Understanding Photovoltaic Cell Dynamic Resistance Behavior with Changing Incident Light Intensity

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Abstract—We present a theoretical examination of the general behavior that should be expected to be displayed by the magnitude of the dynamic resistance of a conventional illuminated photovoltaic device within the power-generating quadrant of its I - V characteristics, when measured in quasi-static conditions from the short-circuit point to the open-circuit point, at various incident illumination intensities. The analysis is based on assuming that the photovoltaic device in question may be adequately described by a simple conventional d-c lumped-element single-diode equivalent circuit solar cell model, which includes significant constant series and shunt resistive losses, but lacks any other secondary effects. Using explicit analytic expressions for the dynamic resistance, we elucidate how its magnitude changes as a function of the terminal variables, the incident illumination intensity and the model's equivalent circuit elements' parameters.

Index Terms—Photovoltaic device, solar cell model, dynamic resistance, Lambert W function.

I. INTRODUCTION

THE reciprocal slope of the I - V characteristics of an illuminated photovoltaic cell is usually called its dynamic resistance, r . This parameter can be used as a convenient tool to analyze the cell's function to optimize its performance [1]. Knowing the behavior of r along a solar cell's I - V characteristics at various levels of light intensity is also essential to calculate the cell's maximum power point (MPP) as it varies under changing illumination conditions. Here we will look into basic issues regarding the r of typical photovoltaic cells, from a purely theoretical point of view. The intention is to develop a clear understanding of the expected fundamental behavior of r as the intensity of the light incident upon the cell changes. The concrete questions that will be considered are:

1) Does the magnitude of the dynamic resistance, r , at any point along the I - V characteristics of typical conventional photovoltaic cells vary considerably when the intensity of the incident light changes?

2) Does the magnitude of r measured under short-circuit and open-circuit conditions change with incident light intensity?

3) If so, how do these dependencies look like?

In addressing these questions about the effect of incident

light intensity on the magnitude of the cell's r we will skip all optical aspects of light reflection and absorption, so as to keep answers basic and simple. Thus, instead of considering light intensity, we will look directly at the dependence of r on the total photo-current, I_{ph} , produced by the electrical separation and collection of photo-generated charge carriers within the body of the cell.

To keep the analysis as straightforward as possible, it will be based on the mathematical equations that define the simplest condensed-element (a.k.a. lumped-parameter) dc equivalent electric circuit [1] that can be possibly used to realistically model the essential physical phenomena present in most typical semiconductor photovoltaic cells. The simplest realistic equivalent circuit model that can be used for this purpose has the uncomplicated configuration shown in Fig. 1. Its electric elements consist of a parallel-connected combination of: a single diode (characterized by the linked pair of lumped parameter values n and I_0 , which together represent the most significant charge carrier transport mechanisms at the cell's junction); a photo-generated current source (with magnitude defined by its lumped parameter I_{ph}) oriented in the direction of the diode's reverse current, which accounts for the photo-current produced by all collected photo-generated carriers; and a shunt resistor (specified by the value of its lumped parameter $R_p=1/G_p$) that includes all resistive shunt losses across the cell's junction. As shown in Fig. 1, this parallel combination is further connected in series to a second resistor (specified by the value of its lumped parameter R_s), which combines all spatially distributed series resistive losses produced by the cell's contacts and terminals. Both resistive elements are assumed, in an acceptable first order approximation, to stay roughly constant

Fig. 1. Equivalent lumped-element dc circuit model used to describe the current-voltage characteristics at the terminals of a conventional illuminated cell.

with respect to the circuit's variables and the light intensity.

We will assume that the magnitude of the dynamic resistance, r , embodied by the value of the illuminated photovoltaic cell I - V characteristics' reciprocal slope, can be determined at any point under quasi-static conditions, either by experimentally performing a small-signal ac measurement at very low frequency, or by numerical calculation from two or more consecutive data points taken around the location of the point of interest on the measured static I - V characteristic [2].

II. METHODS

The first step to theoretically study the behavior of the dynamic resistance is to write the equations that describe the single-diode model whose lumped-element equivalent circuit [1] is shown in Fig. 1. The relationship between its terminals' current I and voltage V is mathematically expressed using Kirchoff's Laws and Shockley's diode equation [3], which results in the following implicit transcendental linear-exponential equation:

$$I = I_{ph} - I_0 \left[\exp\left(\frac{IR_s + V}{n v_{th}}\right) - 1 \right] - (IR_s + V)G_p \quad (1)$$

Notice that in writing (1) we have chosen to define the current in the opposite direction (sign) as that shown in Fig. 1. We do so, without loss of generality, in order to follow the sign convention normally used by photovoltaic power engineers, where the power generating quadrant is presented as the first (instead of as the fourth) quadrant of the illuminated cell's I - V characteristics (see Fig. 2).

A. Explicit solutions

From simulation and analysis points of view, closed-form solutions of the cell's terminal current and voltage as exact explicit functions of each other are usually preferred over the implicit (1). Exact explicit solutions of (1) are possible thanks to the use of the Lambert W function [4].

