Appendix A. List of abbreviations and select terms used in the manuscript in alphabetical order

AIC: Akaike Information Criterion

C: Celsius

cm: centimeters

commercial size trees: Development classes 3-5 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Younger, older, and mature productive forest with satisfactory stand density. Species proportions are reported according to volume in these harvest classes.

complete crown: The function biomass combination from the current study of live crown (LC_{dh}) and dead branches (DB_d).

DB_d: Dead branch biomass single-variable model

dbh: Diameter at breast height (1.3 m)

DW: Dry weight

FW: Fresh weight

H40: Height of tree at 40 years of age

ha: hectare

height-to-live-crown: distance from the ground to the base of the live crown, ignoring one time a single live branch if separated by more than two whorls from the next live branch.

kg: kilogram

LB_d: Live branch biomass single-variable model

LB_{dh}: Live branch biomass two-variable model

LC_d: Live crown biomass single-variable model

LC_{dh}: Live crown biomass two-variable model

LF_d: Leaf biomass single-variable model

m: meter

m.a.s.l.: meters above sea level

m.t.b.: million tons biomass

N: Number

NFI: National Forest Inventory

NLME: Nonlinear mixed-effects model

NNFI: Norwegian National Forest Inventory

NNFI8: Norwegian National Forest Inventory 8th inventory (2000-2004)

NNFI9: Norwegian National Forest Inventory 9th inventory (2005-2009)

older stands: Development classes 4 and 5 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Older and mature productive forest with satisfactory stand density.

p: p-value

RMSE: Root Mean Square Error

sapling size trees: Development classes 1 and 2 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest. Species proportions are reported according to crown cover percentage in these harvest classes.

SB_d: Stem bark biomass single-variable model

SB_{dh}: Stem bark biomass two-variable model

std. error: Standard error

SW_d: Stemwood biomass single-variable model

SW_{dh}: Stemwood biomass two-variable model

TAG_B: Total aboveground biomass component combination from Bollandsås et al. (2009) using: over-bark ("Stem") + total crown ("Tree crown") biomass

$TAG_{combination\ 1}$: Total aboveground component combination using: $TS_{dh} + LC_{dh} + DB_d$

$TAG_{combination\ 2}$: Total aboveground component combination using: $SW_{dh} + SB_d + LB_{dh} + LF_d + DB_d$

TAG_d: Total aboveground biomass single-variable model (model fit with the BM_{ts} + BM_{lc} + BM_{db} biomass estimates (Appendix B))

TAG_{dh}: Total aboveground biomass two-variable model (model fit with the BM_{ts} + BM_{lc} + BM_{db} biomass estimates (Appendix B))

TAG_M: Total aboveground biomass for Marklund using: stemwood (B-5) + stem bark (B-8) + live branch (B-11) + dead branch (B-16) + leaves (where leaf biomass = B-5 * (0.011[a]/0.52[b]) ([a] Factor currently applied by NNFI for UNFCCC reporting; [b] de Wit et al. 2006)

Commented [AS1]: mulitplication

TAG_S: Total aboveground biomass component combination of the current study using: SW_{dh} + SB_d + LB_{dh} + DB_d + LF_d

total crown: Observed crown biomass of the mountain birch sample trees including the live and dead branches (if present) (Bollandsås et al. 2009).

TS_d: Total stem biomass single-variable model

TS_{dh}: Total stem biomass two-variable model

UNFCCC: United Nations Framework Convention on Climate Change

Unprod.: Unproductive birch forest = potential yield < 1 m^3 ha^{-1} yr^{-1}

volume-weighted total stem biomass: The average stem biomass weighted by volume of the stem section from which the sample disk was taken.

young stands: Development classes 1 and 2 definition from Norwegian NFI (Antón-Fernández and Astrup 2012). Young newly regenerating to satisfactorily dense forest.

Appendix B. Detailed methods for the aboveground biomass dataset

Total stem biomass estimate

(1) $\mathrm{DW:FW}_{disk_i} = \frac{\mathrm{DW}_{disk_i}}{\mathrm{FW}_{disk_i}}$

(2) $\mathrm{V}_{s_i} = \frac{l_{s_i}(\mathrm{g}_{1_i} + \mathrm{g}_{2_i})}{2}$ (Smalian's formula)

(3) $\mathrm{V}_{t_j} = \sum_{i=1} \mathrm{V}_{s_i}$

(4) $\mathrm{DW:FW}_{vw_j} = \sum_{i=1} \left(\mathrm{DW:FW}_{disk_i} \left(\frac{\mathrm{V}_{s_i}}{\mathrm{V}_{t_j}} \right)_i \right)$

