This item was submitted to [Loughborough's Research Repository](#) by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

Direct current generation due to wave mixing in semiconductors

PLEASE CITE THE PUBLISHED VERSION

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Alekseev, Kirill N., M.V. Erementchouk, and F.V. Kusmartsev. 2019. "Direct Current Generation Due to Wave Mixing in Semiconductors". figshare. <https://hdl.handle.net/2134/1234>.

Direct current generation due to wave mixing in semiconductors

Kirill N. Alekseev^a, Mikhael V. Erementchouk
*Kirensky Institute of Physics, Russian Academy of Sciences,
 Krasnoyarsk 660036, Russia*

Feodor V. Kusmartsev^{b*}
*School of Mathematical and Physical Sciences ,
 Loughborough University, Loughborough LE11 3TU, UK*

We describe a novel effect of the generation of direct current which may arise in semiconductors or semiconductor microstructures due to a mixing of coherent electromagnetic radiations of commensurate frequencies. The effect is, in essence, due to a nonparabolicity of the electron energy bands and is stronger in systems where this nonparabolicity is greater. We have made exact calculations in the framework of the Kane model, applicable to narrow gap semiconductors and the tight-binding model which we employ for a description of a semiconductor superlattice.

72.20.Ht, 73.61.-r, 42.65.Hw

The problem of emission and reception of electromagnetic radiation has attracted the attention of the scientific community for a long time. Good sources of coherent electromagnetic radiation, its receivers and detectors exist for the radio-frequency, microwave and optical ranges of the spectrum. Many of these devices are based on semiconductor technology. Nowadays, the terahertz range (0.1 - 1 THz, submillimetre wavelengths) is practically the last unexploited *terra incognita*.

In 1970, Esaki and Tsu [1] made a pioneering suggestion to use the semiconductor superlattice (SSL) for generation of Bloch oscillations of frequency $\omega_B = eaE_0/\hbar$, where E_0 is a constant electric field applied along the axis of a SSL with a spatial period a . For typical SSLs and bias E_0 of 1 - 10 kV/cm, the Bloch frequency belongs to the THz range [1]. This can be used for the generation of THz radiation [2]. The work of Esaki and Tsu [1] stimulated enormous theoretical activity devoted to the interaction of a high-frequency electric field with a SSL [3,4]. Moreover, recent progress in the development of THz radiation sources and coupling techniques allow the systematic experimental studies of many of the associated nonlinear effects [5]. Amongst others, one of the most interesting suggestions was that of Esaki and Tsu [6] and Romanov [7] to use the SSL as a new, artificial, nonlinear material for electromagnetic wave mixing and harmonic generation. The theory of wave mixing in SSLs, based on a solution of the Boltzmann equation, has been developed by Romanov and co-workers [8,3].

Very recently, in cooperation with the Urbana group, we showed [9] that the effect inverse to the Bloch oscillations in SSL is possible, namely, an alternating field without constant bias can create Bloch oscillations in a single miniband SSL. The appearance of induced Bloch oscillations means a spontaneous creation of constant voltage and corresponding direct current (DC) [9], that can be considered as a rectification of the THz field in SSLs. The rectification in [9] results from an interplay of dissipation (scattering of ballistic electrons with impurities and phonons) and generation of a self-consistent electric field along the SSL axis, which is a strongly nonlinear effect. Moreover, the generation of DC in [9] is a counterpart of chaos, which arises for the same *ac* field strength and frequency but for much weaker dissipation [10].

Independently, Goychuk and Hänggi [11] suggested another scheme of quantum rectification using a wave mixing of an alternating electric field and its second harmonic in a single miniband SSL. The approach of [11] is based on the theory of quantum ratchets (see [12] and references therein) and therefore the necessary conditions for the appearance of DC include a dissipation (quantum noise) and an extended periodic system.

In this work we find an even more general effect of DC generation, or, a rectification at wave mixing, which may arise in any semiconductor or semiconductor microstructure. The origin of this effect is related to neither dissipation nor quantum noise but rather to a nonparabolicity of the energy spectrum. This nonparabolicity is always present in any semiconductor or semiconductor structure [13,4]. The values of the nonparabolical energy terms may take their largest values in semiconductors with wide bands or small effective masses. It seems that the size of the gaps is not important, however, semiconductors with wide gaps only have small concentrations of current carriers unless, of course, they are doped. Hence, the best candidates for the described effect are semiconductors having wide bands and narrow

^aalso Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow 142432, Russia

gaps or doped semiconductors such as InSb and other analogous compounds. Indeed, wave-mixing has already been observed in such narrow-gap semiconductors [14–16]; its mechanism being mainly related to the nonparabolicity of the energy band [14–18].

We also consider another system, SSLs, which is now under intensive experimental investigation and the possibility of wave mixing there has also been suggested. The description of a SSL in the framework of the tight binding model allows exact solutions to be obtained. Moreover, since the nonparabolic energy term in a SSL takes smaller values than for normal semiconductors, the estimation of the DC generation effect for this system gives a lower bound to the value of the DC which may be observed in semiconductors. We show that the main features of the novel effect, the generation of DC due to the mixing of harmonics, may be well described and understood within the standard semiclassical Boltzmann equation approach [8,3]. We demonstrate that the value of DC is strongly dependent on the product of a characteristic scattering time, τ , and field frequency, Ω . In the experimentally relevant case of weak field strength, the expression for DC can be represented in the form of the generalized Ohm's law with prefactor dependent on $\Omega\tau$. The value of DC is maximal at $\Omega \simeq \tau^{-1}$ (namely, $\Omega\tau \approx 0.7$) and it decreases quadratically for both small and large values of $\Omega\tau$.