The solution of (1) for the terminal current I as an explicit function of the terminal voltage V is [5]:

$$I(V) = -\frac{n v_{th}}{R_s} W_0 \left\{ \frac{I_0 R_s \exp\left[\frac{(I_0 + I_{ph})R_s + V}{n v_{th}(G_p R_s + 1)}\right]}{n v_{th}(G_p R_s + 1)} \right\} - \frac{-\frac{(I_0 + I_{ph})R_s + V}{G_p R_s + 1} + V}{R_s} \quad (2)$$

where W_0 represents the principal branch of the Lambert W function [4].

Substituting $R_s=0$ and $G_p=0$ into (1) yields the corresponding expression for the terminal current, I_i , of an ideal cell, i.e., a cell with negligible series and parallel resistive losses:

$$I_i(V_i) = I_{ph} - I_0 \left[\exp\left(\frac{V_i}{n v_{th}}\right) - 1 \right] \quad (2i)$$

From here onwards, all equations that refer to the ideal case ($R_s=0$ and $G_p=0$) will be identified for convenience using the same number as their corresponding non-ideal counterparts ($R_s \neq 0$ and $G_p \neq 0$) followed by the letter "i" within the right

margin parenthesis. Notice that substituting $R_s=0$ and $G_p=0$ directly into (2) will not yield (2i). Strictly speaking, to get (2i) from (2) we would have to find the limit of (2) as $R_s \rightarrow 0$ and $G_p \rightarrow 0$.

The solution of (1) for the terminal voltage V as an explicit function of the terminal current I can be written also in terms of the Lambert W function [5]:

$$V(I) = -n v_{th} W_0 \left[\frac{I_0 \exp\left(\frac{I_0 + I_{ph} - I}{G_p n v_{th}}\right)}{G_p n v_{th}} \right] + \frac{I_0 + I_{ph} - I}{G_p} - IR_s \quad (3)$$

The corresponding expression for the terminal voltage, V_i , of the ideal cell is quickly found by inverting (2i):

$$V_i(I_i) = n v_{th} \ln\left(\frac{I_{ph} + I_0 - I_i}{I_0}\right) \quad (3i)$$

Again, notice that the substitution of $R_s=0$ and $G_p=0$ directly into (3) would not yield (3i). To get (3i) from (3), the limit of (3) as $R_s \rightarrow 0$ and $G_p \rightarrow 0$ would have to be found.

B. Open-circuit voltage and short-circuit current

Since we will need to evaluate r at the open-circuit ($V=V_{oc}, I=0$) and short-circuit ($V=0, I=I_{sc}$) points of the illuminated cell's I - V characteristics, let us first write the corresponding voltage and current explicit equations at those points.

The explicit expression for the short circuit current, I_{sc} , is written by simply evaluating (2) at $V=0$:

$$I_{sc} = -\frac{n v_{th}}{R_s} W_0 \left\{ \frac{I_0 R_s \exp\left[\frac{(I_0 + I_{ph})R_s}{n v_{th}(G_p R_s + 1)}\right]}{n v_{th}(G_p R_s + 1)} \right\} + \frac{I_0 + I_{ph}}{G_p R_s + 1} \quad (4)$$

The corresponding expression for the short circuit current, I_{isc} , of an ideal cell is found by simply evaluating (2i) at $V_i=0$:

$$I_{isc} = I_{ph} \quad (4i)$$

The explicit expression for the open circuit voltage, V_{oc} , is found by evaluating (3) at $I=0$:

$$V_{oc} = -n v_{th} W_0 \left[\frac{I_0 \exp\left(\frac{I_0 + I_{ph}}{G_p n v_{th}}\right)}{G_p n v_{th}} \right] + \frac{I_0 + I_{ph}}{G_p} \quad (5)$$

The corresponding expressions for the open-circuit voltage, V_{ioc} , of an ideal cell is found by simply evaluating (3i) at $I_i=0$:

$$V_{ioc} = n v_{th} \ln\left(\frac{I_{ph} + I_0}{I_0}\right) \quad (5i)$$

The above equations clearly indicate that the short-circuit current, (4) or (4i), and the open-circuit voltage, (5) or (5i), of a real (non-ideal) ($R_s \neq 0$, $G_p \neq 0$) or ideal ($R_s=0$, $G_p=0$) photovoltaic cell, respectively, are functions of incident light

intensity (represented by the value of the photo-current I_{ph}). In the case of the real (non-ideal) cell, they also depend on the equivalent circuit's lumped elements parameter values, except that V_{oc} (5) does not depend on the value of R_s , since the voltage drop across R_s is zero when $I=0$.

C. An illustrative example

A hypothetical photovoltaic cell, with known equivalent circuit lumped-elements' parameter values, will be used to generate synthetic I - V characteristics that will help us to graphically illustrate the behavior of the dynamic resistance. The example will be the circuit model shown in Fig. 1 with the following lumped elements parameter values: diode's reverse saturation current $I_0=10^{-7}$ A, diode's junction quality factor $n=1.5$, and shunt and series resistive losses, $R_p=1/G_p$ and R_s values indicated in each case. Room temperature operation, i.e. a thermal voltage $v_{th}=k_B T/q=0,02586$ V, is assumed throughout.