(5) $\mathrm{BM}_{ts_j} = \mathrm{DW:FW}_{vw_j} * \mathrm{FW}_{stem_j}$

where:

steps (1), (2), (3), (4), and (5) correspond to the written steps in the manuscript

$\mathrm{DW:FW}_{disk_i}$ = Dry weight to fresh weight ratio of stem disk *i* with bark

DW_{disk_i} = Dry weight of stem disk *i* with bark (g)

FW_{disk_i} = Fresh weight of stem disk *i* with bark (g)

V_{s_i} = Volume of stem section *i* by Smalian's formula (m^3)

l_{s_i} = Length of stem section *i* (cm)

g_{1_i} = Lower surface's cross sectional area of an ellipse of section *i* (mm^2)

g_{2_i} = Upper surface's cross sectional area of an ellipse of section *i* (mm^2)

g = Cross sectional area of an ellipse = $\frac{\pi}{4}(\mathrm{d}_1 * \mathrm{d}_2)$

d_1 = Maximum diameter (mm)

d_2 = Minimum diameter (mm)
V_{t_j} = Total stem volume of tree *j* (m^3)
DW: FW_{vw_j} = Volume-weighted dry weight fresh weight ratio of the stem of tree *j*
FW_{stem_j} = Fresh weight of the stem of tree *j* (total fresh weight of disks + the rest of the stem of tree *j*)(kg)
BM_{ts_j} = The volume-weighted total stem biomass of tree *j* (kg)

Stemwood biomass estimate

(6) A_{ob_i} & $A_{sw_i} = \frac{\pi}{4}(d_{1_i} * d_{2_i})$

(7) $P_{sw_i} = \frac{A_{sw_i}}{A_{ob_i}}$

(8) $P_{sv_i} = \frac{V_{s_i}}{V_{t_j}}$

(9) $P_{sws_i} = P_{sw_i} * P_{sv_i}$

(10) $P_{vwsw_j} = \sum_{i=1} P_{sws_i}$

(11) $BM_{sw_j} = P_{vwsw_j} * BM_{ts_j}$

where:
steps (6), (7), (8), (9), (10), and (11) correspond to the written steps in the manuscript
A_{ob_i}= Cross sectional elliptical over-bark area of stem disk *i* (mm^2)
A_{sw_i} = Cross sectional elliptical stemwood area of stem disk *i* (mm^2)
d_{1_i} = Maximum diameter of stem disk *i* (mm)
d_{2_i} = Minimum diameter of stem disk *i* (mm)
P_{sw_i} = Proportion of stemwood cross sectional area of stem disk *i* assigned to its corresponding stem section
P_{sv_i} = Proportion of the total stem volume that stem section *i* represents
V_{s_i} = Volume of stem section *i* by Smalian's formula (m^3)
V_{t_j} = Total stem volume of tree *j* (m^3)
P_{sws_i}= Proportion of the stemwood in stem section *i*
P_{vwsw_j} = Volume-weighted proportion of stemwood in the stem of tree *j*
BM_{ts_j} = The volume-weighted total stem biomass of tree *j* (kg)
BM_{sw_j} = The volume-weighted stemwood biomass of tree *j* (kg)

Stem bark biomass estimate

(12) $P_{sb_i} = 1 - P_{sw_i}$

(13) $P_{sbs_i} = P_{sb_i} * P_{sv_i}$

(14) $P_{vwsb_j} = \sum_{i=1} P_{sbs_i}$

(15) $BM_{sb_j} = P_{vwsb_j} * BM_{ts_j}$

where:
steps (12), (13), (14), and (15) correspond to written steps in the manuscript
P_{sb_i} = Proportion of stem bark of stem disk *i*
P_{sw_i} = Proportion of stemwood cross sectional area of stem disk *i* assigned to its corresponding stem section
P_{sbs_i} = Proportion of stem bark of section *i*
P_{sv_i} = Proportion of the total stem volume that stem section *i* represents

P_{vwsb_j} = Volume-weighted proportion of stem bark of tree *j*

BM_{ts_j} = The volume-weighted total stem biomass of tree *j* (kg)

BM_{sb_j} = The volume-weighted stem bark biomass of tree *j* (kg)

Live crown biomass estimate

(16) $$DW_{lsb_j} = \sum_{i=1}\left(DW_{lb_i} + DW_{leaf_i} + DW_{catkins_i}\right)$$

