A single Euler Solution Per Anomaly

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1) Summary

We present a new method for selecting a single Euler solution per magnetic anomaly based on the identification of plateaus on the maps of the horizontal coordinate estimates.

2) Introduction

Historically, Thompson (1982) proposed an Euler-based profile depth estimation method called “EULDPH” and Reid et al. (1990) implemented the grid extension suggested by Thompson and coined the term “**Euler deconvolution**.”

Euler deconvolution has gained enormous popularity as a semiautomated method to estimate the 3D position of magnetic sources. By assuming the geometry of the source, Euler deconvolution **estimates the horizontal and vertical source positions** using a **moving-data window scheme**. This moving-data window scheme leads to a large number of Euler solutions (Barbosa and Silva, 2011).

The undesirable broad spray of Euler solutions

To overcome this difficulty, some methods have been developed to select the best Euler solutions and to reduce the number of accepted Euler solutions. These methods were proposed by:

- Thompson (1982)
- Fairhead et al. (1994)
- Mikhailov et al. (2003)
- Li (2003)
- Ugalde and Morris (2010)

We present a new method for selecting the best Euler solutions by using the theoretical basis proposed by Silva and Barbosa (2003). We select a single Euler solution per magnetic anomaly based on the identification of plateaus on the maps of the horizontal coordinate estimates.

3) Methodology

Let's assume a magnetic point (or line) source at the coordinates (x_o, y_o, z_o) referred to a right-hand Cartesian coordinate system with the axis pointing downward. The observed total-field anomaly $h \equiv h(x,y,z)$ at the coordinates (x, y, z) produced by this simple magnetic source obeys Euler's 3D equation (Reid et al., 1990). By considering a discrete set of observations of total-field anomaly, the classical formulation of Euler deconvolution can be written as a linear system of equations given by

$$x_o \frac{\partial h_i}{\partial x} + y_o \frac{\partial h_i}{\partial y} + z_o \frac{\partial h_i}{\partial z} + \eta b = x_i \frac{\partial h_i}{\partial x} + y_i \frac{\partial h_i}{\partial y} + z_i \frac{\partial h_i}{\partial z} + \eta h_i, \quad i = 1, \dots, N,$$

where $h_i \equiv h(x_i, y_i, z_i)$ is the i th observation of the total-field anomaly at the coordinates (x_i, y_i, z_i) , b is a base level (i.e., a constant background value) and η is the structural index related to the geometry of the source. For a given structural index, the 3D Euler deconvolution consists in solving the system of equations (1), in the least-squares sense, for the unknown parameters x_o, y_o, z_o , and b .

In a **moving-data window scheme**, the Euler deconvolution obtains the estimates $\hat{x}_o, \hat{y}_o, \hat{z}_o$ and $\hat{b}$ by using the set of observations inside a data window.

Our method uses the bias and symmetry properties of the **analytical estimators of the horizontal and vertical source positions** (x_o, y_o, z_o) in Euler deconvolution as a function of the x - and y -coordinates of the observations (Silva and Barbosa, 2003).

Our method takes advantage of the **contrasting behavior of the Euler solutions** at the borders and at the neighborhood of the highest absolute values of the anomaly. At the borders of the anomaly, the estimates of the horizontal and vertical coordinates are biased toward the respective horizontal and vertical coordinates of the data window center. Hence, the estimates of the horizontal coordinates form **inclined planes at the anomaly borders**. Conversely, in the neighborhood of the highest absolute values of the anomaly, the estimates of the horizontal coordinates are good and consistent estimates of the correct source coordinates, defining, in this way, **plateau areas**.

Let's exemplify this behavior and our method by a synthetic test.

3.1) Illustrating the steps of our methodology

Here, we illustrate the steps of our methodology. Figure 1 shows the noise-corrupted total-field anomaly produced by four spherical sources emplaced in a nonmagnetic medium. The radii of all sources are 1 km and they are magnetized uniformly by induction only, having a magnetization with intensity of 4 A/m, inclination of 60° and declination of 20°. These sources are at the same depth (2 km). Table 1 shows the horizontal and vertical positions of the four spheres.

Figure 1: Total-field anomaly contaminated with noise with produced by four spherical sources (1-4).

