#### Appendix A. List of abbreviations and select terms used in the manuscript in alphabetical order 4 AIC: Akaike Information Criterion 7 C: Celsius cm: centimeters commercial size trees: Development classes 3-5 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Younger, older, and mature productive forest with satisfactory stand density. Species proportions are reported according to volume in these harvest classes. complete crown: The function biomass combination from the current study of live crown (LC<sub>dh</sub>) and dead branches (DB<sub>d</sub>). DB<sub>d</sub>: Dead branch biomass single-variable model dbh: Diameter at breast height (1.3 m) DW: Dry weight FW: Fresh weight H40: Height of tree at 40 years of age ha: hectare height-to-live-crown: distance from the ground to the base of the live crown, ignoring one time a single live branch if separated by more than two whorls from the next live branch. kg: kilogram LB<sub>d</sub>: Live branch biomass single-variable model LB<sub>dh</sub>: Live branch biomass two-variable model LCd: Live crown biomass single-variable model LC<sub>dh</sub>: Live crown biomass two-variable model LF<sub>d</sub>: Leaf biomass single-variable model m: meter m.a.s.l.: meters above sea level m.t.b.: million tons biomass

51 52	N: Number
52 53	N. Number
54 55	NFI: National Forest Inventory
56 57	NLME: Nonlinear mixed-effects model
58 59	NNFI: Norwegian National Forest Inventory
60 61	NNFI8: Norwegian National Forest Inventory 8th inventory (2000-2004)
62 63	NNFI9: Norwegian National Forest Inventory 9th inventory (2005-2009)
64 65 66 67	older stands: Development classes 4 and 5 definition from Norwegian NFI (Antón- Fernández and Astrup 2012). Older and mature productive forest with satisfactory stand density.
68 69	p: p-value
70 71	RMSE: Root Mean Square Error
72 73 74 75 76	sapling size trees: Development classes 1 and 2 definition from Norwegian NFI (Antón- Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest. Species proportions are reported according to crown cover percentage in these harvest classes.
70 77 78	SB <sub>d</sub> : Stem bark biomass single-variable model
79 80	SB <sub>dh</sub> : Stem bark biomass two-variable model
81 82	std. error: Standard error
83 84	SW <sub>d</sub> : Stemwood biomass single-variable model
85 86	SW <sub>dh</sub> : Stemwood biomass two-variable model
87 88 89	TAG <sub>B</sub> : Total aboveground biomass component combination from Bollandsås et al. (2009) using: over-bark ("Stem") + total crown ("Tree crown") biomass
90 91	$TAG_{combination \ 1} \text{: Total above ground component combination using: } TS_{dh} + LC_{dh} + DB_{d}$
92 93 94	$TAG_{combination2}$ : Total above ground component combination using: $SW_{dh}+SB_d+LB_{dh}+LF_d+DB_d$
95 96 97	TAG <sub>d</sub> : Total above ground biomass single-variable model (model fit with the $BM_{ts} + BM_{lc} + BM_{db}$ biomass estimates (Appendix B))
97 98 99	TAG <sub>dh</sub> : Total aboveground biomass two-variable model (model fit with the $BM_{ts}$ + $BM_{lc}$ + $BM_{db}$ biomass estimates (Appendix B))

101 TAG<sub>M</sub>: Total aboveground biomass for Marklund using: stemwood (B-5) + stem bark 102 (B-8) + live branch (B-11) + dead branch (B-16) + leaves (where leaf biomass = B-5 \* (0.011<sup>a</sup>/0.52<sup>b</sup>) (<sup>a</sup> Factor currently applied by NNFI for UNFCCC reporting; <sup>b</sup> de Wit et 103 104 al. 2006) 105 TAGs: Total aboveground biomass component combination of the current study using: 106 107  $SW_{dh} + SB_d + LB_{dh} + DB_d + LF_d$ 108 total crown: Observed crown biomass of the mountain birch sample trees including the 109 live and dead branches (if present) (Bollandsås et al. 2009). 110 111 TS<sub>d</sub>: Total stem biomass single-variable model 112 113 114 TS<sub>dh</sub>: Total stem biomass two-variable model 115 UNFCCC: United Nations Framework Convention on Climate Change 116 117 Unprod.: Unproductive birch forest = potential yield  $< 1 \text{ m}^3 \text{ ha}^{-1} \text{ yr}^{-1}$ 118 119 volume-weighted total stem biomass: The average stem biomass weighted by volume of 120 121 the stem section from which the sample disk was taken. 122 young stands: Development classes 1 and 2 definition from Norwegian NFI (Antón-123 124 Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest. 125

