

Appendix A. List of abbreviations and select terms used in the manuscript in alphabetical order

AIC: Akaike Information Criterion

C: Celsius

cm: centimeters

commercial size trees: Development classes 3-5 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Younger, older, and mature productive forest with satisfactory stand density. Species proportions are reported according to volume in these harvest classes.

complete crown: The function biomass combination from the current study of live crown (LC_{dh}) and dead branches (DB_d).

DB_d : Dead branch biomass single-variable model

dbh: Diameter at breast height (1.3 m)

DW: Dry weight

FW: Fresh weight

H40: Height of tree at 40 years of age

ha: hectare

height-to-live-crown: distance from the ground to the base of the live crown, ignoring one time a single live branch if separated by more than two whorls from the next live branch.

kg: kilogram

LB_d : Live branch biomass single-variable model

LB_{dh} : Live branch biomass two-variable model

LC_d : Live crown biomass single-variable model

LC_{dh} : Live crown biomass two-variable model

LF_d : Leaf biomass single-variable model

m: meter

m.a.s.l.: meters above sea level

m.t.b.: million tons biomass

N: Number

NFI: National Forest Inventory

NLME: Nonlinear mixed-effects model

NNFI: Norwegian National Forest Inventory

NNFI8: Norwegian National Forest Inventory 8th inventory (2000-2004)

NNFI9: Norwegian National Forest Inventory 9th inventory (2005-2009)

older stands: Development classes 4 and 5 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Older and mature productive forest with satisfactory stand density.

p: p-value

RMSE: Root Mean Square Error

sapling size trees: Development classes 1 and 2 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest. Species proportions are reported according to crown cover percentage in these harvest classes.

SB_d: Stem bark biomass single-variable model

SB_{dh}: Stem bark biomass two-variable model

std. error: Standard error

SW_d: Stemwood biomass single-variable model

SW_{dh}: Stemwood biomass two-variable model

TAG_B: Total aboveground biomass component combination from Bollandsås et al. (2009) using: over-bark (“Stem”) + total crown (“Tree crown”) biomass

TAG_{combination 1}: Total aboveground component combination using: TS_{dh} + LC_{dh} + DB_d

TAG_{combination 2}: Total aboveground component combination using: SW_{dh} + SB_d + LB_{dh} + LF_d + DB_d

TAG_d: Total aboveground biomass single-variable model (model fit with the BM_{ts} + BM_{lc} + BM_{db} biomass estimates (Appendix B))

TAG_{dh}: Total aboveground biomass two-variable model (model fit with the BM_{ts} + BM_{lc} + BM_{db} biomass estimates (Appendix B))

TAG_M: Total aboveground biomass for Marklund using: stemwood (B-5) + stem bark (B-8) + live branch (B-11) + dead branch (B-16) + leaves (where leaf biomass = B-5 * (0.011^a/0.52^b) (^a Factor currently applied by NNFI for UNFCCC reporting; ^b de Wit et al. 2006)

TAG_S: Total aboveground biomass component combination of the current study using: SW_{dh} + SB_d + LB_{dh} + DB_d + LF_d

total crown: Observed crown biomass of the mountain birch sample trees including the live and dead branches (if present) (Bollandsås et al. 2009).

TS_d: Total stem biomass single-variable model

TS_{dh}: Total stem biomass two-variable model

UNFCCC: United Nations Framework Convention on Climate Change

Unprod.: Unproductive birch forest = potential yield < 1 m³ ha⁻¹ yr⁻¹

volume-weighted total stem biomass: The average stem biomass weighted by volume of the stem section from which the sample disk was taken.

young stands: Development classes 1 and 2 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest.

Appendix B. Detailed methods for the aboveground biomass dataset

Total stem biomass estimate

$$\begin{aligned}
 (1) \text{ DW:FW}_{disk_i} &= \frac{DW_{disk_i}}{FW_{disk_i}} \\
 (2) V_{s_i} &= \frac{l_{s_i}(g_{1_i} + g_{2_i})}{2} \text{ (Smalian's formula)} \\
 (3) V_{t_j} &= \sum_{i=1} V_{s_i} \\
 (4) \text{ DW:FW}_{vw_j} &= \sum_{i=1} \left(\text{DW:FW}_{disk_i} \left(\frac{V_{s_i}}{V_{t_j}} \right)_i \right) \\
 (5) \text{ BM}_{ts_j} &= \text{DW:FW}_{vw_j} * \text{FW}_{stem_j}
 \end{aligned}$$

where:

steps (1), (2), (3), (4), and (5) correspond to the written steps in the manuscript