(17) $$FW_{lsb_j} = \sum_{i=1}\left(FW_{lsb_i}\right)$$

(18) $$DW{:}\,FW_{lsb_j} = \frac{DW_{lsb_j}}{FW_{lsb_j}}$$

(19) $$BM_{lc_j} = DW{:}\,FW_{lsb_j} * FW_{tlc_j}$$

where:

steps (16), (17), (18), and (19) correspond to the written steps in the manuscript

DW_{lsb_j} = Sum of the dry weights of live sample branches of tree *j* (kg)

DW_{lb_i} = Dry weight of the woody material of live sample branch *i* (kg)

DW_{leaf_i} = Dry weight of the leaves of live sample branch *i* (kg)

$DW_{catkins_i}$ = Dry weight of the catkins of live sample branch *i* (kg)

FW_{lsb_j} = Sum of the fresh weights of the live sample branches of tree *j* (kg)

FW_{lsb_i} = Fresh weight of live sample branch *i* (kg)

$DW{:}\,FW_{lsb_j}$ = Dry weight to fresh weight ratio of the live sample branches of tree *j*

FW_{tlc_j} = Total fresh weight of the live crown of tree *j* (FW_{lsb_j} + the rest of the live crown)(kg)

BM_{lc_j} = The biomass of the live crown of tree *j* (kg)

Live branch biomass estimate

(20) $$DW_{lb_j} = \sum_{i=1} DW_{lb_i}$$

(21) $$DW_{lsb_j} = \sum_{i=1}\left(DW_{lb_i} + DW_{leaf_i} + DW_{catkins_i}\right)$$

(22) $$BM_{lb_j} = \frac{DW_{lb_j}}{DW_{lsb_j}} * BM_{lc_j}$$

where:

steps (20), (21), and (22) correspond to the written steps in the manuscript

DW_{lb_j} = Sum of the dry weight of the woody material of live sample branches of tree *j* (kg)

DW_{lb_i} = Dry weight of the woody material of live sample branch *i* (kg)

DW_{lsb_j} = Sum of the dry weight of live sample branches of tree *j* (kg)

DW_{leaf_i} = Dry weight of the leaves of live sample branch *i* (kg)

$DW_{catkins_i}$ = Dry weight of the catkins (if present) of live sample branch *i* (kg)

BM_{lc_j} = The biomass of the live crown of tree *j* (kg)

BM_{lb_j} = The biomass of live branches of tree *j* (kg)

Leaf biomass estimate

(23) $$DW_{leaf_j} = \sum_{i=1} DW_{leaf_i}$$

(24) $$DW_{leaf+catkins_j} = DW_{leaf_j} + DW_{catkins_j}$$

(25) $$BM_{leaf_j} = \frac{DW_{leaf+catkins_j}}{DW_{lsb_j}} * BM_{lc_j}$$

where:
steps (23), (24), and (25) correspond to the written steps in the manuscript
DW_{leaf_j} = Sum of the dry weight of leaves of the live sample branches of tree *j* (kg)
DW_{leaf_i} = Dry weight of the leaves of live sample branch *i* (kg)
$DW_{leaf+catkins_j}$ = Dry weight of leaves and catkins (if present) of tree *j* (kg)
$DW_{catkins_j}$ = Dry weight of the catkins (if present) of tree *j* (kg)
DW_{lsb_j} = Sum of the dry weight of live sample branches of tree *j* (kg)
BM_{lc_j} = The biomass of the live crown of tree *j* (kg)
BM_{leaf_j} = The biomass of the leaves and catkins (if present) of tree *j* (kg)

Dead branch biomass estimate

(26) $$DW{:}\,FW_{sdb_j} = \frac{DW_{sdb_j}}{FW_{sdb_j}}$$

(27) $$BM_{db_j} = DW{:}\,FW_{sdb_j} * FW_{tdb_j}$$

where:
steps (26) and (27) correspond to the written steps in the manuscript
$DW{:}\,FW_{sdb_j}$ = Dry weight to fresh weight ratio of sampled dead branches of tree *j*
DW_{sdb_j} = Dry weight of sampled dead branches of tree *j* (kg)
FW_{sdb_j} = Fresh weight of sampled dead branches of tree *j* (kg)
FW_{tdb_j} = Total fresh weight of all dead branches in the crown of tree *j* (FW_{sdb_j} + the rest of the dead branches in the crown of tree *j*)(kg)
BM_{db_j} = The biomass of dead branches (if present) of tree *j* (kg)

Total aboveground biomass estimate

(28) $$BM_{tag_j} = BM_{ts_j} + BM_{lc_j} + BM_{db_j}$$

where:
step (28) corresponds to the written step in the manuscript
BM_{ts_j} = The volume-weighted total stem biomass of tree *j* (kg)
BM_{lc_j} = The biomass of the live crown of tree *j* (kg)
BM_{db_j} = The biomass of the dead branches of tree *j* (kg)
BM_{tag_j} = The total aboveground biomass of tree *j* (kg)

Appendix C. Covariance matrices for single- and two-variable functions

Table A.C.1. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for total aboveground biomass (TAG_d).