The novel effect we find is consistent with results obtained previously in a quantum approach [11] for a special case of quantum noise. However, our predicted effect is much more general and has a much simpler physical picture associated with wave mixing in nonlinear media arising due to a nonparabolicity of the electron bands in semiconductors. In this new framework the results of [9,11], which at first glance look very different, may be unified. The generality of the novel effect highlights new directions for various different applications and uses of semiconductor technology.

Now, we demonstrate how the rectification effect arises due to a nonparabolicity of the spectrum of the semiconductors. Note that, although the dissipative property of a semiconductor or microstructure has an influence on the effect, it only plays a secondary role. To study the generation of a finite DC let us consider, for simplicity, a cubic semiconductor subjected to the electric mixing harmonic fields

$$E(t) = E_1 \cos(\Omega t) + E_2 \cos(2\Omega t + \phi), \quad (1)$$

With the aid of an effective mass method [13] the energy-momentum dispersion relation of a nondegenerate cubic semiconductor in the vicinity of the bottom of the conduction band may be represented as (see, for comparison, Ref. [4])

$$\varepsilon(p_x, p_y, p_z) = \frac{p^2}{2m} + \frac{\eta}{4}(p_x^4 + p_y^4 + p_z^4), \quad \frac{1}{m} = \left. \frac{\partial^2 \varepsilon}{\partial p_i^2} \right|_{p_i=0}, \quad \eta = \left. \frac{1}{6} \frac{\partial^4 \varepsilon}{\partial p_i^4} \right|_{p_i=0}, \quad (2)$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$, m is an effective mass at the bottom of the conduction band, η is a parameter of nonparabolicity. Following [4], for simplicity, we consider electron motion only along one direction, for example, along the $\mathbf{x}$ -direction and therefore, we may limit ourself to the one dimensional approximation. From now on the index x is suppressed. The electron's velocity is given by $v = \partial \varepsilon / \partial p$, the DC related to nonparabolicity is $j_{dc} = en\eta \langle p^3 \rangle$, where n is the number of electrons per unit volume and the angled brackets $\langle \dots \rangle$ indicate the time averaging over a period of the electric field $2\pi/\Omega$. For pure ballistic electron motion without scattering by impurities or phonons, the electron dynamics is determined by the accelerating theorem $\dot{p} = eE(t)$. Combining this formula with Eqs. (1) and (2), we have

$$j_{dc} \propto -en\eta \frac{3}{8} \frac{e^3}{\Omega^3} E_1^2 E_2. \quad (3)$$

Next, we will consider two specific examples where the value of the DC can be expressed explicitly through the well-established parameters of the semiconductor energy bands. The energy-momentum dispersion relations, $\varepsilon(\mathbf{p})$, of a cubic semiconductor is usually obtained within the effective mass method [13]. For a SSL, the dependence $\varepsilon(\mathbf{p})$ was obtained within a tight-binding model [3]. In both cases, in the weak field limit when the excitation energy of electrons is small in comparison with the bandwidth, we may take into account only the first terms in the energy momentum relation (see, the dependence (2)).

We start with the more transparent case of a SSL. Here the interplay of dissipation effects and nonparabolicity of the miniband may be described exactly. Consider the motion of an electron within a single miniband of the SSL with spatial period a and miniband width Δ . In the standard tight-binding approximation, we have

$$\varepsilon = \frac{\Delta}{2} \left[1 - \cos\left(\frac{pa}{\hbar}\right) \right], \quad (4)$$

where p is the momentum of the electron along the SSL axis. For a weak electric field the electrons oscillate in p -space near the center of the Brillouin zone $|p| \ll \pi\hbar/a$ or at the bottom of the miniband. Then, from Eq. (4), we have (2) with $\eta = -a^2/6\hbar^2m$ and $m = (2\hbar^2)/(\Delta a^2)$. The DC [Eq. (3)] takes the form $j_{dc} \simeq \frac{en\hbar}{am} \xi_1^2 \xi_2$, where $\xi_l = \frac{eaE_l}{\hbar\Omega}$, ($l = 1, 2$). It is easy to see that the condition of weak nonparabolicity, $|p| \ll \pi\hbar/a$, corresponds to $\xi_l \ll 1$. We turn to the detailed calculations of the DC taking into account collisions of electrons with impurities and phonons, *i.e.* dissipation effects.

The electron transport properties in narrow miniband SSLs at temperatures above 40K are known [21] to be well described by the semiclassical Boltzmann equation with a constant relaxation time, τ . Starting from a formal solution of the Boltzmann equation with constant relaxation time, Romanov and co-workers found the exact expression for a time-dependent current $j(t)$ in a tight-binding lattice [Eq. (4)] subjected to an electric field with two frequencies ω_1 and ω_2 [8,22] (see also Appendix A). Taking $\omega_1 = \Omega$, $\omega_2 = 2\Omega$ and averaging $j(t)$ over the period of the *ac* field, $2\pi/\Omega$, we get for the rectified DC $j_{dc} = \langle j(t) \rangle$ for $t \gg \tau$ the following formula

$$j_{dc} = j_0 \sum_{\mu_1, \mu_2=-\infty}^{+\infty} \sum_{\nu=-\infty}^{+\infty} \frac{(\mu_1 + 2\mu_2)x \cos(\nu\phi) + \sin(\nu\phi)}{1 + (\mu_1 + 2\mu_2)^2 x^2} J_{\mu_1}(\xi_1) J_{\mu_2}(\xi_2) J_{\mu_1-2\nu}(\xi_1) J_{\mu_2+\nu}(\xi_2), \quad (5)$$

where $x = \Omega\tau$, $j_0 = \frac{\hbar\sigma}{e\tau a}$ with σ being a static, ohmic conductivity along the SSL axis, $J_\mu(\xi)$ is the Bessel function.