Three synthetic I - V characteristics of this hypothetical cell, calculated using the explicit solution for the current given by (2), are presented in Fig. 2, for three intensities of incident light which generate within the cell the three corresponding photo-current magnitudes of I_{ph} indicated on Fig. 2's legend. Matching I - V curves of an ideal cell, similar to the real but without shunt or series resistive losses ($1/R_p=G_p=0$, $R_s=0$), are also shown in Fig. 2 for visual comparison.

The numeric values of short circuit current and open circuit voltage, calculated with (4), (4i), (5) and (5i) for this particular example, at three incident light intensities represented by the three indicated photo-current magnitudes of I_{ph} , are presented in Table I. They correspond to the axes intercepts of the three I - V characteristics shown in Fig.2. It is worthwhile to point out that the sometimes hastily stated assumption that $I_{sc}\approx I_{ph}$ is not always a valid approximation. As (4) clearly implies. Its validity depends on the relative magnitudes of the cell's parallel and series resistive losses.

Fig. 2. Power-generating quadrant of synthetic current-voltage characteristics of hypothetical illuminated real and ideal photovoltaic cells modeled at room temperature by the equivalent circuit of Fig. 1, calculated with (2) and (2i), respectively, using the diode's parameters indicated in the text for the three values of photo-current indicated in the legend. Real cell ($1/G_p=R_p=100\Omega$, $R_s=10\Omega$) (solid lines), deal cell ($G_p=0$, $R_s=0$) (dashed lines). The negative of the curves' reciprocal slope at any point is the dynamic resistance, r .

TABLE I
CALCULATED SHORT-CIRCUIT CURRENT AND OPEN-CIRCUIT VOLTAGE
VALUES OF HYPOTHETICAL REAL AND IDEAL SOLAR CELLS AT THREE
VALUES OF INCIDENT LIGHT INTENSITY

I_{ph} (mA)	I_{sc} (mA)		V_{oc} (V)	
	real	ideal	real	ideal
20	18.2	20	0.463	0.473
30	27.2	30	0.482	0.489
40	35.5	40	0.495	0.500

Equivalent circuit elements' parameters: $v_{th}=0,02586$ V, $n=1.5$, $I_0=10^{-7}$ A, Real: $R_p=100\Omega$, and $R_s=10\Omega$; Ideal: $G_p=0$, and $R_s=0$;

III. RESULTS

A. Behavior of the dynamic resistance along the I - V curves

The magnitude of the dynamic resistance, r , was defined in Section I as the negative of the reciprocal slope of the I - V characteristics. Explicit analytic expressions of dynamic resistance as a function of the terminal current, $r(I)$, and of its reciprocal the dynamic conductance as a function of the terminal voltage, $g(V)$, are obtained by taking the corresponding derivatives of the explicit solutions of the voltage (3) and of the current (2) with respect to current and voltage, respectively. These derivatives are found without difficulty because the Lambert W function is readily differentiable [4]. The resulting explicit expressions for $r(I)$ and $g(V)$ are:

$$r(I) = \frac{dV(I)}{dI} = -\frac{1}{G_p} \frac{1}{1+W_0 \left[\frac{I_0 \exp\left(\frac{I_0+I_{ph}-I}{G_p n v_{th}}\right)}{G_p n v_{th}} \right]} - R_s. \quad (6)$$

and

$$g(V) = \frac{dI(V)}{dV} = -\frac{W_0 \left[\frac{I_0 R_s \exp\left(\frac{(I_0+I_{ph})R_s+V}{n v_{th}(G_p R_s+1)}\right)}{n v_{th}(G_p R_s+1)} \right] + \frac{G_p R_s}{(G_p R_s+1)}}{R_s \left\{ 1+W_0 \left[\frac{I_0 R_s \exp\left(\frac{(I_0+I_{ph})R_s+V}{n v_{th}(G_p R_s+1)}\right)}{n v_{th}(G_p R_s+1)} \right] \right\}}. \quad (7)$$

Fig. 3 presents the resulting evolution of the dynamic resistance as a function of terminal voltage $r(V)$, from the short-circuit point ($V=0, I=I_{sc}$) to the open circuit point ($V=V_{oc}, I=0$) along the I - V curve of the illustrative example cell, for three levels of photo-current, I_{ph} . Fig. 3 indicates that the magnitude of the dynamic resistance r , for any illumination level, decreases as the terminal voltage increases from the short-circuit point to the open-circuit point.

Alternatively, the dynamic resistance, r , can be obtained by direct differentiation of the implicit current equation (1):

$$1/r(V, I) = \frac{dI}{dV} = -\left(1 + \frac{dI}{dV} R_s\right) \left[G_p + \frac{I_0}{n v_{th}} \exp\left(\frac{I R_s + V}{n v_{th}}\right)\right]. \quad (8)$$

Solving (8) for dI/dV and rearranging yields the dynamic conductance as a function of both the terminal voltage and terminal current $g(V, I)$:

Fig. 3. Magnitude of the dynamic resistance $|r|$ of the hypothetical illuminated cell with $R_p=100\Omega$, and $R_s=10\Omega$, calculated from either (6) or (7) at the three values of I_{ph} indicated in the legend, from the short-circuit point ($V=0$) to the open-circuit point ($V=V_{oc}$). These values of r correspond to the negative reciprocals of the I - V curves' slopes shown in Fig. 2.