DW:FW_{disk_i} = Dry weight to fresh weight ratio of stem disk *i* with bark

DW_{disk_i} = Dry weight of stem disk *i* with bark (g)

FW_{disk_i} = Fresh weight of stem disk *i* with bark (g)

V_{s_i} = Volume of stem section *i* by Smalian's formula (m³)

l_{s_i} = Length of stem section *i* (cm)

g_{1_i} = Lower surface's cross sectional area of an ellipse of section *i* (mm²)

g_{2_i} = Upper surface's cross sectional area of an ellipse of section *i* (mm²)

g = Cross sectional area of an ellipse = $\frac{\pi}{4} (d_1 * d_2)$

d₁ = Maximum diameter (mm)

d_2 = Minimum diameter (mm)

V_{t_j} = Total stem volume of tree j (m³)

DW: FW_{vw_j} = Volume-weighted dry weight fresh weight ratio of the stem of tree j

FW_{stem_j} = Fresh weight of the stem of tree j (total fresh weight of disks + the rest of the stem of tree j)(kg)

BM_{ts_j} = The volume-weighted total stem biomass of tree j (kg)

Stemwood biomass estimate

$$(6) A_{obi} \& A_{swi} = \frac{\pi}{4} (d_{1i} * d_{2i})$$

$$(7) P_{swi} = \frac{A_{swi}}{A_{obi}}$$

$$(8) P_{svi} = \frac{V_{si}}{V_{t_j}}$$

$$(9) P_{swsi} = P_{swi} * P_{svi}$$

$$(10) P_{vwsw_j} = \sum_{i=1} P_{swsi}$$

$$(11) BM_{sw_j} = P_{vwsw_j} * BM_{ts_j}$$

where:

steps (6), (7), (8), (9), (10), and (11) correspond to the written steps in the manuscript

A_{obi} = Cross sectional elliptical over-bark area of stem disk i (mm²)

A_{swi} = Cross sectional elliptical stemwood area of stem disk i (mm²)

d_{1i} = Maximum diameter of stem disk i (mm)

d_{2i} = Minimum diameter of stem disk i (mm)

P_{swi} = Proportion of stemwood cross sectional area of stem disk i assigned to its corresponding stem section

P_{svi} = Proportion of the total stem volume that stem section i represents

V_{si} = Volume of stem section i by Smalian's formula (m³)

V_{t_j} = Total stem volume of tree j (m³)

P_{swsi} = Proportion of the stemwood in stem section i

P_{vwsw_j} = Volume-weighted proportion of stemwood in the stem of tree j

BM_{ts_j} = The volume-weighted total stem biomass of tree j (kg)

BM_{sw_j} = The volume-weighted stemwood biomass of tree j (kg)

Stem bark biomass estimate

$$(12) P_{sbi} = 1 - P_{swi}$$

$$(13) P_{sbis_i} = P_{sbi} * P_{svi}$$

$$(14) P_{vwsb_j} = \sum_{i=1} P_{sbis_i}$$

$$(15) BM_{sb_j} = P_{vwsb_j} * BM_{ts_j}$$

where:

steps (12), (13), (14), and (15) correspond to written steps in the manuscript

P_{sbi} = Proportion of stem bark of stem disk i

P_{swi} = Proportion of stemwood cross sectional area of stem disk i assigned to its corresponding stem section

P_{sbis_i} = Proportion of stem bark of section i

P_{svi} = Proportion of the total stem volume that stem section i represents

P_{vwsbj} = Volume-weighted proportion of stem bark of tree j
 BM_{tsj} = The volume-weighted total stem biomass of tree j (kg)
 BM_{sbj} = The volume-weighted stem bark biomass of tree j (kg)

Live crown biomass estimate

$$(16) \quad DW_{lsbj} = \sum_{i=1} (DW_{lb_i} + DW_{leaf_i} + DW_{catkins_i})$$

$$(17) \quad FW_{lsbj} = \sum_{i=1} (FW_{lsb_i})$$

$$(18) \quad DW:FW_{lsbj} = \frac{DW_{lsbj}}{FW_{lsbj}}$$

$$(19) \quad BM_{lcj} = DW:FW_{lsbj} * FW_{tlcj}$$

where:

steps (16), (17), (18), and (19) correspond to the written steps in the manuscript