Table 1 – The true horizontal and vertical coordinates (x_o, y_o, z_o) of the centers of the four spheres (1-4).

# Spheres	x_o (km)	y_o (km)	z_o (km)
1	10	20	2
2	18	12	2
3	30	25	2
4	35	15	2

3.3) How to identify the plateaus

To identify the plateaus in the plots of $\hat{x}_o$ and $\hat{y}_o$ we need to differentiate the plateaus from inclined planes on the estimates. To do this, we fit a first-degree polynomial to $\hat{x}_o$ in a moving-window scheme and plot the estimated angular coefficients, $\hat{C}_x$, against the x - and y -coordinates of the moving-window center (Figure 3a). Analogously, this procedure is used to identify the plateaus in the map of estimates $\hat{y}_o$. The places where the estimated angular coefficients $\hat{C}_x$ and $\hat{C}_y$ are closest to zero (red areas in Figures 3a and 3b) identify the plateaus of $\hat{x}_o$ and $\hat{y}_o$.

To define automatically the positions of the four plateaus in the maps of $\hat{x}_o$ and $\hat{y}_o$ we applied a cluster analysis which identifies those x - and y -coordinates where the estimated angular coefficients are closest to zero. If the sources are horizontally well separated, then the plateaus are also horizontally well separated from each other and thus their horizontal positions can be easily recognized through a cluster analysis. After discriminating each plateau, we compute the means ($\tilde{x}_o$ and $\tilde{y}_o$) of the estimates of the horizontal source positions ($\hat{x}_o$ and $\hat{y}_o$) over each plateau

3.2) First step: Selecting the best horizontal coordinate estimates in Euler deconvolution

First, we apply the 3D **Euler deconvolution with any structural index**.

For each position of the moving-data window, we produce the plots of $\hat{x}_o$ and $\hat{y}_o$ against the x - and y -coordinates of the center of the moving-data window. Figures 2a and 2b show the estimates $\hat{x}_o$ and $\hat{y}_o$ against the x - and y -coordinates of the center of the moving-data window.

The most striking features in these maps are the overall trend of an inclined plane and the presence of four local plateaus.

Note that the estimates $\hat{x}_o$ (Figure 2a) and $\hat{y}_o$ (Figure 2b) over these plateaus are very close to the respective true source positions x_o and y_o presented in Table 1. Here, we need to identify these plateaus.

Figure 2: Estimates (a) $\hat{x}_o$ and (b) $\hat{y}_o$ against the x - and y -coordinates of the center of the moving-data window in 3D Euler deconvolution assuming any structural index.

Figure 3: Estimates of angular coefficients (a) $\hat{C}_x$ and (b) $\hat{C}_y$ against the x - and y -coordinates of the center of the moving window. The red areas in (a) and (b) highlight the places where these estimates are closest to zero. These areas identify the plateaus in the plots of $\hat{x}_o$ and $\hat{y}_o$ (Figure 2)

3.4) Second step: Selecting the structural index η

To determine the correct η we first obtain estimates $\hat{b}$ by using the classical Euler deconvolution for a number of tentative structural indices. Next, we select the subsets of the total-field anomaly and estimates $\hat{b}$ that approximately fall within the plateaus identified in the plots of $\hat{x}_o$ and $\hat{y}_o$. Finally, for each tentative η , we compute the **correlation coefficient between these subsets of the total-field anomaly and of the estimates $\hat{b}$** . The tentative η that produces the **minimum correlation coefficient** (in modulus) is the **best estimate of the structural index** (Barbosa et al., 1999). For all sources, the best estimate of the structural index is 3 indicating that the sources are dipoles (spheres).

3.6) The single Euler solution per magnetic anomaly

Table 2 summarizes the best estimates ($\tilde{x}_o, \tilde{y}_o, \tilde{z}_o$) of the horizontal and vertical coordinates of the centers of the four spheres.