#### 126 Appendix B. Detailed methods for the aboveground biomass dataset

Total stem biomass estimate 127

(1) DW:  $FW_{disk_i} = \frac{DW_{disk_i}}{FW_{disk_i}}$ 128

(2)  $V_{s_i} = \frac{l_{s_i}(g_{1_i} + g_{2_i})}{2}$  (Smalian's formula) (3)  $V_{t_j} = \sum_{i=1}^{2} V_{s_i}$ 129

130 (3) 
$$V_{t_j} = \sum_{i=1}^{N} V_{t_i}$$

131 (4) DW: 
$$FW_{vw_j} = \sum_{i=1} \left( DW: FW_{disk_i} \left( \frac{v_{s_i}}{v_{t_j}} \right)_i \right)$$
  
132 (5)  $BM_{ts_j} = DW: FW_{vw_j} * FW_{stem_j}$ 

133 where:

steps (1), (2), (3), (4), and (5) correspond to the written steps in the manuscript 134

- 135 DW:  $FW_{disk_i}$  = Dry weight to fresh weight ratio of stem disk *i* with bark
- $DW_{disk_i} = Dry$  weight of stem disk *i* with bark (g) 136

 $FW_{disk_i}$  = Fresh weight of stem disk *i* with bark (g) 137

- $V_{s_i}$  = Volume of stem section *i* by Smalian's formula (m<sup>3</sup>) 138
- 139  $l_{s_i}$  = Length of stem section *i* (cm)
- $g_{1i}$  = Lower surface's cross sectional area of an ellipse of section *i* (mm<sup>2</sup>) 140
- $g_{2i}$  = Upper surface's cross sectional area of an ellipse of section *i* (mm<sup>2</sup>) 141

g = Cross sectional area of an ellipse =  $\frac{\pi}{4}$  (d<sub>1</sub> \* d<sub>2</sub>) 142

 $d_1 = Maximum diameter (mm)$ 143

Commented [AS1]: mulitplication

- 144 d<sub>2</sub> = Minimum diameter (mm)
- 145  $V_{t_i}$  = Total stem volume of tree *j* (m<sup>3</sup>)
- 146 DW:  $FW_{vw_i}$  = Volume-weighted dry weight fresh weight ratio of the stem of tree j
- $FW_{stem_{j}} = Fresh$  weight of the stem of tree j (total fresh weight of disks + the rest of the 147
- 148 stem of tree *j*)(kg)
- $BM_{ts_i}$  = The volume-weighted total stem biomass of tree *j* (kg) 149
- 150
- 151 Stemwood biomass estimate

152 (6) 
$$A_{ob_i} \& A_{sw_i} = \frac{\pi}{4} (d_{1_i} * d_{2_i})$$

153 (7) 
$$P_{sw_i} = \frac{aw_i}{A_{ob_i}}$$

154 (8) 
$$P_{sv_i} = \frac{v_i}{v_{t_i}}$$

(9)  $P_{sws_i} = P_{sw_i} * P_{sv_i}$ 155

156 (10) 
$$P_{vwsw_j} = \sum_{i=1}^{j} P_{sws_i}$$

157 (11) 
$$BM_{sw_j} = P_{vwsw_j} * BM_{ts_j}$$

159 steps (6), (7), (8), (9), (10), and (11) correspond to the written steps in the manuscript