	β_0	β_d
β_0	0.00011	
β_d	-0.00044	0.00195

Table A.C.2. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for total stem biomass (TS_d).

	β_0	β_d
β_0	0.00012	
β_d	-0.00055	0.00291

Table A.C.3. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for stemwood biomass (SW_d).

	β_0	β_d
β_0	0.00009	
β_d	-0.00049	0.00310

Table A.C.4. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for stem bark biomass (SB_d).

	β_0	β_d
β_0	7.53085 [10^{-6}]	
β_d	-0.00020	0.00590

Table A.C.5. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for live crown biomass (LC_d).

	β_0	β_d
β_0	0.00003	
β_d	-0.00050	0.01042

Table A.C.6. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for live branch biomass (LB_d).

	β_0	β_d
β_0	7.72835 [10^{-6}]	
β_d	-0.00026	0.00938

Table A.C.7. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for leaf biomass (LF_d).

	β_0	β_d
β_0	5.95136 [10^{-6}]	
β_d	-0.00030	0.01888

Table A.C.8. Parameter covariance matrix (Ψ_f) of the single-variable biomass function for dead branch biomass (DB_d).

	β_0	β_d
β_0	5.21287 [10^{-6}]	
β_d	-0.00063	0.08046

Table A.C.9. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for total aboveground biomass (TAG_{dh}).

	β_0	β_d	β_h
β_0	0.00006		
β_d	0.00020	0.00489	
β_h	-0.00070	-0.00673	0.01279

Table A.C.10. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for total stem biomass (TS_{dh}).

	β_0	β_d	β_h
β_0	7.63909 [10^{-6}]		
β_d	0.00005	0.00333	
β_h	-0.00019	-0.00440	0.00799

Table A.C.11. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for stemwood biomass (SW_{dh}).

	β_0	β_d	β_h
β_0	4.30251 [10^{-6}]		
β_d	0.00004	0.00332	
β_h	-0.00014	-0.00438	0.00782

Table A.C.12. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for stem bark biomass (SB_{dh}).

	β_0	β_d	β_h
β_0	2.55211 [10^{-6}]		
β_d	0.00008	0.01434	
β_h	-0.00029	-0.02115	0.04571

Table A.C.13. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for live crown biomass (LC_{dh}).

	β_0	β_d	β_h
β_0	0.00049		
β_d	0.00148	0.03068	
β_h	-0.00542	-0.04366	0.08934

Table A.C.14. Parameter covariance matrix (Ψ_f) of the two-variable biomass function for live branch biomass (LB_{dh}).

	β_0	β_d	β_h
β_0	0.00011		
β_d	0.00069	0.03005	
β_h	-0.00237	-0.04188	0.08060

A.C.15. Residual covariance matrix ∑ for single-variable biomass functions.

	Res. TAG_d	Res. TS_d	Res. SW_d	Res. SB_d	Res. LC_d	Res. LB_d	Res. LF_d	Res. DB_d
Res. TAG_d	456.92051							
Res. TS_d	357.45628	488.03138						
Res. SW_d	255.96758	331.84240	244.35479					
Res. SB_d	90.75776	134.78428	75.04395	52.84070				
Res. LC_d	107.11887	-94.82201	-60.62131	-28.14675	190.72404			
Res. LB_d	133.43144	-46.20378	-33.96514	-9.53215	176.88983	172.50598		
Res. LF_d	0.29437	-7.36793	-4.88840	-1.90201	7.30314	5.79553	1.55523	
Res. DB_d	1.80268	5.30791	3.43170	1.50237	-2.67981	-1.79410	-0.28645	0.42875

A.C.16. Residual covariance matrix ∑ for two-variable biomass functions.

	Res. TAG_{dh}	Res. TS_{dh}	Res. SW_{dh}	Res. SB_{dh}	Res. LC_{dh}	Res. LB_{dh}
Res. TAG_{dh}	480.29573					
Res. TS_{dh}	296.22257	261.88310				
Res. SW_{dh}	243.53965	195.76021	182.12349			
Res. SB_{dh}	61.66899	71.02619	19.03552	52.66629		
Res. LC_{dh}	195.01797	42.66572	63.31380	-15.55137	160.62961	
Res. LB_{dh}	207.77599	60.74303	69.85139	-3.73878	153.77029	151.42379