# Spheres	x_o (km)	y_o (km)	z_o (km)	$\tilde{x}_o$ (km)	$\tilde{y}_o$ (km)	$\tilde{z}_o$ (km)
1	10	20	2	9.95	20.00	2.07
2	18	12	2	17.94	11.99	2.08
3	30	25	2	29.99	25.04	2.10
4	35	15	2	34.99	14.97	2.08

3.5) Third step: Selecting the best depth estimates in Euler deconvolution

To determine the best depth estimates, we use the best estimate of the structural index (η) to produce a plot of all estimates $\hat{z}_o$ against the x- and y-coordinates of the center of the moving-data window (Figure 4). Next, we extract the subset of the estimates $\hat{z}_o$ that fall within the mapped plateaus. Finally, we compute the sample mean $\tilde{z}_o$ of the extracted subset of the estimates $\hat{z}_o$. This sample mean $\tilde{z}_o$ is taken as the best depth estimate of the source.

Figure 4: Estimates $\hat{z}_o$ against the x- and y-coordinates of the center of the moving-data window in 3D Euler deconvolution by using the $\eta = 3$.

4) Application to Real Data

Figure 5 shows the aeromagnetic total-field anomaly over mafic–ultramafic alkaline bodies, in central Brazil (Dutra and Marangoni, 2009). The strong magnetic anomalies have been named Morro do Engenho (ME) and A2 anomalies. The ME anomaly is produced by an alkaline outcropping body intruded in sediments. The A2 anomaly is a possible buried alkaline body.

Figure 5: Alkaline bodies, Brazil: (a) Total-field anomaly. Morro do Engenho (ME) and A2 anomalies.

By assuming any structural index in Euler deconvolution, we estimate $\hat{x}_o$ (Figure 6a) and $\hat{y}_o$ (Figure 6b) against the x- and y-coordinates of the center of the moving-data window.

Figure 6: Alkaline bodies, Brazil: Euler horizontal estimates $\hat{x}_o$ (a) and $\hat{y}_o$ (b).

Two plateaus are easily detected by visual inspection of the maps of the horizontal coordinate estimates (Figure 6). We fit first-degree polynomials to the estimates $\hat{x}_o$ and $\hat{y}_o$. Figures 7a and 7b show the estimated angular coefficients $\hat{c}_x$ and $\hat{c}_y$ where we can easily identify areas (in red) closest to zero. These red areas define the two subsets of the x- and y-coordinates which will be used to define the positions of the two plateaus on the maps of the horizontal coordinate estimates (Figure 6).

Figure 7: Alkaline bodies, Brazil: Angular coefficient estimates $\hat{c}_x$ (a) and $\hat{c}_y$ (b).

After locating these plateaus, we computed the two sample means $\tilde{x}_o$ and $\tilde{y}_o$ (Table 3) of the two subsets of the estimates $\hat{x}_o$ and $\hat{y}_o$ lying over each plateau (Figure 6). We estimated the structural index of each body (Barbosa et al., 1999). Both bodies are interpreted as dipoles (spheres). Figure 8 shows the depth estimates. We computed the sample means $\tilde{z}_o$ of the corresponding depth estimates $\hat{z}_o$ located over the intersections of the pairs of plateaus of $\hat{x}_o$ and of $\hat{y}_o$. Hence, we found that ME and A2 bodies are two sphere-like sources whose centers are located at depths of 3 and 3.2 km, respectively.

Figure 8: Alkaline bodies, Brazil: Euler depth estimates $\hat{z}_o$ by using the structural index 3.

Table 3 – The best estimates of the horizontal and vertical coordinates ($\tilde{x}_o, \tilde{y}_o, \tilde{z}_o$) of two alkaline bodies, Brazil.

Bodies	$\tilde{x}_o$ (km)	$\tilde{y}_o$ (km)	$\tilde{z}_o$ (km)
ME	11.50	15.40	3.20
A2	23.83	36.35	2.97

5) Conclusions

We have presented a new method for selecting the best source location estimates in 3D Euler deconvolution. Our approach has drastically reduced the number of the selected 3D Euler solutions to only one per anomaly. This is possible because our method does not select Euler solutions based on their statistical consistency after a cluster analysis. Rather, in our method this selection is grounded on the theoretical analysis of the estimators for the horizontal and vertical source positions in 3D Euler deconvolution as a function of the x- and y-coordinates of the observations. Our approach consists in detecting automatically the regions of the anomaly producing consistent estimates of the source horizontal coordinates.

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