The calculation of the DC with the use of Eq. (5) shows that the DC strongly depends on the product $x = \Omega\tau$. Fig. 1 illustrates the dependence of the DC on ξ_1 and ξ_2 for different values of x . For small x , the absolute value of the DC increases monotonically with an increase of both ξ_1 and ξ_2 (Fig. 1a). The increasing slope of the DC graph changes dramatically when the value of x increases. For instance, this slope can increase by almost five orders of magnitude with an increase in x of only one order (compare Figs. 1a and 1b). However, for further change of x from $x = 0.2$ (Fig. 1b) to $x = 1$ (Fig. 1c), the corresponding increase in DC slows down and becomes nonmonotonical. The DC reaches its maximal value for some optimal value of the relationship between ξ_1 and ξ_2 . So for the case $x = 1$, the DC is maximal when $\xi_1/\xi_2 \simeq 1.0$ (see Fig. 1c).

When the electric field amplitudes are small, $\xi_{1,2} \ll 1$, we can use the Bessel function approximation $J_n(\xi) \approx (\xi/2)^n (1/n!)$ and obtain from (5) the following analytic expression for the DC (see Appendix B)

$$j_{dc} = -1.5j_0 \frac{x^3}{4x^4 + 5x^2 + 1} \xi_1^2 \xi_2 \cos \phi + O(\xi^5). \quad (6)$$

Substituting usual Drude conductivity $\sigma_{\text{drude}} = ne^2\tau/m$ into j_0 , we get factor $(en\hbar)/(am)$, which is similar to the prefactor for j_{dc} obtained within collisionless approximation. The equation (6) agrees well with the asymptotic dependence of the DC on the field amplitudes, $j_{dc} \propto \xi_1^2 \xi_2 \cos \phi$ as obtained in reference [11] in the framework of a different approach. However, our result (6) also gives the dependence of DC on the parameter $x = \Omega\tau$ as well as indicate that the next significant contributions to the current appear only when the parameter, ξ , is of the order of ξ^5 .

Let us estimate the value of the predicted DC for the experimental conditions of THz field driven SSLs [5]. For the miniband width $\Delta \simeq 10$ meV, superlattice period $a \simeq 10$ nm, electron density $n \simeq 10^{16} \text{ cm}^{-3}$, sample area $S \simeq 10 (\mu\text{m})^2$, electric field of amplitude $E \simeq 1$ kV/cm and frequency Ω of several THz ($\xi_1 = \xi_2 \simeq 0.1$) and the characteristic relaxation time of several picoseconds ($x = \Omega\tau \simeq 1$), we get the DC $I_{dc} = j_{dc}S \simeq 0.1 \mu\text{A}$. This value is in very good agreement with an estimate for the DC obtained by Goychuk and Hänggi [11] from numerical calculations of the integral representations for DC within a different approach based on the ratchet theory.

Note that DC density (6) can be represented in the form of the generalized nonlinear Ohm's law incorporating only total electric field $E(t)$ [Eq.(1)]. Really, using the property $\langle E^3 \rangle = 3/4(E_1^2 E_2) \cos \phi$, we have from (6) the following expression for the DC

$$j_{dc} = -\sigma f(x) \langle \overline{E}^2 E \rangle, \quad f(x) = \frac{x^2}{4x^4 + 5x^2 + 1}, \quad \overline{E} \equiv \frac{eaE(t)}{\hbar\Omega}. \quad (7)$$

The nontrivial prefactor $f(x)$ increases as x^2 in the limit $x \ll 1$, reaches its maximal value $f(x^*) \approx 0.11$ at $x^* \approx 0.71$, and finally decreases as $(4x^2)^{-1}$ for high frequencies ($x = \Omega\tau \gg 1$). Note that in order to use the single miniband approximation and neglect the interminiband transitions, the electric field frequency, in units of the energy $\hbar\Omega$, should be less than the interminiband distance or, in other words, it should be of the order of or less than the miniband width $\hbar\Omega \lesssim \Delta$. The case $\hbar\Omega \simeq \Delta$ is the typical case in experiments [5] (see also our estimates above). Thus, the dimensionless electric field, $\overline{E}$, (involved in the expression (7)) is of the order of eaE/Δ . To apply the Boltzmann

equation to the description of miniband transport in a quantum superlattice, the parameter eaE/Δ should always be small [3], *i.e.*, in our notation, $\bar{E} \ll 1$. This remark shows that the DC generated in the SSL under the action of the high-frequency electric field $E(t)$ is less than the corresponding current $j = \sigma E$ generated by a constant bias of the same strength E in the factor $\simeq 0.1\bar{E}^2$. The numerical value of this factor is only $\simeq 10^{-3}$ for typical experiments in SSLs, which implies that the semiclassical approach is valid and at the same time gives serious grounds for the observation of DC generation due to wave mixing.