- 160  $A_{ob_i}$  = Cross sectional elliptical over-bark area of stem disk *i* (mm<sup>2</sup>)
- 161  $A_{sw_i}$  = Cross sectional elliptical stemwood area of stem disk *i* (mm<sup>2</sup>)
- 162  $d_{1_i}$  = Maximum diameter of stem disk *i* (mm)
- $d_{2_i}$  = Minimum diameter of stem disk *i* (mm) 163
- 164  $P_{sw_i}$  = Proportion of stemwood cross sectional area of stem disk *i* assigned to its
- corresponding stem section 165
- 166  $P_{sv_i}$  = Proportion of the total stem volume that stem section *i* represents
- 167  $V_{s_i}$  = Volume of stem section *i* by Smalian's formula (m<sup>3</sup>)
- 168  $V_{t_i}$  = Total stem volume of tree j (m<sup>3</sup>)
- 169  $P_{sws_i}$  = Proportion of the stemwood in stem section *i*
- $P_{vwsw_i}$  = Volume-weighted proportion of stemwood in the stem of tree *j* 170
- 171  $BM_{ts_i}$  = The volume-weighted total stem biomass of tree *j* (kg)
- 172  $BM_{sw_i}$  = The volume-weighted stemwood biomass of tree *j* (kg)
- 173
- 174 Stem bark biomass estimate

175 (12) 
$$P_{sb_i} = 1 - P_{sw}$$

- 176 (13)
- $P_{sbs_i} = P_{sb_i} * P_{sv_i}$   $P_{vwsb_j} = \sum_{i=1}^{N_{sbs_i}} P_{sbs_i}$ (14)177
- $BM_{sb_i} = P_{vwsb_i} * BM_{ts_i}$ 178 (15)
- 179 where:
- steps (12), (13), (14), and (15) correspond to written steps in the manuscript 180
- $P_{sb_i}$  = Proportion of stem bark of stem disk *i* 181
- $P_{sw_i}$  = Proportion of stemwood cross sectional area of stem disk *i* assigned to its 182
- 183 corresponding stem section
- $P_{sbs_i}$  = Proportion of stem bark of section *i* 184
- 185  $P_{sv_i}$  = Proportion of the total stem volume that stem section *i* represents

