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“I’m still making dots for them”: Mathematics lecturers’ views on their mathematical modelling practices

Paul Hernandez-Martinez^{a*}, Stephanie Thomas^b, Olov Viirman^{c,d} and Yuriy Rogovchenko^d

^aDepartment of Mathematics, Swinburne University of Technology, Melbourne, Australia;

^bMathematics Education Centre, Loughborough University, Loughborough, UK, ^cDepartment of Electrical Engineering, Mathematics and Science, University of Gävle, Gävle, Sweden,

^dDepartment of Mathematical Sciences, University of Agder, Kristiansand, Norway

* phernandezmartinez@swin.edu.au Department of Mathematics, Swinburne University of Technology, PO Box 218 (Internal post H38), Hawthorn, VIC 3122, Australia.

“I’m still making dots for them”: Mathematics lecturers’ views on their mathematical modelling practices

Using Cultural-Historical Activity Theory, we analyse lecturers’ views on the aims and teaching practices of mathematical modelling (MM) education in Norway and England. We aim to expose the tensions that exist within the activity of teaching MM at university, such as those that exist between multiple, sometimes competing, aims for teaching MM, or between the lecturers’ professional identities and the structure of university degrees. Our conceptualisation of these tensions might help lecturers consider how to overcome obstacles in their own contexts.

Keywords: mathematical modelling; undergraduate teaching; Cultural-Historical Activity Theory; tensions and contradictions

Background

Mathematical modelling (MM) is an important topic in mathematics education. Blum and Niss [1] present several reasons why MM should be part of mathematics instruction. They point out, for example, that mathematics is now taught to an increasing proportion of the population who will have a need to apply mathematical concepts and modelling processes to problems that arise in home or work situations and that lie outside the realm of mathematics itself. They argue that learning MM takes practice and "requires a degree of preparation and training" (ibid, p.43) and hence should be included in any school and higher education curricula. Despite its importance, MM teaching practice at all levels of education is far less prominent than is desirable perhaps because, as it is reported, modelling is difficult for students [2, 3]. In relation to this, Vorhölter et al. [4] point out that the role of the teacher within MM has not been sufficiently researched, and there is a lack of understanding of how teachers can develop practices that help foster their students’ MM skills. Furthermore, Schukajlow et al. [5] found that most of the research on MM has been from a cognitive viewpoint, without considering other important factors such as emotions or beliefs, or wider

socio-cultural issues that are inherent to all modelling activity [6,7].

In this paper, we ground our research on a socio-cultural theoretical framework with the aim of better understanding the MM teaching practices of mathematics university lecturers from their own perspective. Our research questions (RQs) are:

RQ1 What are lecturers' views on the aims of MM education?

RQ2 How do lecturers describe their use of MM in their teaching practices?

RQ3 How do lecturers describe their ideal use of MM in teaching, and what are their perceived obstacles in implementing these ideal practices?

Theoretical framework

We employ Cultural-Historical Activity Theory (CHAT) to conceptualise the views of mathematics lecturers in relation to their MM teaching practices. CHAT has been used in post-secondary mathematics education research to study, for example, the contradictions that emerge in a university mathematics flipped classroom in Norway [8], teachers' conceptions and constraints for changing their higher education teaching practices in Brazil [9], or to conceptualise technology mediated post-compulsory instruction in Australia [10].

CHAT provides a framework for analysing a social activity such as teaching MM. According to this theory, an activity consists of a person (subject), an aim for the subject to engage (object) and the tools used to achieve the object of the activity. The activity happens within a community, which has certain rules and division of labour [11]. For example, a lecturer (subject) engages in the activity of teaching with the aim of developing students' skills (object) and uses tools such as a whiteboard or a project task to achieve that aim. The university community has certain rules, such as students attending lectures and working in groups, and there is a division of labour between the lecturer who instructs students what to

do and the students who follow what their lecturer asks of them. This activity system forms a non-divisible entity that is the unit of analysis for research purposes.