It is interesting that in another scheme of rectification as suggested in Ref. [9] only a single harmonic of a THz field is required. However, in this case there is actually an *implicit* wave mixing. Indeed, even in the simplest case of rectification, the self-consistent field generated in a SSL has a second harmonic of the fundamental frequency, Ω , so that the *total electric field* acting on the electrons has at least two components with frequencies Ω and 2Ω [9,10]. Consequently, the effect proposed in Ref. [9] may arise due to the wave mixing of the induced self-consistent field and the applied external THz field. This effect will be similar to the effect of DC generation arising at the wave mixing of two commensurate THz harmonics as described in Ref. [9,11]. Thus, in spite of obvious differences, the approaches to rectification of THz radiation developed in Refs [9,11] are not contradictory but rather complement each other.

Now, let us consider the generation of DC at harmonic mixing in the III-V type semiconductors with narrow gaps, where the nonparabolicity is also strong. Mixing of mm-waves in n-doped InAs, InSb and GaAs semiconductors has been studied experimentally in [14–16] and discussed theoretically in [17,18] a long time ago. However, we do not know of any reports devoted to the observation of DC generation at wave mixing in these semiconductors. This is probably because the experiments [14–16] were devoted only to the mixing of waves with similar frequencies or third harmonic generation.

Let us estimate the DC that could be generated in such semiconductors at harmonic mixing. Semiconductors such as InAs, InSb can be described [17,15] by the Kane four-band model [19,13]. Of course, for a proper description of the electron-hole dynamics we have to take into account all four bands, which is, in fact, a very tedious task. However, in a weak field assuming that, due to dominant donor doping, the electron concentration is larger than the hole concentration, we may take into account only one of the energy bands: the electron branch with the energy-momentum dispersion relation [19,4,13] in the form

$$\varepsilon(p) = \frac{\varepsilon_g}{2} \left[\left(1 + \frac{2p^2}{m\varepsilon_g} \right)^{1/2} - 1 \right], \quad (8)$$

where ε_g is the width of the gap. Note that GaAs cannot be described by the dependence (8), however in the weak field limit, the nonparabolicity, η , is roughly twice as great as follows from the Kane model [15]. Note also that, due to a diamond structure of the III-V type semiconductors, into a nonparabolicity coefficient the cubic invariants will also contribute, however, these contributions should not exceed those which follow from the Kane model. In the limiting case $p/\sqrt{m\varepsilon_g} \ll 1$, we have the dependence (2) with $\eta = -1/(m^2\varepsilon_g)$ and the DC generated at wave mixing (3) is of the order of

$$j_{dc} \simeq \frac{3}{4} en \left(\frac{\varepsilon_g}{m} \right)^{1/2} \tilde{E}_1^2 \tilde{E}_2, \quad \tilde{E}_l \equiv \frac{eE_l}{l\Omega(m\varepsilon_g)^{1/2}}, \quad (l = 1, 2). \quad (9)$$

The condition of weak field $p/(m\varepsilon_g)^{1/2} \ll 1$ takes the form $\tilde{E}_l \ll 1$ ($l = 1, 2$). For the donor doped n-InSb with the gap width $\varepsilon_g \approx 0.2$ eV, effective mass $m \approx 0.016m_e$, level of doping $n \simeq 10^{16} \text{ cm}^{-3}$, sample area $S \simeq 10^{-4} \text{ cm}^2$, field frequency $\Omega \simeq 10^{11} \text{ sec}^{-1}$ and field strength of $\simeq 100 \text{ V/cm}$, we get the DC $I_{dc} = j_{dc}S$ of the order of several mA. At the same time, the condition $\tilde{E} \simeq 0.1 \ll 1$ guarantees the applicability of the weak field limit here.

It is important to note that for gapless or narrow-gap semiconductors the current carriers may be excited by microwave radiation. That is their concentration may be changed by the applied field. On one hand the nonparabolicity of the energy spectrum is larger in wide band semiconductors, while on the other hand the concentration of electrons which may be excited by microwave radiation may be larger in narrow gap or gapless semiconductors or semimetals. Therefore, the most optimal conditions for the described effect would be in semiconductors having narrow gaps and wide bands, like InSb and other analogous compounds.

In this estimation of the DC in InSb we actually neglect a scattering of the electrons by impurities, defects and phonons; which is in itself an important process in semiconductors. In this respect, we should make two remarks. First of all, if the frequency Ω is of the same order as the inverse characteristic relaxation time τ^{-1} , which is the case of experiments [14–16], the DC should be of the same order as estimated in Eq. (9) [20]. Otherwise, the dependence of the DC will have a form similar to that obtained above for the case of a SSL, Eq. (7).

Secondly, several other mechanisms of nonlinearity could also be responsible for wave mixing in semiconductors [14,17,18]. One of the most important of these is the so-called heating mechanism where the nonlinearity is related to the dependence of a relaxation constant on the field [16,18]. This is mainly responsible for the mixing of mm-waves at low temperatures ($\simeq 4\text{K}$) [16]. On other hand, the experiments [14–16] and theories [17,18] demonstrate that at temperatures $\simeq 80\text{K}$ the nonlinearity related to the nonparabolicity of the conduction band becomes more important. The estimates presented in [18] show that the relative influence of these two different types of nonlinearity is dependent not only on the temperature but also on the type of wave mixing. For the mixing of waves with similar frequencies (the case of experiments [14]), the nonparabolicity is the dominant nonlinearity for temperatures $\simeq 80\text{K}$. However, for the mixing of waves with an integer ratio of frequencies, *e.g.* for a third harmonic generation, the influence of both types of nonlinearity is comparable [16,18]. In any case, the effect of DC generation arising due to the mixing of mm-waves should exist in narrow gap III-V and even some II-VI semiconductors. We hope that our results describing this new effect will attract the attention of experimentalists to this very intriguing issue.