186  $P_{vwsb_i}$  = Volume-weighted proportion of stem bark of tree *j*  $BM_{ts_i}$  = The volume-weighted total stem biomass of tree *j* (kg) 187  $BM_{sb_i}$  = The volume-weighted stem bark biomass of tree *j* (kg) 188 189 Live crown biomass estimate 190  $DW_{lsb_i} = \sum_{i=1} (DW_{lb_i} + DW_{leaf_i} + DW_{catkins_i})$ 191 (16) $FW_{lsb_i} = \sum_{i=1} (FW_{lsb_i})$ 192 (17) $DW: FW_{lsb_j} = \frac{DW_{lsb_j}}{FW_{lsb_j}}$ (18)193  $BM_{lc_i} = DW: FW_{lsb_j} * FW_{tlc_j}$ 194 (19)195 where: 196 steps (16), (17), (18), and (19) correspond to the written steps in the manuscript 197  $DW_{lsb_i} = Sum of the dry weights of live sample branches of tree j (kg)$  $DW_{lb_i} = Dry$  weight of the woody material of live sample branch *i* (kg) 198 199  $DW_{leaf_i} = Dry$  weight of the leaves of live sample branch *i* (kg) 200  $DW_{catkins_i} = Dry$  weight of the catkins of live sample branch *i* (kg) 201  $FW_{lsb_i} = Sum of the fresh weights of the live sample branches of tree j (kg)$ 202  $FW_{lsb_i}$  = Fresh weight of live sample branch *i* (kg) 203 DW:  $FW_{lsb_i} = Dry$  weight to fresh weight ratio of the live sample branches of tree j  $FW_{tlc_i}$  = Total fresh weight of the live crown of tree *j* ( $FW_{lsb_j}$  + the rest of the live 204 205 crown )(kg) 206  $BM_{lc_i}$  = The biomass of the live crown of tree *j* (kg) 207 208 Live branch biomass estimate 209  $DW_{lb_i} = \sum_{i=1} DW_{lb_i}$ (20) $DW_{lsb_{j}} = \sum_{i=1}^{N_{l}} (DW_{lb_{i}} + DW_{leaf_{i}} + DW_{catkins_{i}})$  $BM_{lb_{j}} = \frac{DW_{lb_{j}}}{DW_{lsb_{j}}} * BM_{lc_{j}}$ 210 (21)211 (22)212 where: 213 steps (20), (21), and (22) correspond to the written steps in the manuscript 214  $DW_{lb_i} = Sum of the dry weight of the woody material of live sample branches of tree j$ 215 (kg) 216  $DW_{lb_i} = Dry$  weight of the woody material of live sample branch *i* (kg) 217  $DW_{lsb_i} = Sum of the dry weight of live sample branches of tree j (kg)$ 218  $DW_{leaf_i} = Dry$  weight of the leaves of live sample branch *i* (kg) 219  $DW_{catkins_i} = Dry$  weight of the catkins (if present) of live sample branch *i* (kg) 220  $BM_{lc_i}$  = The biomass of the live crown of tree *j* (kg) 221  $BM_{lb_i}$  = The biomass of live branches of tree *j* (kg) 222 Leaf biomass estimate 223 224 (23) $DW_{leaf_i} = \sum_{i=1} DW_{leaf_i}$  $DW_{leaf+catkins_{j}} = DW_{leaf_{j}} + DW_{catkins_{j}}$  $BM_{leaf_{j}} = \frac{DW_{leaf+catkins_{j}}}{DW_{lsb_{j}}} * BM_{lc_{j}}$ 225 (24)(25) 226

227 where:

- 228 steps (23), (24), and (25) correspond to the written steps in the manuscript
- 229  $DW_{leaf_i}$  = Sum of the dry weight of leaves of the live sample branches of tree *j* (kg)
- $DW_{leaf_i} = Dry$  weight of the leaves of live sample branch *i* (kg) 230
- 231  $DW_{leaf+catkins_i} = Dry$  weight of leaves and catkins (if present) of tree j (kg)
- 232  $DW_{catkins_i} = Dry$  weight of the catkins (if present) of tree *j* (kg)
- 233  $DW_{lsb_i} = Sum of the dry weight of live sample branches of tree j (kg)$
- 234  $BM_{lc_i}$  = The biomass of the live crown of tree *j* (kg)

 $BM_{leaf_i}$  = The biomass of the leaves and catkins (if present) of tree j (kg) 235

236

248

237

Dead branch biomass estimate DW:  $FW_{sdb_j} = \frac{DW_{sd}}{FW_{sd}}$ 238 (26)

$$BM_{db_i} = DW; FW_{cdb_i} *$$

239 (27) 
$$BM_{db_j} = DW: FW_{sdb_j} * FW_{tdb}$$

240 where:

241 steps (26) and (27) correspond to the written steps in the manuscript

242 DW:  $FW_{sdb_i}$  = Dry weight to fresh weight ratio of sampled dead branches of tree *j* 

243  $DW_{sdb_i} = Dry$  weight of sampled dead branches of tree *j* (kg)

- 244  $FW_{sdb_i}$  = Fresh weight of sampled dead branches of tree *j* (kg)
- 245  $FW_{tdb_i}$  = Total fresh weight of all dead branches in the crown of tree j ( $FW_{sdb_i}$  + the
- 246 rest of the dead branches in the crown of tree *j*)(kg)
- 247  $BM_{db_i}$  = The biomass of dead branches (if present) of tree j (kg)