$$r(V, I) = \frac{1}{dI/dV} = \frac{dV}{dI} = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{IR_s + V}{n v_{th}}\right)}, \quad (9)$$

which can be easily converted to the form of a sigmoid type of bi-variate function:

$$r(V, I) = -R_s - \frac{1}{G_p \left[1 + \exp\left(\frac{IR_s + V}{n v_{th}} + \ln\left(\frac{I_0}{G_p n v_{th}}\right)\right) \right]}. \quad (9s)$$

Substituting $R_s=0$ and $G_p=0$ into (9) yields the expression for the dynamic resistance of an ideal photovoltaic cell as a function of voltage, $r_i(V_i)$:

$$r_i(V_i) = -\frac{n v_{th}}{I_0 \exp\left(\frac{V_i}{n v_{th}}\right)}. \quad (9i)$$

Another form of the dynamic resistance expression as a function of both terminal variables, $r(V, I)$, can be obtained by substitution of (1) into (9):

$$r(V, I) = \frac{dV}{dI} = -R_s - \frac{n v_{th}}{G_p (n v_{th} - V) + I_{ph} + I_0 - I(G_p R_s + 1)}. \quad (10)$$

Substitution of $R_s=0$ and $G_p=0$ into (10) yields another form of the expression of the dynamic resistance of an ideal cell as a function of current, $r_i(I_i)$, which is totally equivalent to (9i):

$$r_i(I_i) = -\frac{n v_{th}}{I_0 + I_{ph} - I_i}. \quad (10i)$$

B. Dynamic resistance dependence on incident light intensity

The most important conclusion that can be drawn from Fig. 3 is that the magnitude of the dynamic resistance r , at any voltage in the power-generating quadrant, always decreases as incident light intensity increases. The other behavior that is obvious

from Fig. 3 is that at any intensity of incident light, r decreases as voltage increases from the short-circuit point ($V=0$) to the open-circuit point ($V=V_{oc}$). Furthermore, although this is not fully visible in Fig. 3, because of the insufficient variation of light intensities presented (not wide enough range of values of I_{ph}), Fig. 3 seems to suggest that the high and low values of r occur at the short-circuit ($V=0$) and open-circuit ($V=V_{oc}$) points, respectively. Also, the high and low limiting values of r at any voltage seem to be approximately given by $R_p + R_s$ and R_s , respectively. This just described behavior of r will be more dramatically visualized by looking at the behavior of the magnitude of the dynamic resistance measured at the open-circuit point, $|r_{oc}|$, and at the short-circuit point, $|r_{sc}|$, of the cell's I - V curves, which will be examined next.

C. Dynamic resistance at the open-circuit point

To calculate the magnitude of the dynamic resistance at the open-circuit point, we evaluate (9) at $V=V_{oc}$ and $I=0$:

$$r_{oc}(V_{oc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}, \quad (11)$$

which can be converted, as before, to a logistic symmetric function (sigmoid type, "S" form):

$$|r_{oc}|(V_{oc}) = R_s + \frac{R_p}{1 + \exp\left[\frac{V_{oc} - n v_{th} \ln\left(\frac{n v_{th}}{I_0 R_p}\right)}{n v_{th}}\right]}. \quad (11s)$$

In the case of an ideal cell, evaluation of (9i) at $V=V_{oc}$, or substituting $R_s=0$ and $G_p=0$ into (11) yields:

$$r_{ioc}(V_{oc}) = -\frac{n v_{th}}{I_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}. \quad (11i)$$

Alternatively, evaluating (10) at $V=V_{oc}$ and $I=0$ yields an expression equivalent to (11):

$$r_{oc}(V_{oc}, I_{ph}) = -R_s - \frac{n v_{th}}{G_p (n v_{th} - V_{oc}) + I_{ph} + I_0}. \quad (12)$$

Again, in the case of an ideal cell, evaluation of (10i) at $I=0$, or substitution of $R_s=0$ and $G_p=0$ into (12) yields an expression which, recalling (2i), is equivalent to (11i):

$$r_{ioc}(I_{ph}) = -\frac{n v_{th}}{I_{ph} + I_0}. \quad (12i)$$

Table II presents some values of $|r_{ioc}|$ and $|r_{oc}|$ as calculated with (12i) and (12), respectively, for different levels of incident light intensity, represented by corresponding values of I_{ph} .

The variation of the dynamic resistance's magnitude, r_{oc} , calculated with (11) or (12) at the open-circuit point ($V=V_{oc}$, $I=0$), is presented in Fig. 4 as a function of photo-generated current intensity, I_{ph} , (directly corresponding to incident light intensity), for one value of $R_p=100\Omega$ and three values of series resistance: $R_s=1, 4$, and 10Ω .