In summary, with the use of the semiclassical Boltzmann equation we have found the novel effect of DC generation in semiconductors and semiconductor microstructures driven by two pure coherent electromagnetic waves of commensurate frequencies. The described effect originates from the nonlinearity associated with the nonparabolicity of the energy band and therefore, it is universal and may be observed in any semiconductors. Thus, our findings indicate that any semiconductor may be considered as a particular type of nonlinear medium serving as a generator of DC [23]. This effect is very different from the photogalvanic effect arising in semiconductors without a center of symmetry [24]. Finally, we propose that experiments be set up for the mixing of millimetre- and submillimetre-waves in semiconductors and semiconductor superlattices to observe the predicted generation of direct current.

We thank David Campbell, Ethan Cannon, Laurence Eaves, Holger Grahn, Peter Kleinert, Eckehard Schöll, Andreas Wacker for useful discussions on THz field driven semiconductor superlattices and Yuri Romanov for bringing our attention to refs. [7,8]. KNA thanks the Department of Physics, Loughborough University for hospitality and Krasnoyarsk Regional Science Foundation for partial support.

APPENDIX A:

In this Appendix we will derive Eq. (5). Starting from the Boltzmann equation for a distribution function $f(p, t)$

$$\frac{\partial f}{\partial t} + eE \frac{\partial f}{\partial p} = -\frac{f - f^{\text{eq}}}{\tau} \quad (\text{A1})$$

with an equilibrium distribution function $f^{\text{eq}}(p)$ and electric field $E(t) = E_1 \cos(\omega_1 t) + E_2 \cos(\omega_2 t + \phi)$, Orlov and Romanov [8] have found the exact expression for time-dependent current

$$j(t) = en \int v(p) f(p, t) dp \quad (\text{A2})$$

in the form

$$j(t) = j_0 \left[\frac{i}{2} \sum_{\mu_1, \mu_2 = -\infty}^{+\infty} \sum_{\nu_1, \nu_2 = -\infty}^{+\infty} J_{\mu_1}(\xi_1) J_{\mu_2}(\xi_2) J_{\mu_1 + \nu_1}(\xi_1) J_{\mu_2 + \nu_2}(\xi_2) \frac{1}{1 + i(\mu_1 \omega_1 + \mu_2 \omega_2) \tau} \exp(-i\nu_1 \omega_2 t - i\nu_2 (\omega_2 t + \phi)) \right] + \text{c.c.}, \quad (\text{A3})$$

where $\xi_k = \frac{e a E_k}{\hbar \omega_k}$ ($k = 1, 2$) and $j_0 = \frac{\hbar \sigma}{e a \tau}$. In this appendix, we first present simple derivation¹ of Eq.(A3), and then will obtain the expression for rectified DC.

Using periodicity of distribution function in p -space

¹The simplest way is to solve the Boltzmann equation in the Fourier representation. For the case $E(t) = E_0 \cos(\omega t)$, this method is presented in clear form in: Bass F. G., Rubinshtein E. A., *Fiz. Tverd. Tela*, **19** (1977) 1379 [*Sov. Phys. Solid State*, **19** (1977) 800].

$$f(p, t) = \sum_{l=-\infty}^{+\infty} f_l(t) \exp\left(i \frac{apl}{\hbar}\right), \quad (\text{A4})$$

the Boltzmann equation for l th harmonics could be represented as

$$\frac{\partial f_l}{\partial t} - \frac{iel a}{\hbar c} \frac{\partial A}{\partial t} f_l = -\frac{f_l - f_l^{\text{eq}}}{\tau}, \quad E = -\frac{1}{c} \frac{\partial A}{\partial t}. \quad (\text{A5})$$

Assuming $f(p, t = -\infty) = f^{\text{eq}}(p)$, the solution of (A5) is

$$f_l = \tau^{-1} \exp\left(-\frac{t}{\tau}\right) \exp\left[\frac{iel a A(t)}{\hbar c}\right] f_l^{\text{eq}} \int_{-\infty}^t \exp\left(\frac{t'}{\tau}\right) \exp\left[-\frac{iel a A(t')}{\hbar c}\right] dt'. \quad (\text{A6})$$

This solution is valid for arbitrary $A(t)$; in our case $A(t) = -\frac{E_1 c}{\omega_1} \sin(\omega_1 t) - \frac{E_2 c}{\omega_2} \sin(\omega_2 t + \phi)$. Using the Bessel functions expansion

$$\exp(\pm i \xi \sin \theta) = \sum_{m=-\infty}^{+\infty} J_m(\xi) \exp(\pm i m \theta), \quad (\text{A7})$$

we can expand $\exp[\pm \frac{iel a A(t)}{\hbar c}]$ in (A6) and get after integration:

$$f_1 = f_1^{\text{eq}} \sum_{k_1, k_2} \sum_{n_1, n_2} J_{n_1}(\xi_1) J_{n_2}(\xi_2) J_{k_1}(\xi_1) J_{k_2}(\xi_2) \frac{e^{i(n_1 - k_1)\omega_1 t} e^{i(n_2 - k_2)\omega_2 t}}{1 + i(n_1 \omega_1 + n_2 \omega_2) \tau} e^{i(n_2 - k_2)\phi}. \quad (\text{A8})$$

Now, introducing new summation indexes $\nu_l = k_l - n_l$ ($l = 1, 2$) and substituting Eq. (A8) into the expression for the current $j = \frac{i q}{2} i (f_1 - f_{-1})$, we get the formula (A3). The Fourier transform of equilibrium distribution function f_1^{eq} determines the temperature dependence of conductivity.

