

$$O'B = cT - v_s T, \quad (24)$$

$$O'A = [(cT)^2 - (v_s T)^2]^{\frac{1}{2}}, \quad (25)$$

thus:

$$f_w \approx \left(\frac{c + v_s}{c}\right) \times \left(\frac{c - v_s}{c + v_s}\right)^{\frac{1}{2}} = \left(\frac{c^2 - v_s^2}{c^2}\right)^{\frac{1}{2}}, \quad (26)$$

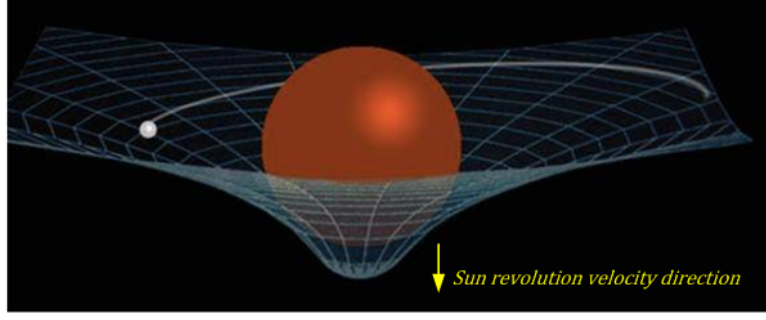


Figure 6: GR space-time depression model

4.4 Calculation of the Influence Factor of Gravitational Waves on the Reverse of the Sun's Velocity

Behind the vertical plane (to the left of the red line), Figure 5 shows that the density of the gravitational waves will continue to decrease and reach a minimum in the opposite direction of the sun's velocity. At this time $\frac{O'B}{O'C} = \frac{c-v_s}{c+v_s}$, the gravitational wave influence factor is as below:

$$f_w \approx \left(\frac{c+v_s}{c}\right) \times \frac{c-v_s}{c+v_s} = \frac{c-v_s}{c}. \quad (27)$$

Substituting $v_s = c$ into (26) and (27), it can be determined that when the speed of the sun reaches c , the orbital surface of the planet perpendicular to the sun's velocity (the position of the red line) and the position behind it (the left side of the red line) is no longer affected by gravitational waves.

4.5 Calculation of the Influence Factor of Gravitational Waves at any Position

As shown in Figure 5, assuming that the angle between $O'D$ and the red line is θ (with D at any position), then

$$OD^2 = O'D^2 + OO'^2 - 2O'D \times OO' \cos\left(\frac{\pi}{2} - \theta\right), \quad (28)$$

we get:

$$O'D = \frac{2OO' \cos(\frac{\pi}{2} - \theta) + [4(OO' \cos(\frac{\pi}{2} - \theta))^2 - 4(OO'^2 - OD^2)]^{\frac{1}{2}}}{2},$$

then,

$$\frac{O'B}{O'D} = \frac{2O'B}{2OO' \cos(\frac{\pi}{2} - \theta) + [4(OO' \cos(\frac{\pi}{2} - \theta))^2 - 4(OO'^2 - OD^2)]^{\frac{1}{2}}}, \quad (29)$$

thus:

$$f_w \approx \left(\frac{c+v_s}{c}\right) \times \frac{c-v_s}{v_s \cos(\frac{\pi}{2}-\theta) + [(v_s \cos(\frac{\pi}{2}-\theta))^2 - (v_s^2 - c^2)]^{\frac{1}{2}}}. \quad (30)$$

4.6 The Influence of Gravitational Waves on Gravity

Assuming that the gravitational force of an object under the influence of gravitational waves is F_w , F_w can be regarded as two parts:

Part 1: Newtonian gravity $F = \frac{G_0 M m}{r^2}$.

Part 2: The gravity contributed by the gravitational wave $r_w f_w F$. (r_w is the ratio of the gravitational increase caused by gravitational waves to Newtonian gravitation.

Thus, we get:

$$F_w = F + r_w \times f_w \times F. \quad (31)$$

Let us take the orbital position as an example to illustrate the calculation of gravity under the influence of gravitational waves:

$$F_w = F + r_w \times f_w \times F = F \times \left(1 + r_w \times \left(\frac{c^2 - v_s^2}{c^2}\right)^{\frac{1}{2}}\right), \quad (32)$$

As there is also a general Doppler effect between planets and gravitational waves, it is also necessary to consider the influence of this factor. Assuming that the speed of the planet is v_p and the speed of the planet in the direction of the gravitational wave is v_{pw} , then the chase factor $\frac{c-v_{pw}}{c}$ between the planet and the gravitational wave can be obtained and this factor is put into (32) to get:

$$F_w = F \times \left(1 + r_w \times \left(\frac{c^2 - v_s^2}{c^2}\right)^{\frac{1}{2}} \times \frac{c - v_{pw}}{c}\right), \quad (33)$$

substituting F , we get:

$$F_w = \frac{G_0 M m}{r^2} \times \left(1 + r_w \times \left(\frac{c^2 - v_s^2}{c^2}\right)^{\frac{1}{2}} \times \frac{c - v_{pw}}{c}\right). \quad (34)$$

here $r_w \approx 0.00042$; this value was derived from a program simulation.