In this paper, we are interested in analysing the contradictions that arise within the activity of university MM teaching from the perspective of the lecturers. Contradictions are inherent in all activity systems and explain “historically accumulating structural tensions within and between activity systems” [12, p. 137]. Contradictions in CHAT are not mere conflicting circumstances but situations deeply rooted in the sociocultural context. For example, Jaworski et al. [13] explained how contradictions between the activity systems of lecturers and students emerge when the objects of activity differ: the aim of the lecturers is for their students to have conceptual understanding while the aim of the students is to memorise knowledge to pass the exam. This tension did not occur overnight; it is deeply rooted in how educational systems in capitalist societies have developed historically and more explicitly as a product of a system of accountability, audit and performativity [14,15,16].

Engeström [11] describes four levels of contradictions: primary, secondary, tertiary and quaternary. In this paper, we are interested in primary and secondary contradictions because these occur within the activity system. We do not consider multiple activity systems in our data. Primary contradictions in all activities centre on the exchange value and use value. We refer to a use value when the product of activity is of use to the person that produces it (e.g. learning for the sake of it), and an exchange value when such product is of value to someone else, usually someone who is willing to pay for the product (learning in order to obtain a university degree or a job). Secondary contradictions manifest themselves in particular tensions between the constituents of the central activity system, for example, between the tools and the division of labour.

Contradictions can serve as tools for development when the community finds new ways of organising the activity in order to solve the tensions experienced. By collectively reconfiguring their activity system, subjects can innovate. In this paper, we analyse the contradictions that exist within the activity system of university MM teaching from the perspective of the subjects of the activity.

Methodology

The data for this paper were collected through an online questionnaire and interviews with mathematics lecturers in England and Norway, the countries in which the authors resided at the time of data collection.

Data from the online questionnaire came from 221 university lecturers that responded to questions about their aims and practices of MM teaching. Data from interviews came from four English (Charles¹, Diana, Ernest and Fred) and five Norwegian (Gunnar, Henrik, Lars, Linnea and Odin) university mathematics lecturers that had completed the questionnaire and volunteered to be interviewed.

Online questionnaire results from the Norwegian case have been published in Treffert-Thomas et al. [17]. In this paper, we add analyses of the English online questionnaire data and the interview data for both countries. Our aim in this paper is to aggregate results rather than to make a comparative analysis with the data from the two countries. However, we did look out for noticeable differences that might be due to cultural or other factors, and therefore we present some of the English and Norwegian data separately. As it turned out, there was not a great diversity in participants' responses based on their country of residence.

The online questionnaire consisted of questions and statements that probed lecturers' views on the aims of teaching MM and on their MM teaching practices, and were based on

1. All names are pseudonyms

the perspectives developed in Kaiser and Sriraman [18] who identified five aims in connection with MM education: realistic or applied, contextual, educational, socio-critical and epistemological. Within a realistic (or applied) perspective, the aims of MM are utilitarian and based on solving authentic problems, that is, problems perceived to lie outside mathematics. The aims of MM in the contextual perspective relate to psychological development, that is, MM should elicit invention and refinement of mathematical constructs. In the educational perspective, all MM activity is pedagogical and aimed at fostering understanding of mathematics; here the modelling cycle described by e.g. Blum and Leiß [19] is considered a tool for the learning of mathematics. The aims of MM in the socio-critical perspective are to develop a person's critical understandings of the world and the role mathematics and MM play in societal decisions. In the epistemological perspective, the aims of MM are to develop (mathematical) theory; thus, all mathematical activity - including problems that lie entirely within pure mathematics - is considered MM.

To this characterisation, we added an enjoyment perspective [20], a perspective that had so far not been explored in the MM literature per se. In this perspective, the aims of MM relate to the intrinsic satisfaction gained from engaging in mathematical activity - often reported by mathematicians in self-reflections - without any reference to utilitarian aims.

The online questionnaire was distributed by email to university mathematics departments in Norway and England. 119 questionnaires were returned in Norway and 102 in England, a response rate of 24% and 8%, respectively, although a small number of participants did not respond to all questions.