Finally, taking $\omega_1 = \Omega$, $\omega_2 = 2\Omega$ and averaging $j(t)$ [Eq. (A3)] over period of ac field $2\pi/\Omega$, we get for the rectified DC $j_{\text{dc}} = \langle j(t) \rangle$ the formula (5).

APPENDIX B:

In this Appendix we derive Eq. (6). We start with the expression for DC j_{dc} , Eq. (5), which now is convenient to rewrite in the form

$$j_{\text{dc}} = j_0 \sum_{\mu_1, \mu_2 = -\infty}^{+\infty} \sum_{\nu = -\infty}^{+\infty} [A(\mu_1, \mu_2, \nu) + B(\mu_1, \mu_2, \nu)] J_{\mu_1}(\xi_1) J_{\mu_2}(\xi_2) J_{\mu_1 - 2\nu}(\xi_1) J_{\mu_2 + \nu}(\xi_2), \quad (\text{B1})$$

where

$$A(\mu_1, \mu_2, \nu) = \frac{(\mu_1 + 2\mu_2)x \cos(\nu\phi)}{1 + (\mu_1 + 2\mu_2)^2 x^2}, \quad B(\mu_1, \mu_2, \nu) = \frac{\sin(\nu\phi)}{1 + (\mu_1 + 2\mu_2)^2 x^2} \quad (\text{B2})$$

Using the Bessel function property $J_n(\xi) \simeq \xi^n$ for $\xi \ll 1$, we will consider only nonvanishing lowest order in $\xi_{1,2} \ll 1$ terms in (B1). We start with terms of $[\mu_1 = 0, \mu_2 = 0, \nu = \pm 1]$:

$$\begin{aligned} & [A(0, 0, 1) + B(0, 0, 1)] J_0(\xi_1) J_0(\xi_2) J_{-2}(\xi_1) J_1(\xi_2) + \\ & [A(0, 0, -1) + B(0, 0, -1)] J_0(\xi_1) J_0(\xi_2) J_2(\xi_1) J_{-1}(\xi_2) = \\ & 2 \sin(\phi) J_0(\xi_1) J_0(\xi_2) J_2(\xi_1) J_1(\xi_2). \end{aligned} \quad (\text{B3})$$

Term with $[0, -1, 1]$ plus term with $[0, 1, -1]$:

$$\begin{aligned} & [A(0, -1, 1) + B(0, -1, 1)] J_0(\xi_1) J_{-1}(\xi_2) J_{-2}(\xi_1) J_0(\xi_2) + \\ & [A(0, 1, -1) + B(0, 1, -1)] J_0(\xi_1) J_1(\xi_2) J_2(\xi_1) J_0(\xi_2) = \end{aligned}$$

$$2 \left[\frac{2x \cos \phi}{1 + 4x^2} - \frac{\sin \phi}{1 + 4x^2} \right] J_0(\xi_1) J_1(\xi_2) J_2(\xi_1) J_0(\xi_2). \quad (\text{B4})$$

Term with $[1, 0, 1]$ plus term with $[-1, 0, -1]$:

$$\begin{aligned} & [A(1, 0, 1) + B(1, 0, 1)] J_1(\xi_1) J_0(\xi_2) J_{-1}(\xi_1) J_1(\xi_2) + \\ & [A(-1, 0, -1) + B(-1, 0, -1)] J_{-1}(\xi_1) J_0(\xi_2) J_1(\xi_1) J_{-1}(\xi_2) = \\ & -2 \left[\frac{x \cos \phi}{1 + x^2} + \frac{\sin \phi}{1 + x^2} \right] J_1^2(\xi_1) J_0(\xi_2) J_1(\xi_2). \end{aligned} \quad (\text{B5})$$

Term with $[1, -1, 1]$ plus term with $[-1, 1, -1]$:

$$\begin{aligned} & [A(1, -1, 1) + B(1, -1, 1)] J_1(\xi_1) J_{-1}(\xi_2) J_{-1}(\xi_1) J_0(\xi_2) + \\ & [A(-1, 1, -1) + B(-1, 1, -1)] J_{-1}(\xi_1) J_1(\xi_2) J_1(\xi_1) J_0(\xi_2) = \\ & 2 \left[\frac{-x \cos \phi}{1 + x^2} + \frac{\sin \phi}{1 + x^2} \right] J_1^2(\xi_1) J_0(\xi_2) J_1(\xi_2). \end{aligned} \quad (\text{B6})$$