Fig. 4 indicates that as I_{ph} becomes high (i.e around $I_{ph} > 0.05\text{A}$ in this example), the curves start to flatten, with the magnitude of r_{oc} tending to values near those of R_s ($\approx 1, 4$, and 10Ω in this example). This observation is consistent with Fig. 3, which shows the case for $R_s=10\Omega$, where we see that as $V \rightarrow V_{oc} \approx 0.5\text{V}$ (open-circuit point) all three curves of r

TABLE II

MAGNITUDE OF THE DYNAMIC RESISTANCE AT OPEN-CIRCUIT CONDITIONS ($V=V_{oc}$, $I=0$) OF THE HYPOTHETICAL IDEAL AND REAL SOLAR CELLS, FOR THREE VALUES OF INCIDENT LIGHT INTENSITY

Incident light intensity, expressed as photo-current I_{ph} (mA)	Ideal $ r_{ioc} $ (Ω) $G_p=0$, $R_s=0$	Real $ r_{oc} $ (Ω) $R_p=100\Omega$, $R_s=10\Omega$
20	1.939	12.462
30	1.293	11.517
40	0.970	11.095

Equivalent circuit model diode element's parameters:

$v_{th}=0.02586\text{V}$, $n=1.5$, $I_0=10^{-7}\text{A}$.

Fig. 4. Log-log plot of the magnitude of the dynamic resistance $|r_{oc}|$, calculated with (11) or (12) at the open-circuit point as a function of photo-current I_{ph} (upper plot), and the value of $|r_{oc}| - R_s$ (lower plot), for three values of R_s and $R_p=100\Omega$. Also shown for comparison as a black dash straight line is the magnitude of the dynamic resistance $|r_{ioc}|$ for the corresponding ideal cell ($G_p=0$ and $R_s=0$) as calculated with (12i).

(calculated for the three values of $I_{ph}=0.02, 0.03$, and 0.04A) approach the single value of $R_s=10\Omega$.

Fig. 4 also reveals that the magnitudes of r_{oc} increase as I_{ph} decreases, until they saturate to constant values around $R_p + R_s$ ($\approx 101, 104$, and 110Ω in the present example) when I_{ph} reaches very low values (about an order of magnitude lower in Fig. 4). Notice that the saturation condition ($r_{oc} \approx R_p + R_s \approx 110\Omega$ in this example) cannot be observed at the open-circuit point of the curves shown in Fig.3, since none of those three r_{oc} curves correspond to a low enough value of I_{ph} . The lowest value of I_{ph} in Fig. 3 is 0.02A , and according to the curve for $R_s=10\Omega$ in Fig. 4, the value of r_{oc} at $I_{ph}=0.02\text{A}$ is only slightly larger than R_s , still far from reaching saturation at $r_{oc} \approx R_p + R_s$.

The three curves of $|r_{oc}|$ shown in the upper pane of Fig. 4 for different vales of R_s , are made to collapse into a single curve by subtracting from $|r_{oc}|$ the corresponding value of R_s , as (11) and (12) suggest, and the lower pane of Fig.4 illustrates.

We do not present plots of $|r_{oc}|$ vs I_{sc} because the difference with the plots of $|r_{oc}|$ vs I_{ph} shown in Fig. 4 would be almost imperceptible, because only the horizontal axis would need to be shifted by the small difference that exists between I_{sc} and I_{ph} , which is given by (4).

In the case of the ideal cell that difference does not exist since (4i) compels $I_{sc}=I_{ph}$. Instead, Fig. 5 presents the magnitude of the dynamic resistance $|r_{oc}|$ at the open-circuit point, as a function of the reciprocal short-circuit current, $1/I_{sc}$, calculated with (11) or (12) for the same three values of R_s and a single value of R_p . The value of I_{sc} is calculated using (4). Also shown in Fig. 5 for comparison is the $|r_{ioc}|$ vs $1/I_{sc}$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i) or (12i), with the value of $I_{sc}=I_{ph}$ according to (4i).

The variation of the dynamic resistance's magnitude, $|r_{oc}|$, at the open-circuit point is shown in Fig. 6 also as a function of

Fig. 5. Linear plots of the magnitude of the dynamic resistance $|r_{oc}|$ as a function of reciprocal short-circuit current $1/I_{sc}$, calculated at the open-circuit point with (11) or (12) for three values of R_s and $R_p=100\Omega$. The value of $1/I_{sc}$ is calculated from I_{ph} using (4). Also shown as a black dash straight line, is the $|r_{ioc}|$ of the ideal cell ($G_p=0$ and $R_s=0$) vs $1/I_{sc}$, calculated with (11i) or (12i), using $I_{sc}=I_{ph}$ according to (4i).

Fig. 6. Semi-logarithmic plot of the magnitude of the dynamic resistance $|roc|$ at the open-circuit point, as a function of open-circuit voltage, V_{oc} , corresponding to increasing values of I_{ph} , calculated using (11) or (12) for three values of R_s and $R_p=100\Omega$. The value of V_{oc} is calculated using (5). Also shown for comparison as a black dash straight line is the dynamic resistance $|rioc|$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i).

open-circuit voltage, V_{oc} , whose value according to (5) increases with increasing levels of incident light intensity. $|roc|$ is calculated using (11) with $R_p=100\Omega$ at the three values of $R_s=1, 4$, and 10Ω . All three curves shown in Fig. 6 are seen to saturate to a value of $\approx R_p+R_s$ at low V_{oc} (very low levels of incident light intensity), and to a value of $\approx R_s$ at high V_{oc} (very high levels of incident light intensity).