In the online questionnaire, participants were invited to choose three statements that best described their view on teaching MM and to rank these in order of importance. There were two statements pertaining to each perspective of Kaiser and Sriraman [18] plus two

statements for the enjoyment perspective (see figure 1). Analyses were both qualitative and quantitative.

<i>The aim of teaching modelling is for students</i>	
<i>to solve real-life problems.</i>	
<i>to increase their understanding of the real world.</i>	
<i>to gain mathematical insights.</i>	
<i>to advance their understanding of mathematical theory.</i>	
<i>to become more critically aware as citizens.</i>	
<i>to gain conceptual understandings of mathematics.</i>	
<i>to have/add structure in their learning of mathematics.</i>	
<i>to encourage critical reflection and discussion of the standing of mathematics in the world.</i>	
<i>To elicit models from solving word problems.</i>	
<i>to learn to apply models and modelling to different situations.</i>	
<i>to enjoy learning mathematics.</i>	
<i>to become interested in mathematics.</i>	

Figure 1. Online questionnaire statements about the aim of teaching MM

All participants who had responded to the online questionnaire and indicated their willingness to be interviewed (by ticking a box) were contacted by email. Those who replied were interviewed. The interviews consisted of questions about the interviewees' aims for teaching MM, about their MM teaching practices, both actual and ideal, and whether there were obstacles in implementing any desired changes to these practices. We shared the questions with the lecturers beforehand and gave them the opportunity to add anything they felt was important in relation to the questions posed during the interview. All interviews were conducted in English, mostly via Skype, and lasted approximately 30 minutes. They were transcribed and analysed qualitatively through a thematic analysis [21]. Three interviews out of nine were analysed independently by all four researchers and these analyses were subsequently shared and discussed with the aim of agreeing on a set of coding categories. The rest of the interviews were coded using these categories, which comprised: (i) aims of MM in teaching including 'definitions' of MM that lecturers provided during the interviews, (ii) lecturers' descriptions of their actual practices, including reasons why these practices took the form they did, and (iii) lecturers' descriptions of their ideal practices, including perceived

obstacles to their implementation. To answer the research questions in this paper, we draw on the responses from both the questionnaire and interviews.

Results

Lecturers' views on the aims of mathematical modelling in teaching

Data from the online questionnaire showed a similar pattern of responses in both countries (table 1). Slightly less than half of respondents in each country selected two statements from the same category, which we interpreted as an indication of commitment to a particular view on the aims of MM in teaching. In both countries, the realistic perspective was represented most often, with one third of participants selecting two realistic statements. In Norway, all perspectives were represented by at least one respondent whereas in England the socio-critical and educational perspectives were not represented. However, more than half of respondents in both countries chose all three statements from different categories, showing that many lecturers hold a wider variation in their views concerning the aims of MM in teaching.

Perspective statements	Norway	England
2 realistic	28 (35%)	22 (30%)
2 epistemological	3 (4%)	4 (5%)
2 socio-critical	2 (2%)	0 (0%)
2 educational	1 (1%)	0 (0%)
2 contextual	1 (1%)	2 (3%)
2 enjoyment	3 (4%)	2 (3%)
No two from same perspective	43 (53%)	43 (59%)
Total responses (percentage)	81 (100%)	73 (100%)

Table 1. Participants' choice of perspective

Data from the interviews provided rich descriptions of lecturers' views on the aims of MM in teaching. Six of the interviewees expressed realistic views and mentioned that one of the aims of MM in teaching should be to "bridge the gap between knowing the maths and applying it to problems" (Charles). This utilitarian view was nearly always accompanied by

other aims such as (1) the need to motivate or engage students in the study of mathematics by making it relevant, (2) the need to prepare students for employment by showing them where and how mathematics can be applied, (3) the need to develop students' problem-solving skills for future employment, and (4) (ideally) a way to develop creativity for and "pure enjoyment" (Gunnar) of the subject.

