Blocking with Aliasing of Quadratic Main Effects

The DSD blocking scheme for continuous factors and fixed blocks requires that one center run be added to $\max(B - k, 0)$ of the blocks. This may, at times, lead to designs that are too costly. If the experimenter is willing take the risk of confounding potential block effects with potential quadratic effects, the following blocking scheme, which we refer to as the *Greedy* blocking scheme, can be implemented. With this scheme, each additional block after the first is aliased with a linear combination of the quadratic main effects. The motivation for such a design is economic and analogous to the use of a resolution IV design. When practitioners employ a resolution IV design, they do so with the realization that some pairs of two-factor interactions are directly confounded. If such a pair is determined to be active, the experimenter will either have to decouple the terms with followup experimentation, or use domain knowledge coupled with the results of the experiment to argue that the observed effect was due to one or the other of the coupled interaction terms. The same risk is assumed with the Greedy blocking scheme, wherein quadratic effects are aliased with block effects. We describe the Greedy blocking scheme below.

Greedy Blocking Scheme. Assume that all m factors are quantitative and that an $m-$factor DSD has been constructed, so that $n = 2m + 1$. Assume further that the design is to be partitioned into B blocks, where $2 \leq B \leq m$. Any partitioning of the n runs into B blocks such that:

1. The two treatment combinations in each fold-over pair are assigned to the same block, and

2. The single center-run is added to one block

produces a blocked design in which block effects are orthogonal to linear main effects. Each block effect is confounded with a linear combination of quadratic main effects.

To illustrate the use of this blocking scheme, assume that the 13 runs in the six factor DSD is to be run in blocks of size 5, 4, and 4. The design in Table 1 was constructed using this blocking scheme. If the *a priori* model is (2) (from the paper) the complex aliasing between blocks and quadratic effects is shown in Table 2. The large values in Table 2 provide more support for our preference for the blocking approach that guarantees estimability of quadratic effects.

Table 1: Six-factor DSD in three blocks of size 4, 4, and 5

Run	Block	X_1	X_2	X_3	X_4	X_5	X_6
1	1	0	1	1	1	1	1
2	1	0	-1	-1	-1	-1	-1
3	1	1	0	1	-1	-1	1
4	1	-1	0	-1	1	1	-1
5	2	1	1	0	1	-1	-1
6	2	-1	-1	0	-1	1	1
7	2	1	-1	1	0	1	-1
8	2	-1	1	-1	0	-1	1
9	3	1	-1	-1	1	0	1
10	3	-1	1	1	-1	0	-1
11	3	1	1	-1	-1	1	0
12	3	-1	-1	1	1	-1	0
13	3	0	0	0	0	0	0

Table 2: Alias matrix for block factors and quadratic main effects

Blocking Factor	X_1^2	X_2^2	X_3^2	X_4^2	X_5^2	X_6^2
Z_1	0.5	0.5	1.0	1.0	1.0	1.0
Z_2	1.0	1.0	0.5	0.5	1.0	1.0
Z_3	0.8	0.8	0.8	0.8	0.4	0.4