ELASTIC BODY SPLINE BASED IMAGE SEGMENTATION

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CONTRIBUTION: INTERACTIVE IMAGE SEGMENTATION

- Elastic body splines (EBS) belonging to the family of 3D splines and were introduced for non-rigid biomedical image registration [1].
- We propose a novel extension of using elastic body splines for learning driven figure-ground segmentation.
- Interactive segmentation with user defined seeds is formulated as learning an interpolating pixel classification function that is then used to assign labels for all unlabeled pixels in the image.
- In our work, we chose to model the supervised pixel classifier is the Gaussian elastic body spline (GEBS) which can use sparse scribbles from the user and has a closed form solution enabling a fast on-line implementation.
- The GEBS method for interactive foreground image labeling shows promise and outperforms a previous approach using the thin-plate spline model [2].

GAUSSIAN ELASTIC BODY SPLINES

Davis et al [3] used elastic body splines for registration of biomedical images. Given the displacement for landmark points $\vec{p}_i$, $i = 0, \dots, N$ between source and target images a transformation is determined that maps the corresponding landmarks from the source to the target image and interpolates the displacement field for all the remaining points in the transformed image. The transformation is given as,

$$\vec{d}(\vec{x}) = \mathbf{A}\vec{x} + \vec{b} + \sum_{i=0}^N \mathbf{G}(\vec{x} - \vec{p}_i) \vec{c}_i. \quad (1)$$

Here $\mathbf{G}$ is the spline function:

$$\mathbf{G}(\vec{x}) = \left[\left((4(1-\nu) - 1) \frac{\text{Erf}(\hat{r})}{r} - \sqrt{\frac{2}{\pi}} \sigma \frac{e^{-\hat{r}^2}}{r^2} + \sigma^2 \frac{\text{Erf}(\hat{r})}{r^3} \right) \mathbf{I} + \left(\frac{\text{Erf}(\hat{r})}{r^3} + 3\sqrt{\frac{2}{\pi}} \sigma \frac{e^{-\hat{r}^2}}{r^4} - 3\sigma^2 \frac{\text{Erf}(\hat{r})}{r^5} \right) \vec{x} \vec{x}^T \right]. \quad (2)$$

Here $\text{Erf}(\cdot)$ is the standard error function, $\hat{r} = |\vec{x}|/\sqrt{2}\sigma$, $\mathbf{I}$ is the identity matrix, $\vec{x} \vec{x}^T$ is an outer product, $\vec{d}(\vec{x})$ is the displacement vector for the landmark points, $\vec{p}_i$, $\mathbf{A}$ and $\vec{b}$ are the coefficients for the affine transformation and $\vec{c}_i$ the set of coefficients for the non-linear elastic terms.

For an interactive image segmentation task we learn the interpolating pixel classification function $\vec{d}(\vec{x})$ in (1). From the user supplied seed points we set

- $\vec{d}(\vec{x}) = [+1 \ +1 \ +1]^T$ for foreground pixels
- $\vec{d}(\vec{x}) = [-1 \ -1 \ -1]^T$ for background pixels
- $\vec{x}$ is a feature vector comprised of spatial location and grayscale value of the pixel

Coefficients $\mathbf{A}$, $\vec{b}$, and $\vec{c}_i$ are computed using (1) from the seed points. Let $\mathbf{A}\vec{x} + \vec{b}$, be the affine part of the GEBS interpolating function,

$$\mathbf{A} = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3]. \quad (3)$$

Let $\mathbf{W}$ be the vector of all the GEBS coefficients given as,

$$\mathbf{W} = [\mathbf{C}_F \ \mathbf{C}_B \ \vec{a}_1^T \ \vec{a}_2^T \ \vec{a}_3^T \ \vec{b}^T]^T \quad (4)$$

where $\mathbf{C}_F$ and $\mathbf{C}_B$ are the elastic coefficients corresponding to, f , foreground seed pixels and, b , background seed pixels,

$$\mathbf{C}_F = [\vec{c}_1^T \ \dots \ \vec{c}_f^T], \quad \mathbf{C}_B = [\vec{c}_1^T \ \dots \ \vec{c}_b^T].$$

The Gaussian Elastic Body Spline (GEBS) mapping determined by the weights $\mathbf{W}$ that we are solving for is given by the relationship,

$$\mathbf{Y} = \mathbf{L} \mathbf{W}$$

where $\mathbf{Y}$ is the set of displacement vectors for the user defined seeds.