DW_{lsbj} = Sum of the dry weights of live sample branches of tree j (kg)

DW_{lb_i} = Dry weight of the woody material of live sample branch i (kg)

DW_{leaf_i} = Dry weight of the leaves of live sample branch i (kg)

$DW_{catkins_i}$ = Dry weight of the catkins of live sample branch i (kg)

FW_{lsbj} = Sum of the fresh weights of the live sample branches of tree j (kg)

FW_{lsb_i} = Fresh weight of live sample branch i (kg)

$DW:FW_{lsbj}$ = Dry weight to fresh weight ratio of the live sample branches of tree j

FW_{tlcj} = Total fresh weight of the live crown of tree j (FW_{lsbj} + the rest of the live crown) (kg)

BM_{lcj} = The biomass of the live crown of tree j (kg)

Live branch biomass estimate

$$(20) \quad DW_{lbj} = \sum_{i=1} DW_{lb_i}$$

$$(21) \quad DW_{lsbj} = \sum_{i=1} (DW_{lb_i} + DW_{leaf_i} + DW_{catkins_i})$$

$$(22) \quad BM_{lbj} = \frac{DW_{lbj}}{DW_{lsbj}} * BM_{lcj}$$

where:

steps (20), (21), and (22) correspond to the written steps in the manuscript

DW_{lbj} = Sum of the dry weight of the woody material of live sample branches of tree j (kg)

DW_{lb_i} = Dry weight of the woody material of live sample branch i (kg)

DW_{lsbj} = Sum of the dry weight of live sample branches of tree j (kg)

DW_{leaf_i} = Dry weight of the leaves of live sample branch i (kg)

$DW_{catkins_i}$ = Dry weight of the catkins (if present) of live sample branch i (kg)

BM_{lcj} = The biomass of the live crown of tree j (kg)

BM_{lbj} = The biomass of live branches of tree j (kg)

Leaf biomass estimate

$$(23) \quad DW_{leafj} = \sum_{i=1} DW_{leaf_i}$$

$$(24) \quad DW_{leaf+catkinsj} = DW_{leafj} + DW_{catkinsj}$$

$$(25) \quad BM_{leafj} = \frac{DW_{leaf+catkinsj}}{DW_{lsbj}} * BM_{lcj}$$

where:

steps (23), (24), and (25) correspond to the written steps in the manuscript

DW_{leaf_j} = Sum of the dry weight of leaves of the live sample branches of tree j (kg)

DW_{leaf_i} = Dry weight of the leaves of live sample branch i (kg)

$DW_{leaf+catkins_j}$ = Dry weight of leaves and catkins (if present) of tree j (kg)

$DW_{catkins_j}$ = Dry weight of the catkins (if present) of tree j (kg)

DW_{lsb_j} = Sum of the dry weight of live sample branches of tree j (kg)

BM_{lc_j} = The biomass of the live crown of tree j (kg)

BM_{leaf_j} = The biomass of the leaves and catkins (if present) of tree j (kg)

Dead branch biomass estimate

$$(26) \quad DW:FW_{sdb_j} = \frac{DW_{sdb_j}}{FW_{sdb_j}}$$

$$(27) \quad BM_{db_j} = DW:FW_{sdb_j} * FW_{tdb_j}$$

where:

steps (26) and (27) correspond to the written steps in the manuscript

$DW:FW_{sdb_j}$ = Dry weight to fresh weight ratio of sampled dead branches of tree j

DW_{sdb_j} = Dry weight of sampled dead branches of tree j (kg)

FW_{sdb_j} = Fresh weight of sampled dead branches of tree j (kg)

FW_{tdb_j} = Total fresh weight of all dead branches in the crown of tree j (FW_{sdb_j} + the rest of the dead branches in the crown of tree j)(kg)

BM_{db_j} = The biomass of dead branches (if present) of tree j (kg)

Total aboveground biomass estimate

$$(28) \quad BM_{tag_j} = BM_{ts_j} + BM_{lc_j} + BM_{db_j}$$

where:

step (28) corresponds to the written step in the manuscript

BM_{ts_j} = The volume-weighted total stem biomass of tree j (kg)

BM_{lc_j} = The biomass of the live crown of tree j (kg)

BM_{db_j} = The biomass of the dead branches of tree j (kg)

BM_{tag_j} = The total aboveground biomass of tree j (kg)

Appendix C. Covariance matrices for single- and two-variable functions

Table A.C.1. Parameter covariance matrix (Ψ_f) of the single-variable biomass

function for total aboveground biomass (TAG_a).