Notice that although $|roc|$ is presented in Fig. 6 in semi-logarithmic scale, it is evident that its shape confirms the fact that $|roc|$ is a symmetric “S” shaped logistic function of V_{oc} , raised on a pedestal of magnitude of $\approx R_s$, as (11s) clearly indicates.

Also shown in Fig. 6 for comparison purposes is the magnitude of the dynamic resistance $|rioc|$ of the corresponding ideal cell ($G_p=0$ and $R_s=0$), calculated with (11i). It is easy to visually imagine how all three curves of $|roc|$ in Fig. 6 with $G_p \neq 0$ and $R_s \neq 0$ would join into the single straight line representing $|rioc|$ if their resistive losses vanished, $G_p \rightarrow 0$ and $R_s \rightarrow 0$.

The semi-logarithmic nature of Fig. 6 indicates that since the plot of the magnitude of the ideal cell's $|rioc|$ as a function of V_{ioc} is a straight line with negative slope, the value of $|rioc|$ must vary (decrease) as an exponential function of negative open-circuit voltage, V_{ioc} , which is what in fact it does, according to (11i).

On the other hand, Fig. 6 also indicates that, in the case of a real cell ($G_p \neq 0$ and $R_s \neq 0$), the observed dependence of $|roc|$ on V_{oc} happens to be in general considerably different from a purely exponential behavior. How much it differs depends on how significant the magnitudes of R_s and G_p are, as clearly implied by (11) or better yet, by (11s), as illustrated in Fig. 6 by the observed saturations to $|roc| \rightarrow R_p+R_s$ at low V_{oc} and to $|roc| \rightarrow R_s$ at high V_{oc} . Obviously, the smaller the values of R_s and G_p are, the nearer (11) becomes to (11i), and thus, the

closer the behavior of $|roc|$ would be to that of $|rioc|$, which decreases exponentially with increasing V_{oc} .

D. Dynamic resistance at the sort-circuit point

Let us now look at the other end of the power-generating quadrant, i.e., the short-circuit point, defined by the coordinates ($V=0, I=I_{sc}$). The magnitude of the dynamic resistance at the sort-circuit point, $|r_{sc}|$, is also a useful parameter to examine. To calculate it we need to evaluate (9) at $V=0$ and $I=I_{sc}$:

$$r_{sc}(I_{sc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{I_{sc} R_s}{n v_{th}}\right)}. \quad (13)$$

This expression represents an “S” shaped curve in the form of a logistic function raised on a pedestal. In fact, (13) can be easily recast in the usual logistic form, so that the magnitude $|r_{sc}|$ of the dynamic resistance at the short-circuit point ($V=0$) is:

$$|r_{sc}| = R_s + R_p \frac{1}{1 + \exp[k(I_{sc} - I_d)]}, \quad (13s)$$

where the fraction that multiplies R_p in the r.h.s. of (13s) is a symmetric “S” shaped (sigmoid) function of I_{sc} , which varies between a minimum value of 0 as $I_{sc} \rightarrow +\infty$ and a maximum value of 1 as $I_{sc} \rightarrow -\infty$, with the center of symmetry at the midpoint value = $1/2$. The multiplier factor $k = \frac{R_s}{n v_{th}}$ (A^{-1}) in the exponent is called the logistic growth rate, or steepness of the sigmoid curve, and the coefficient $I_d = \frac{1}{k} \ln\left(\frac{n v_{th}}{I_0 R_p}\right)$ (A) subtracted from I_{sc} in (13s) is a positive number that represents the displacement of the logistic midpoint location from the origin $I_{sc}=0$.

In the case of an ideal cell, substitution of $R_s=0$ and $G_p=0$ into (13) yields a usually high constant value of the dynamic resistance's magnitude:

$$|r_{isc}| = \frac{n v_{th}}{I_0}, \quad (13i)$$

Alternatively, evaluation of (10) at $V=0$ and $I=I_{sc}$ yields an expression equivalent to (13):

$$r_{sc} = r(I_{ph}, I_{sc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} - I_{sc}(G_p R_s + 1) + I_{ph} + I_0}, \quad (14)$$

In the case of an ideal cell, substitution of $R_s=0$ and $G_p=0$ into (14) yields exactly the same constant as in (13i):

$$|r_{isc}| = \frac{n v_{th}}{I_0}. \quad (14i)$$

The light intensity I_{ph} dependence of the magnitude of the dynamic resistance calculated at the sort-circuit point, $|r_{sc}|$, is presented in Fig. 7, for the same example cell with a single value of $R_p=100\Omega$ and three values of $R_s=1, 4$, and 10Ω . Fig. 7 quickly reveals that $|r_{sc}|$ is a function of light intensity, I_{ph} , in a way that qualitatively resembles that of $|roc|$ shown in Fig. 4. But this similarity refers only to the fact that when I_{ph} reaches

Fig. 7. Semi-logarithmic plot of the magnitude of the dynamic resistance r_{sc} calculated at the short-circuit point ($V=0$, $I=I_{sc}$) on the I - V characteristics of the illuminated hypothetical cell, as a function of photo-current I_{ph} , for $R_p=100\Omega$ and three values of R_s . The plot of the dynamic resistance $|r_{sc}|$ of the ideal cell ($G_p=0$ and $R_s=0$), calculated with (13i), may not be included here for comparison, since it is a very large constant (see Table III) that falls outside the bounds of this figure.

very large values, $|r_{sc}|$ exhibits saturated values near those of R_s (≈ 1 , 4, and 10Ω in this example); and that when the value of I_{ph} starts to decrease, $|r_{sc}|$ starts to increase until at low values of I_{ph} , $|r_{sc}|$ saturates to constant values around those of $R_p + R_s$ (≈ 101 , 104, and 110Ω in the present example).