Term with $[2, 0, 1]$ plus term with $[-2, 0, -1]$:

$$\begin{aligned} & [A(2, 0, 1) + B(2, 0, 1)] J_2(\xi_1) J_0(\xi_2) J_0(\xi_1) J_1(\xi_2) + \\ & [A(-2, 0, -1) + B(-2, 0, -1)] J_{-2}(\xi_1) J_0(\xi_2) J_0(\xi_1) J_{-1}(\xi_2) = \\ & 2 \left[\frac{2x \cos \phi}{1 + 4x^2} + \frac{\sin \phi}{1 + 4x^2} \right] J_2(\xi_1) J_0(\xi_2) J_0(\xi_1) J_1(\xi_2). \end{aligned} \quad (\text{B7})$$

Term with $[2, -1, 1]$ plus term with $[-2, 1, -1]$:

$$\begin{aligned} & [A(2, -1, 1) + B(2, -1, 1)] J_2(\xi_1) J_{-1}(\xi_2) J_0(\xi_1) J_1(\xi_2) + \\ & [A(-2, 1, -1) + B(-2, 1, -1)] J_{-2}(\xi_1) J_1(\xi_2) J_0(\xi_1) J_0(\xi_2) = \\ & = -2 \sin(\phi) J_2(\xi_1) J_1(\xi_2) J_0(\xi_1) J_0(\xi_2). \end{aligned} \quad (\text{B8})$$

All other terms in Eq. (B1) either compensate each other or are of order which is greater than ξ^3 . Combining (B3)-(B8), we get following formula for the DC

$$\begin{aligned} \frac{j_{dc}}{j_0} &= \frac{8x \cos \phi}{1 + 4x^2} J_0(\xi_1) J_1(\xi_2) J_2(\xi_1) J_0(\xi_2) - \frac{4x \cos \phi}{1 + x^2} J_1^2(\xi_1) J_0(\xi_2) J_1(\xi_2) \\ &\approx \frac{1}{2} \left(\frac{x}{1 + 4x^2} - \frac{x}{1 + x^2} \right) \xi_1^2 \xi_2 \cos \phi = -\frac{3}{2} \frac{x^3}{4x^4 + 5x^2 + 1} \xi_1^2 \xi_2 \cos \phi, \end{aligned} \quad (\text{B9})$$

where we have used the expansion

$$J_n(\xi) \approx (\xi/2)^n (1/n!), \quad \xi \ll 1. \quad (\text{B10})$$

As follows from Eq. (B9), the absolute value of the DC as a function of $x \equiv \Omega\tau$ at fixed ξ_1 , ξ_2 and ϕ increases like x^3 for small x , then reaches maximum at $x^* \approx 1.3$ ($|j_{dc}(x = x^*)/j_0| \approx 0.16 \xi_1^2 \xi_2 \cos \phi$), and finally decreases as $3/(8x)$ for large x .

To check these calculations and understand what is the order in $\xi_{1,2}$ for the next terms in expression for DC, we perform the symbolic computations for Eq. (B1) with account of (B10) using MAPLE. As example, we present here the form of DC obtained by the summation in (B1) over intervals $-2 \leq (\mu_1, \mu_2, \nu) \leq 2$:

$$\begin{aligned}
& \left(\left(\frac{1}{294912} \frac{\cos(\phi) x}{1+4x^2} + \frac{1}{589824} \frac{\sin(\phi)}{1+4x^2} \right) \xi_2^5 \right. \\
& + \left(\frac{1}{4096} \frac{\cos(\phi) x}{1+36x^2} - \frac{1}{24576} \frac{\sin(\phi)}{1+36x^2} + \frac{1}{24576} \sin(\phi) \right) \xi_2^3 \\
& + \left(\frac{1}{768} \frac{\cos(\phi) x}{1+16x^2} - \frac{1}{3072} \frac{\sin(\phi)}{1+16x^2} - \frac{1}{1536} \frac{\cos(\phi) x}{1+4x^2} + \frac{1}{3072} \frac{\sin(\phi)}{1+4x^2} \right) \xi_2 \Big) \xi_1^6 + \left(\right. \\
& \left(\frac{1}{6144} \frac{\cos(\phi) x}{1+9x^2} + \frac{1}{18432} \frac{\sin(\phi)}{1+9x^2} \right) \xi_2^5 \\
& + \left(\frac{1}{768} \frac{\cos(\phi) x}{1+x^2} + \frac{1}{768} \frac{\sin(\phi)}{1+x^2} + \frac{5}{768} \frac{\cos(\phi) x}{1+25x^2} - \frac{1}{768} \frac{\sin(\phi)}{1+25x^2} \right) \xi_2^3 \\
& + \left(\frac{1}{32} \frac{\cos(\phi) x}{1+9x^2} - \frac{1}{96} \frac{\sin(\phi)}{1+9x^2} - \frac{1}{96} \frac{\cos(\phi) x}{1+x^2} + \frac{1}{96} \frac{\sin(\phi)}{1+x^2} \right) \xi_2 \Big) \xi_1^4 + \left(\left(\frac{1}{256} \frac{\cos(\phi) x}{1+36x^2} \right. \right. \\
& + \frac{1}{1536} \frac{\sin(\phi)}{1+36x^2} - \frac{5}{768} \frac{\cos(\phi) x}{1+25x^2} - \frac{1}{768} \frac{\sin(\phi)}{1+25x^2} + \frac{1}{384} \frac{\cos(\phi) x}{1+16x^2} + \frac{1}{1536} \frac{\sin(\phi)}{1+16x^2} \Big) \\
& \left. \xi_2^5 + \left(-\frac{3}{16} \frac{\cos(\phi) x}{1+9x^2} + \frac{1}{8} \frac{\cos(\phi) x}{1+16x^2} + \frac{1}{16} \frac{\cos(\phi) x}{1+4x^2} \right) \xi_2^3 + \left(\frac{1}{2} \frac{\cos(\phi) x}{1+4x^2} - \frac{1}{2} \frac{\cos(\phi) x}{1+x^2} \right) \xi_2 \right) \xi_1^2
\end{aligned}$$

As is evident from this result, next after (B9) contribution do DC is of order of ξ^5 (i.e., of order of $\xi_1^l \xi_2^k$ with $k+l=5$).

