Reply to the Referee 2

Re: (Manuscript Number: GAPA-2019-0596) “Nonlinear Separation Methods and Applications for Vector Equilibrium Problems Using Improvement Sets” by Yang Dong Xu, Cheng Yu Lei and Chun Yu Sun

We are thankful to this referee for the comments on the paper. The paper has been revised following the report. The detailed revisions are listed below.

(1) Page 1, lines 1-2: According to your suggest, the title of this paper has been revised, that is “Nonlinear Separation Methods and Applications for Unified Vector Equilibrium Problems with Matrix Inequality Constraints” has been replaced by “Nonlinear Separation Methods and Applications for Vector Equilibrium Problems Using Improvement Sets”.

(2) Page 1, line 5: “under an improvement set and with matrix inequality constraints” has been replaced by “using improvement sets and with matrix inequality constraints”.

(3) Throughout this paper, the abbreviation “UVEP” has been replaced by “E-VEP”.

(4) Page 2, line 25: “inequalitiy” has been replaced by “inequality”.

(5) Page 3, line 3: “in the image space (IS)” has been replaced by “in the IS”.

(6) Page 3, line 4: “ linear and nonlinear regular weak separation functions” has been replaced by “ linear or nonlinear regular weak separation functions”.

(7) Page 3, line 6: “improvement sets may be not convex or not a cone” has been replaced by “improvement sets may be not convex or not cone”.

(8) Page 4, line 5:

$$\text{“}P^* := \{z \in \mathbb{R}^n : \langle z, y\rangle \geq 0,\ \forall\, y \in P\}.$$

$$P^\sharp := \{z \in \mathbb{R}^n : \langle z, y\rangle > 0,\ \forall\, y \in P\backslash\{0_{\mathbb{R}_m}\}\}.\text{”}$$

has been replaced by

$$\text{“}P^* := \{z \in \mathbb{R}^n : \langle z, y\rangle \geq 0,\ \forall\, y \in P\},$$

$$P^\sharp := \{z \in \mathbb{R}^n : \langle z, y\rangle > 0,\ \forall\, y \in P\backslash\{0_{\mathbb{R}_m}\}\},\text{”}$$

(9) Page 4, line -1: the comma “,” has been replaced by the full stop “.”.

(10) Page 6, line -1: the semicolon ";" has been replaced by the full stop ".".

(11) Page 7, line 17: we have added the full stop "." after the following expression

$$\hat{w}(u_0, v_0; \theta, \mathcal{A}) > 0, \ \ \forall\, (\theta,\, \mathcal{A}) \in E^\sharp \times S^l_+$$

(12) Page 9, line 22: we have added the full stop "." after the following expression

$$\forall\, v \notin D, \ \exists \gamma_v \in \Gamma \ s.t. \ \underline{w}(v; \gamma_v) < 0$$

(13) Page 10, lines 1-8: We have added Example 3.3 to show that the closedness assumption of $E \cup \{0_{\mathbb{R}^m}\}$ is necessary for Proposition 3.1 to hold.

(14) Page 14, line 3:

$$\text{``} \inf_{(\alpha,\mathcal{A})\in\mathbb{R}_+\times S^l_+} \Big[-\alpha\Delta_{E\cup\{0_{\mathbb{R}^m}\}}(u) + \|u\| + +\langle \mathcal{A}, A\rangle \Big] \leq 0, \ \ \forall\, (u, A) \in \mathcal{K}(x)\text{''}.$$

has been replaced by

$$\text{``} \inf_{(\alpha,\mathcal{A})\in\mathbb{R}_+\times S^l_+} \Big[-\alpha\Delta_{E\cup\{0_{\mathbb{R}^m}\}}(u) + \|u\| + \langle \mathcal{A}, A\rangle \Big] \leq 0, \ \ \forall\, (u, A) \in \mathcal{K}(x)\text{''}.$$

(15) Page 14, lines 8-20: We have added Example 3.4 to show that the closedness assumption of $E \cup \{0_{\mathbb{R}^m}\}$ is necessary for Proposition 3.1 to hold.

(16) Page 16, line 5: the semicolon "$\Upsilon = \langle N, (\Lambda_i)_{i\in N}, (u_i)_{i\in N}\rangle$" has been replaced by the full stop "$\Upsilon = \langle N, (\Lambda_i)_{i\in N}, (\phi_i)_{i\in N}\rangle$".

(17) Page 16, lines 13-14:

$$\text{``}(a(b_i))_j = \begin{cases} a_j & \text{if } j \neq i, \\ b_j, & \text{if } j = i. \end{cases} \text{''}$$

has been replaced by

$$\text{``}(a(b_i))_j = \begin{cases} a_j, & \text{if } j \neq i, \\ b_j, & \text{if } j = i. \end{cases} \text{''}$$

(18) Page 21, line 12: "Luo, H.Z., Wu, H.X., Liu, J.Z." has been replaced by "Luo, H.Z. Wu, H.X., Liu, J.Z.".

(19) According to your suggest, we have added the following references in the literature:

(1) Chen, J.W., Kobis, E., K obis, M., Yao, J.C.: Image space analysis for constrained inverse vector variational inequalities via multiobjective optimization. J. Optim. Theory Appl. 177, 816-834 (2018)

(2) Chen, J.W., Li, S.J., Wan, Z., Yao, J.C.: Vector variational-like inequalities with constraints: separation and alternative. J. Optim. Theory Appl. 166, 460-479 (2015)

(3) Mai, T.T., Luu, D.V.: Optimality conditions for weakly efficient solutions of vector variational inequalities via convexificators. J. Nonlinear Var. Anal. 2, 379-389 (2018)

(4) Li, J.L.: Constrained ordered equilibrium problems and applications, J. Nonlinear Var. Anal. 1, 357-365 (2017)