Table III presents values of $|r_{sc}|$ and $|r_{sc}|$ for three values of I_{ph} . Notice that in the case of the ideal cell, the value of $|r_{sc}|$ is very large and constant at any incident light intensity (represented by I_{ph}).

Fig. 7 is perfectly compatible with the behavior observed in Fig. 3. Notice, e.g., that when $I_{ph}=0.02A$, the r_{sc} curve corresponding to $R_s=10\Omega$ (solid red line in Fig. 7) reaches a constant value of $\approx R_p + R_s$ ($\approx 107\Omega$ in this example). This is the same value the $|r|$ corresponding to $I_{ph}=0.02A$ (dashed green line in Fig.3) reaches when $V \rightarrow 0$ (short-circuit point) in Fig. 3.

Next we want to look at the magnitude of the dynamic resistance $|r_{sc}|$ at the short-circuit point ($V=0$, $I=I_{sc}$) as a function of short-circuit current, I_{sc} . It is plotted in Fig.8 as calculated using (13s) for a single value of $R_s=4\Omega$ and three values of R_p . The corresponding tangents at the midpoints (centers of symmetry) of the logistic functions are also shown in Fig. 8. They are calculated using the following equation:

TABLE III

MAGNITUDE OF THE DYNAMIC RESISTANCE AT SHORT-CIRCUIT CONDITIONS ($V=0$, $I=I_{sc}$) OF THE HYPOTHETICAL IDEAL AND REAL SOLAR CELL, FOR THREE VALUES OF INCIDENT LIGHT INTENSITY

Incident light intensity, expressed as photo-current I_{ph} (mA)	Ideal $ r_{sc} $ (k Ω) $G_p=0$, $R_s=0$	Real $ r_{sc} $ (Ω) $R_p=100\Omega$, $R_s=10\Omega$
20	387.9	107.28
30	387.9	87.87
40	387.9	39.11

Equivalent circuit model diode element's parameters:
 $v_{th}=0.02586V$, $n=1.5$, $I_0=10^{-7}A$.

$$|r_{sc}| = R_s + \frac{R_p}{2} \left[1 - \frac{k}{2} (I_{sc} - I_d) \right] \quad (15)$$

where k and I_d have been previously defined under (13s). Thus, the midpoints (center of symmetry) are located in Fig. 8 at ($I_{sc}=I_d$, $|r_{sc}|=R_s+R_p/2$) and the slope of $|r_{sc}|$ at the midpoint is given by:

$$slope = -\frac{R_p}{2} \left(\frac{k}{2} \right) \frac{R_s}{n v_{th}} = -\frac{R_p R_s}{4 n v_{th}} \quad (16)$$

Fig.9 again presents the same $|r_{sc}|$ as a function of short-circuit current I_{sc} , also calculated using (13s), but now for a single value of $R_p=100\Omega$ and three values of R_s . The corresponding tangents at the midpoints are calculated, as before, with (15), and are also included in Fig. 9.

IV. DISCUSSION

We have established that the magnitudes of the dynamic resistance at the short-circuit and the open-circuit points, $|r_{sc}|$ and $|r_{oc}|$, both experience a transition when I_{ph} changes as a consequence of a change of the incident light intensity. They both decrease from $\sim R_p + R_s$ at very low incident light intensities (very low I_{ph}) to $\sim R_s$ at very high incident light intensities (very high I_{ph}). Although the observed behavior of these two magnitudes appear to follow similar sigmoid-function type of behavior, a more careful comparison of Figs. 4 and 7 clearly reveals that their transitions from high to low values as the light intensity increases are different from each other. Several differences can be visualized in the present example. The first observation is that the transition of $|r_{oc}|$ (Fig. 4) occurs at lower values of I_{ph} than the transition of $|r_{sc}|$ (Fig. 7), and this is so for all values of R_s .

The second observation is that a similar change of the value

Fig. 8. The magnitude of the dynamic resistance $|r_{sc}|$ at the short-circuit point ($V=0$, $I=I_{sc}$) of the illuminated hypothetical cell, as a function of short-circuit current I_{sc} , calculated with (13s) for one value of $R_s=4\Omega$ and the three indicated values of R_p . Also shown are dotted straight lines representing the corresponding tangents at the midpoints of the three sigmoid curves, calculated with (15).