^a E-mail: kna@vist.krascience.rssi.ru, kna@iph.krasnoyarsk.su

^b E-mail: F.Kusmartsev@lboro.ac.uk

- [1] Esaki L. and Tsu R., *IBM J. Res. Dev.*, **14** (1970) 61.
- [2] Bouchard A. M. and Luban M., *Phys. Rev. B*, **47** (1993) 6815; Martini R. *et al.*, *Phys. Rev. B*, **54** (1996) R14325.
- [3] Bass F. G. and Tetervov A. P., *Phys. Rep.*, **140** (1986) 237.
- [4] Bass F. G. and Vatova L. B., *Phys. Rep.*, **241** (1994) 219.
- [5] Keay B. J. *et al.*, *Phys. Rev. Lett.*, **75** (1995) 4098; **75**, (1995) 4102; Unterrainer K. *et al.*, *Phys. Rev. Lett.*, **76** (1996) 2973; Schomburg E. *et al.*, *Appl. Phys. Lett.*, **68** (1996) 1096; Zeuner S. *et al.*, *Phys. Rev. B*, **53** (1996) R1717; Ignatov A. A. *et al.*, *Ann. der Physik (Leipzig)*, **3** (1994) 137; Winnerl S. *et al.*, *Phys. Rev. B*, **56** (1997) 10303.
- [6] Esaki L. and Tsu R., *Appl. Phys. Lett.*, **19** (1971) 246.
- [7] Romanov Yu. A., *Optika i Spekt.*, **33** (1972) 917 (*Sov. Phys. Optics and Spectroscopy*).
- [8] Orlov L. K. and Romanov Yu. A., *Fiz. Tverd. Tela* **19** (1977) 726 (*Sov Phys. Solid State*); Romanov Yu. A., Orlov L. K. and Bovin V. P., *Fiz. Tekhn. Polupr.*, **9**, (1978) 1665 (*Sov. Phys. Semicond.*).
- [9] Alekseev K. N. *et al.* *Phys. Rev. Lett.*, **80** (1998) 2669 [also cond-mat/9709026]
- [10] Alekseev K. N. *et al.*, *Phys. Rev. B*, **54** (1996) 10625 [also cond-mat/9604173]; *Physica D*, **113** (1998) 129.
- [11] Goychuk I. and Hänggi P., *Europhys. Lett.*, **43** (1998) 503.
- [12] Goychuk I., Grifoni M. and Hänggi P., *Phys. Rev. Lett.*, **81** (1998) 649.
- [13] Bir G.L. and Pikus G. E., *Symmetry and deformational effects in semiconductors*, (Nauka, Moscow) 1972.
- [14] Patel C. K. N., Slusher R. E. and Fleury P. A., *Phys. Rev. Lett.*, **17** (1966) 1011.
- [15] Wynne J. J., *Phys. Rev.*, **178** (1969) 1295.
- [16] Belyantsev A. M. *et al.*, *Zh. Eksp. Teor. Fiz.*, **61** (1971) 886 (*Sov. Phys. JETP*); Genkin V. N., Kozlov V. A. and Piskarev V. I., *Fiz. Tekh. Polupr.*, **8** (1974) 2013 (*Sov. Phys. Semicond.*).
- [17] Wolff P. A. and Pearson G. A., *Phys. Rev. Lett.*, **17** (1966) 1015.
- [18] Belyantsev A. M., Kozlov V. A. and Trifonov B. A., *phys. status sol. (b)*, **48** (1971) 581.
- [19] Kane E. O., *J. Phys. Chem. Solid.*, **1** (1957) 249.
- [20] Alekseev K. N. and Kusmartsev F. V., (unpublished).
- [21] Grahn H. T., von Klitzing K., Ploog K. and Döhler G. H., *Phys. Rev. B*, **43** (1991) 12094; Sibille A. *et al.*, *Superlatt. Microstr.*, **13** (1993) 247.
- [22] See also ref. [3], p. 310, Eq (8.15).

- [23] Breymayer H.-J., Risken H., Vollmer H. D. and Wonneberger W., *Appl. Phys. B*, **28** (1982) 335.
- [24] Belinicher V. I. and Sturman B. I., *Uspekhi. Fiz. Nauk*, **130** (1980) 415 (*Soviet Phys. Uspekhi*, **23** (1980) 199).

FIG. 1. The dependence of DC on amplitudes E_1 and E_2 [Eq. (5)]. The values E_1 and E_2 are expressed in units of $\hbar\omega/(ea)$ and $2\hbar\omega/(ea)$, respectively, and DC is normalized to j_0 . The product $\Omega\tau$ has the values: $x = 0.01$ (a), $x = 0.2$ (b), and $x = 1.0$ (c). Phase $\phi = 0$.