249 Total aboveground biomass estimate

- 250  $BM_{tag_i} = BM_{ts_i} + BM_{lc_i} + BM_{db_i}$ (28)
- 251 where:
- step (28) corresponds to the written step in the manuscript 252
- 253  $BM_{ts_i}$  = The volume-weighted total stem biomass of tree *j* (kg)
- 254  $BM_{lc_i}$  = The biomass of the live crown of tree *j* (kg)
- $BM_{db_i}$  = The biomass of the dead branches of tree *j* (kg) 255
- 256  $BM_{tag_i}$  = The total aboveground biomass of tree *j* (kg)
- 257

#### 258 Appendix C. Covariance matrices for single- and two-variable functions

Table A.C.1. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass 259

#### function for total aboveground biomass (TAGd). 260

	$\beta_0$	$\beta_d$
$\beta_0$	0.00011	
$\beta_d$	-0.00044	0.00195

# 262 **Table A.C.2. Parameter covariance matrix** $(\Psi_f)$ of the single-variable biomass

263 function for total stem biomass (TS<sub>d</sub>).

$$\begin{array}{c|c} & \beta_0 & \beta_d \\ \hline \beta_0 & 0.00012 \\ \beta_d & -0.00055 & 0.00291 \end{array}$$

264

265 Table A.C.3. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass

266 function for stemwood biomass (SWd).

$$\begin{array}{c|c} \hline \beta_0 & \beta_d \\ \hline \beta_0 & 0.00009 \\ \hline \beta_d & -0.00049 & 0.00310 \\ \hline \end{array}$$

267

268 Table A.C.4. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass

269 function for stem bark biomass (SBd).

$$\begin{array}{c|c} \beta_0 & \beta_d \\ \hline \beta_0 & 7.53085 \ [10^{-6}] \\ \beta_d & -0.00020 & 0.00590 \end{array}$$

270

271 Table A.C.5. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass

272 function for live crown biomass (LC<sub>d</sub>).

$$\begin{array}{c|c} & \beta_0 & \beta_d \\ \hline \beta_0 & 0.00003 \\ \hline \beta_d & -0.00050 & 0.01042 \\ \end{array}$$

273

274 Table A.C.6. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass

275 function for live branch biomass (LB<sub>d</sub>).

	$\beta_0$	$\beta_d$
$\beta_0$	7.72835 [10-6]	
$\beta_d$	-0.00026	0.00938

# 277 Table A.C.7. Parameter covariance matrix $(\Psi_f)$ of the single-variable biomass

#### 278 function for leaf biomass (LF<sub>d</sub>).

$$\begin{array}{c|c} \hline \beta_0 & \beta_d \\ \hline \beta_0 & 5.95136 \, [10^{-6}] \\ \beta_d & -0.00030 & 0.01888 \end{array}$$

279

280 Table A.C.8. Parameter covariance matrix  $(\Psi_f)$  of the single-variable biomass

#### 281 function for dead branch biomass (DB<sub>d</sub>).

$$\begin{array}{c|c} \hline \beta_0 & \beta_d \\ \hline \beta_0 & 5.21287 \ [10^{-6}] \\ \hline \beta_d & -0.00063 & 0.08046 \end{array}$$

282

283 Table A.C.9. Parameter covariance matrix  $(\Psi_f)$  of the two-variable biomass

#### 284 function for total aboveground biomass (TAG<sub>dh</sub>).

	$\beta_0$	$\beta_d$	$\beta_h$
$\beta_0$	0.00006		
$\beta_d$	0.00020	0.00489	
$\beta_h$	-0.00070	-0.00673	0.01279

285

# 286 Table A.C.10. Parameter covariance matrix $(\Psi_f)$ of the two-variable biomass

#### 287 function for total stem biomass (TS<sub>dh</sub>).

	$\beta_0$	$\beta_d$	$\beta_h$
$\beta_0$	7.63909 [10 <sup>-6</sup> ]		
$\beta_d$	0.00005	0.00333	
$\beta_h$	-0.00019	-0.00440	0.00799