Fig. 9. The magnitude of the dynamic resistance $|r_{sc}|$ calculated at the short-circuit point ($V=0$, $I=I_{sc}$) of the illuminated hypothetical cell, as a function of short-circuit current I_{sc} , calculated with (13s) for a single value of $R_p=100\Omega$ and three values of R_s . Also shown are dotted straight lines representing the corresponding tangents at the centers of symmetry of the three sigmoid curves.

of R_s causes a much larger shift on the I_{ph} axis (wider spread) of the transition of $|r_{sc}|$ (Fig. 7) than that of $|r_{oc}|$ (Fig. 4). The shift produced by increasing the value of R_s by the same amount is smaller in the case of $|r_{oc}|$ (Fig. 4) than in the case of $|r_{sc}|$ (Fig. 7).

The third and most obvious observation refers to how the magnitudes of $|r_{oc}|$ and $|r_{sc}|$ vary in different ways. In the case of $|r_{sc}|$ that difference is a consequence of the crossovers of the curves, which can mean a total reversal of the I_{ph} dependence direction, as can be seen in the central region of Fig. 7. For example, assuming a value of I_{ph} of 0.08A, the curves presented in Fig. 7 for $|r_{sc}|$ indicate that if the value R_s increases from 1, to 4, and then to 10Ω , the corresponding magnitude of $|r_{sc}|$ experiences a direct reduction from ~ 101 , to ~ 62 , and then to $\sim 12\Omega$. This is the opposite dependence that occurs at very high and low values of I_{ph} , shown at the right and left sides, respectively, of Fig. 7. Even stranger behavior of $|r_{sc}|$ can be expected when R_s changes at certain values of I_{ph} , e.g. at $I_{ph}=0.2A$ in Fig. 7. This type of effect does not exist in the case of $|r_{oc}|$, whose value always increases as R_s increases at any value of I_{ph} , as shown Fig. 4.

A common approximation mistake that unfortunately is often made about the dynamic resistance of illuminated solar cells is to naively assume that it is always approximately equal to R_s when measured at the open-circuit point, or that when measured at the short-circuit point it is always approximately equal to R_p [6]. While these two approximations could be perfectly assumable in some instances, the hazard is, as we have shown here, to assume that these approximations are always justified, regardless of the magnitude of the parasitic resistive losses and the intensity of the incident light, as Figs. 4 and 7 clearly indicate. The results of this analysis, therefore, call attention to the need to handle these assumptions about the dynamic

resistance with care, always carefully checking their validity before proceeding to draw any conclusions.

V. CONCLUSIONS

The following very general conclusions may be summoned up about the theoretically expected behavior of the dynamic resistance of typical illuminated solar cells:

- 1) Varying the illumination level incident on a photovoltaic cell, in general not only changes the value of its I_{sc} (4) but also of its V_{oc} (5).
- 2) The magnitude of the dynamic resistance $|r|$ of an illuminated photovoltaic cell measured (or calculated) at any point of its static I - V characteristic, depends in general on the illumination intensity incident on the cell's surface (Fig. 3). This includes, of course, the value of the dynamic resistance measured at the short circuit, open circuit, and MPP points of the I - V curve.
- 3) Specifically, the magnitude of the dynamic resistance measured (or evaluated) at the open-circuit and short-circuit points, $|r_{oc}|$ and $|r_{sc}|$ respectively, behave as symmetric logistic functions of V_{oc} and I_{sc} , respectively, which decrease from a maximum value of $\sim(R_s+R_p)$ to a minimum value of $\sim R_s$ on going from very low to very high values of V_{oc} and I_{sc} , respectively (Figs. 8 & 9).
- 4) The dependence of the magnitude of the dynamic resistance on the changing level of photo-generated current I_{ph} , can have a considerable impact on the ability to effectively track the MPP ($R_{load}=V_{mpp}/I_{mpp}=-r_{mpp}$) of a solar cell or module, when the incident solar radiation density (and its spectral content) experiences seasonal, daily, hourly, or arbitrary climatic and spurious changes.
- 5) Whenever the intensity of the incident light is increased considerably, e.g. through some light concentration means, so that the resulting photo-generated current I_{ph} is allowed to reach high enough levels, the magnitude of the dynamic resistance $|r|$ could eventually reach an almost constant value, close to that of the equivalent lumped series resistive loss (R_s), at every point along the I - V characteristic, including, of course, the short-circuit, open-circuit, and MPP points. Such very high I_{ph} condition would mean that the value of the cell's $FF \rightarrow 0.5$, in the best case if the load is kept at that constant value of $R_{load} \approx R_s$. This is without considering other significant second order effects, such as photo-induced phenomena or rising cell temperature.
- 6) On the other end, whenever the level of illumination decreases drastically, e.g. with approaching nighttime, or under cloud cover in the case of terrestrial solar cells. In such circumstances the intensity of the photocurrent I_{ph} could become low enough so that the value of $|r|$ would approach, and eventually saturate to, a value approximately equal to (R_s+R_p) at every operating point along the I - V characteristic, including, of course, the short-circuit, open-circuit, and MPP points. Therefore, under vanishing incident light conditions the value of $FF \rightarrow 0.5$, although this fact might be irrelevant once the available energy to be converted drops to insignificant levels.

ACKNOWLEDGMENT

Abundant conversations with countless unnamed students and colleagues have contributed over time to a better understanding of the complex implications of this issue.

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