In the same way, the gravity of other positions can be calculated. We write the gravity equation of any position:

$$F_w = \frac{G_0 M m}{r^2} \times \left(1 + r_w \times \frac{c + v_s}{c} \times \left(\frac{c - v_s}{v_s \cos(\frac{\pi}{2} - \theta) + [(v_s \cos(\frac{\pi}{2} - \theta))^2 - (v_s^2 - c^2)]^{\frac{1}{2}}} \times \frac{c - v_{pw}}{c}\right)\right) \quad (35)$$

4.7 Gravitational Waves Caused by the Rotation of the Sun

The sun's rotation can also cause gravitational waves, however, the sun's revolution speed of 240km/s is much greater than its rotation speed of 2km/s. As such, this physical model does not consider the influence of gravitational waves caused by rotation. To obtain more precise calculations, we must consider this factor.

5 Analysis of the Influence of Gravitational Waves on Planetary Orbits

If the planet's orbital surface is not completely perpendicular to the velocity of the sun and the orbit is split over both sides of the red line, then the impact of gravitational waves on planets is also irregular, which affects the orbit and contributes part of the force to planetary precession. The closer the planet's orbit is to the sun, the greater the gravitational wave density gradient and the more obvious the effect of precession; the farther the distance, the less obvious.

Based on formula (35), we have briefly simulated the contribution of gravitational waves to the planet's precession per century. The data are as follows:

Mercury 43"

Venus 240"

Earth 3"

Mars 1"

Jupiter 0.8"

Saturn 0.1"

Let us examine the calculation formula of GR.

$$\delta\dot{\varphi} \simeq \frac{0.0383}{RT} \quad (36)$$

From the formula, GR does not consider the angle between the planet's orbital plane and the sun's vertical plane (the red line in Figure 5) nor the eccentricity of the orbit when calculating the planetary precession. However, we must consider them as the main factors in the data calculated by formula (35). These may be the biggest differences between the two. Below, we substitute the R and T values of each planet (see Figure 7) for GR calculation.

The calculated precession data of each planet per century is as follows:

Mercury 41.06"

Venus 8.6"

Earth 3.83"

Mars 1.34"

Planet	M/M_0	T(yr)	R(au)
Mercury	1.66×10^{-7}	0.241	0.387
Venus	2.45×10^{-6}	0.615	0.723
Earth	3.04×10^{-6}	1.000	1.00
Mars	3.23×10^{-7}	1.881	1.52
Jupiter	9.55×10^{-4}	11.86	5.20
Saturn	2.86×10^{-4}	29.46	9.54
Uranus	4.36×10^{-5}	84.01	19.19
Neptune	5.18×10^{-5}	164.8	30.07

Figure 7: Data for the major planets in the solar system, giving the planetary mass relative to that of the sun, the orbital period in years, and the mean orbital radius relative to that of Earth.

Jupiter 0.062"

Saturn 0.0136"

Uranus 0.00238"

Except for Venus's precession data of 240" vs 8.6", the data of other planets are relatively close to GR.

Let us examine the characteristics of Venus: Venus's eccentricity is abnormally low ($e = 0.0068$), which makes its perihelion extremely sensitive to small disturbances. However, the angle between its orbit and the vertical plane of the sun is very large (3.39°); thus, we have reason to believe that gravitational waves will have a significant influence on the orbital precession of Venus.

Why is the data of Venus (240" vs 8.6") so different? From formula (36), it can be determined that GR does not consider the eccentricity and the angle between the orbital surface and the vertical surface of the sun. Under different eccentricities and angles, the precession data calculated by GR remains the same. This may be the reason for the large difference between the two.

In addition to causing planetary precession, gravitational waves also cause planets to move away from the sun. We know there is also a general Doppler effect between the

planet's revolution velocity and the gravitational waves caused by the sun. The previous 3.2 "Energy Conservation" has analyzed the influence of the general Doppler effect on orbital energy. Gravitational waves also cause the planetary orbital mechanical energy to continue to increase; this causes planets to gradually move away from the sun.

6 Conclusion

The discovery of gravitational waves provides a new way for us to understand the universe, however, the speed of gravitational waves does not represent the speed of gravitational fields. The speed of action of gravitational fields is much greater than the speed of gravitational waves. As stated by Newton: Gravity is an action-at-a-distance force. Gravitational waves caused by the revolution of the sun affect the orbits of planets and provide some planetary precession data. The general Doppler effect of gravitational waves also causes the planetary orbital mechanical energy to continue to increase slowly until the planet escapes from the solar system. Gravitational waves exist; the gravitational model under the influence of gravitational waves that we constructed was a physical model. Through the calculation of planetary orbital precession, the correctness of the gravitational equation under the influence of gravitational waves is verified, which shows that the gravitational physical model has a certain research value and also provides strong evidence to supplement GR. GR provides ideas for the correction of Newton's gravitational equation.

Finally, we also ask the following questions:

Is the acceleration of planetary orbits caused by the gravitational wave general Doppler effect related to the accelerated expansion of the universe?

Is there an association between the action-at-a-distance of the gravitational field and that in quantum mechanics?

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