	β_0	β_d
β_0	0.00011	
β_d	-0.00044	0.00195

Table A.C.2. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for total stem biomass (TS_d).

	β_0	β_d
β_0	0.00012	
β_d	-0.00055	0.00291

Table A.C.3. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for stemwood biomass (SW_d).

	β_0	β_d
β_0	0.00009	
β_d	-0.00049	0.00310

Table A.C.4. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for stem bark biomass (SB_d).

	β_0	β_d
β_0	7.53085 [10 ⁻⁶]	
β_d	-0.00020	0.00590

Table A.C.5. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for live crown biomass (LC_d).

	β_0	β_d
β_0	0.00003	
β_d	-0.00050	0.01042

Table A.C.6. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for live branch biomass (LB_d).

	β_0	β_d
β_0	7.72835 [10 ⁻⁶]	
β_d	-0.00026	0.00938

Table A.C.7. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for leaf biomass (LF_d).

	β_0	β_d
β_0	5.95136 [10 ⁻⁶]	
β_d	-0.00030	0.01888

Table A.C.8. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for dead branch biomass (DB_d).

	β_0	β_d
β_0	5.21287 [10 ⁻⁶]	
β_d	-0.00063	0.08046

Table A.C.9. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for total aboveground biomass (TAG_{dh}).

	β_0	β_d	β_h
β_0	0.00006		
β_d	0.00020	0.00489	
β_h	-0.00070	-0.00673	0.01279

Table A.C.10. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for total stem biomass (TS_{dh}).

	β_0	β_d	β_h
β_0	7.63909 [10 ⁻⁶]		
β_d	0.00005	0.00333	
β_h	-0.00019	-0.00440	0.00799

Table A.C.11. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for stemwood biomass (SW_{dh}).

	β_0	β_d	β_h
β_0	4.30251 [10 ⁻⁶]		
β_d	0.00004	0.00332	
β_h	-0.00014	-0.00438	0.00782

Table A.C.12. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for stem bark biomass (SB_{dh}).

	β_o	β_d	β_h
β_o	2.55211 [10 ⁻⁶]		
β_d	0.00008	0.01434	
β_h	-0.00029	-0.02115	0.04571

Table A.C.13. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for live crown biomass (LC_{dh}).

	β_o	β_d	β_h
β_o	0.00049		
β_d	0.00148	0.03068	
β_h	-0.00542	-0.04366	0.08934

Table A.C.14. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for live branch biomass (LB_{dh}).

	β_o	β_d	β_h
β_o	0.00011		
β_d	0.00069	0.03005	
β_h	-0.00237	-0.04188	0.08060

A.C.15. Residual covariance matrix Σ for single-variable biomass functions.

	Res. TAG _d	Res. TS _d	Res. SW _d	Res. SB _d	Res. LC _d	Res. LB _d	Res. LF _d	Res. DB _d
Res. TAG _d	456.92051							
Res. TS _d	357.45628	488.03138						
Res. SW _d	255.96758	331.84240	244.35479					
Res. SB _d	90.75776	134.78428	75.04395	52.84070				
Res. LC _d	107.11887	-94.82201	-60.62131	-28.14675	190.72404			
Res. LB _d	133.43144	-46.20378	-33.96514	-9.53215	176.88983	172.50598		
Res. LF _d	0.29437	-7.36793	-4.88840	-1.90201	7.30314	5.79553	1.55523	
Res. DB _d	1.80268	5.30791	3.43170	1.50237	-2.67981	-1.79410	-0.28645	0.42875

A.C.16. Residual covariance matrix Σ for two-variable biomass functions.

	Res. TAG _{dh}	Res. TS _{dh}	Res. SW _{dh}	Res. SB _{dh}	Res. LC _{dh}	Res. LB _{dh}
Res. TAG _{dh}	480.29573					
Res. TS _{dh}	296.22257	261.88310				
Res. SW _{dh}	243.53965	195.76021	182.12349			
Res. SB _{dh}	61.66899	71.02619	19.03552	52.66629		
Res. LC _{dh}	195.01797	42.66572	63.31380	-15.55137	160.62961	
Res. LB _{dh}	207.77599	60.74303	69.85139	-3.73878	153.77029	151.42379