INTERACTIVE IMAGE SEGMENTATION

The GEBS coefficients are estimated by solving the matrix equation,

$$\mathbf{W} = \mathbf{L}^{-1} \mathbf{Y} \quad (5)$$

where,

$$\mathbf{L} = \begin{bmatrix} \mathbf{K} & \mathbf{P} \\ \mathbf{P}^T & \mathbf{O} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{G}_{FF} & \mathbf{G}_{FB} \\ \mathbf{G}_{BF} & \mathbf{G}_{BB} \end{bmatrix} \quad (6)$$

$$\mathbf{G}_{FF}(\vec{r}) = \begin{bmatrix} \mathbf{G}_{11}(\vec{r}_{11}) & \dots & \mathbf{G}_{1f}(\vec{r}_{1f}) \\ \vdots & & \vdots \\ \mathbf{G}_{f1}(\vec{r}_{11}) & \dots & \mathbf{G}_{ff}(\vec{r}_{ff}) \end{bmatrix} \quad (7)$$

with $\vec{r}_{ij} = \vec{x}_i - \vec{x}_j$ and $\vec{x}_i$ is the feature vector for the i^{th} foreground seed point. Hence, $\mathbf{G}_{FF}$ is the matrix of GEBS (2) functions defined only over foreground pixels. $\mathbf{G}_{FB}$, $\mathbf{G}_{BB}$ and $\mathbf{G}_{BF}$ are similarly defined,

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_F & \mathbf{I}_F \\ \mathbf{P}_B & \mathbf{I}_B \end{bmatrix}, \quad (8)$$

where the identity matrices are given by,

$$\mathbf{I}_F = [\mathbf{I}_1 \ \dots \ \mathbf{I}_f], \quad \mathbf{I}_B = [\mathbf{I}_1 \ \dots \ \mathbf{I}_b], \quad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (9)$$

for f foreground seed points, b background seed points and,

$$\mathbf{P}_F = \begin{bmatrix} x_{11}\mathbf{I} & x_{12}\mathbf{I} & x_{13}\mathbf{I} \\ \vdots & \vdots & \vdots \\ x_{f1}\mathbf{I} & x_{f2}\mathbf{I} & x_{f3}\mathbf{I} \end{bmatrix} \quad (10)$$

where x_{ij} is the j^{th} feature for i^{th} pixel. $\mathbf{P}_B$ is similarly defined. After solving (5) the GEBS classifier can be estimated and used to interpolate the labels of unlabeled pixels in the image:

$$\ell(\vec{x}) = \begin{cases} \text{foreground, if majority } \vec{d}(\vec{x}) \text{ elements } \geq 0 \\ \text{background, if majority } \vec{d}(\vec{x}) \text{ elements } < 0. \end{cases} \quad (11)$$

EXPERIMENTAL RESULTS: SEGMENTATION

(a) Input with seeds (b) GT (c) Our GEBS result (d) TPS [2] result

Comparison of segmentation results on three BSDS 500 images along with one manual ground-truth. (a) Input image with user provided foreground (green) and background (red) seed points. (b) Sample human annotated ground truth (GT) from BSDS500. (c) Our GEBS segmentation results. (d) TPS segmentation results [2].

CLUSTERS SEPARATION

(a) Input with seeds (b) Our GEBS results

Semi-supervised two-class clustering using GEBS.

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- Lab: <http://cell.missouri.edu>