288

289 **Table A.C.11. Parameter covariance matrix**  $(\Psi_f)$  of the two-variable biomass

#### 290 function for stemwood biomass (SW<sub>dh</sub>).

	$\beta_0$	$\beta_d$	$\beta_h$
$\beta_0$	4.30251 [10 <sup>-6</sup> ]		
$\beta_d$	0.00004	0.00332	
$\beta_h$	-0.00014	-0.00438	0.00782

# 292 Table A.C.12. Parameter covariance matrix $(\Psi_f)$ of the two-variable biomass

# 293 function for stem bark biomass (SB<sub>dh</sub>).

	$\beta_0$	$\beta_d$	$\beta_h$
$\beta_0$	2.55211 [10-6]		
$\beta_d$	0.00008	0.01434	
$\beta_h$	-0.00029	-0.02115	0.04571

294

295 Table A.C.13. Parameter covariance matrix  $(\Psi_f)$  of the two-variable biomass

#### 296 function for live crown biomass (LC<sub>dh</sub>).

$\begin{array}{ccc} \beta_0 & 0.00049 \\ \beta_d & 0.00148 & 0.03068 \end{array}$		$\beta_h$	$\beta_d$	$\beta_0$	
$\beta_d$ 0.00148 0.03068				0.00049	$\beta_0$
			0.03068	0.00148	$\beta_d$
$\beta_h$ -0.00542 -0.04366 0.089	34	0.0893	-0.04366	-0.00542	$\beta_h$

297

298 Table A.C.14. Parameter covariance matrix  $(\Psi_f)$  of the two-variable biomass

#### 299 function for live branch biomass (LB<sub>dh</sub>).

	$\beta_0$	$\beta_d$	$\beta_h$
$\beta_0$	0.00011		
$\beta_d$	0.00069	0.03005	
$\beta_h$	-0.00237	-0.04188	0.08060

301 A.C.15. Residual covariance matrix  $\sum$  for single-variable biomass functions.

	Res. TAG <sub>d</sub>	Res. TS <sub>d</sub>	Res. SW <sub>d</sub>	Res. SB <sub>d</sub>	Res. LC <sub>d</sub>	Res. LB <sub>d</sub>	Res. LF <sub>d</sub>	Res. DB <sub>d</sub>
Res. TAG <sub>d</sub>	456.92051							
Res. TS <sub>d</sub>	357.45628	488.03138						
Res. SW <sub>d</sub>	255.96758	331.84240	244.35479					
Res. SB <sub>d</sub>	90.75776	134.78428	75.04395	52.84070				
Res. LCd	107.11887	-94.82201	-60.62131	-28.14675	190.72404			
Res. LB <sub>d</sub>	133.43144	-46.20378	-33.96514	-9.53215	176.88983	172.50598		
Res. LF <sub>d</sub>	0.29437	-7.36793	-4.88840	-1.90201	7.30314	5.79553	1.55523	
Res. DB <sub>d</sub>	1.80268	5.30791	3.43170	1.50237	-2.67981	-1.79410	-0.28645	0.42875

303 A.C.16. Residual covariance matrix  $\boldsymbol{\Sigma}$  for two-variable biomass functions.

	Res. TAG <sub>dh</sub>	Res. TS <sub>dh</sub>	Res. SW <sub>dh</sub>	Res. SB <sub>dh</sub>	Res. LC <sub>dh</sub>	Res. LB <sub>dh</sub>
Res. TAG <sub>dh</sub>	480.29573					
Res. TS <sub>dh</sub>	296.22257	261.88310				
Res. SW <sub>dh</sub>	243.53965	195.76021	182.12349			
Res. SB <sub>dh</sub>	61.66899	71.02619	19.03552	52.66629		
Res. LC <sub>dh</sub>	195.01797	42.66572	63.31380	-15.55137	160.62961	
Res. LB <sub>dh</sub>	207.77599	60.74303	69.85139	-3.73878	153.77029	151.42379