Lars, for example, argued that students "find it more interesting when they see directly some practical applications or at least pseudo-applications", and Odin expressed the view that students "[first] should know how to model and second, because it's an excellent way to introduce theory". Fred, who teaches first year students, stated that they (the department) "want [the students] engaged, we want them happy".

Most lecturers expressed aims in using MM in their teaching directed towards mathematics itself and presented MM as a way to solve problems much as applied mathematicians would do in their professional practice. However, there were other aims that mediated this realistic view related to the more general educational system (prepare students for work, engagement/retention/attainment) or the institution (departmental needs for student satisfaction or the number of students that complete their university studies).

Personal views that mediate lecturers' aims in using MM in their teaching were also expressed. Six of the interviewees mentioned their research or previous educational experiences as a reason for using MM in their teaching. Fred, for example, said that "it is enjoyable teaching something that you yourself find interesting and exciting, and this then shows". Diana stated that "everything I do from a research point of view is mathematical modelling... it is my research area. So, of course, I use modelling for my teaching". These lecturers seemed to imply that they use MM in their teaching because they enjoy doing MM or using models in their own research, or because they realised at some point in their education that MM was important, and they enjoy teaching it. These views are interrelated to

other aims, such as motivating students' learning, showing the relevance of mathematics, or developing employability skills ("I've been educated to think of mathematics as a problem-solving tool" – Ernest).

How do these aims translate into teaching practices from the perspective of the lecturers themselves?

Lecturers' descriptions of their mathematical modelling teaching practices

All English and nearly all Norwegian questionnaire respondents had current or past experience (often 16 years or more) of teaching at tertiary level. Most Norwegian and English respondents taught undergraduates (87% and 95%, respectively), and many also taught master level (72% and 82%, respectively) and PhD students (52% and 75%, respectively).

In the questionnaire, participants were asked to select the statement that best represented how they use(d) models or MM in their teaching. They could select one of four categories provided, if applicable, or to add their own description. 74% (61 out of 82) of Norwegian respondents and 86% (71 out of 83) of English respondents indicated that they used models or modelling in their teaching. Categories and participants' choices are summarised in table 2.

	Norway	England
To illustrate theory	25 (41%)	18 (25%)
To motivate new theory	13 (21%)	14 (20%)
As a separate course	13 (21%)	20 (28%)
Small pure units, then show applications	7 (11%)	3 (4%)
Own description	2 (3%)	9 (13%)
None of the above	1 (2%)	7 (10%)
Total	61 (100%)	71 (100%)

Table 2. How MM is used in teaching

Eleven participants gave their own descriptions, using phrases such as "as part of a course", "to show uses or usefulness", "when mathematically appropriate" and "for motivation and as a precursor for conceptual understanding".

The data from the interviews helped to complete the picture. We found that the interviewees' practices were strongly shaped by the educational level at which they taught MM. There were pronounced differences between lecturers who described their teaching of first year undergraduate students and those who taught second or third year undergraduate, master or PhD students.

From lecturers' descriptions, it seems that first year undergraduate MM teaching takes the form of examples-driven practices, where the lecturers present a model as an example of a mathematical concept (illustrating theory) or develop models themselves in order to introduce a new mathematical concept (motivating new theory). For example, describing his teaching of engineers, Lars noted that "it's most[ly] [about] established models or models that I invent to suit the thing I'm going to teach". Odin described his teaching of Calculus and Linear Algebra as "trying to do it as inductive as I can somehow, and I hope that they get an idea what modelling is after having seen some examples. But that's difficult because the examples that I introduce are usually rather well digested".

Fred used MM as a way to introduce new theory; in the interview he described how he derived the wave equation:

It takes almost two weeks to do but there is sort of a highlight in the end which is surprising. So, I start with trying to model the vibrations on an elastic string. [...] I bring in a rubber band; I stretch it out, I pull it like a guitar string and then it vibrates [...] I basically say, okay, we want to write down the equation that governs the vibrations of the elastic string. And I derive it from scratch.

On the other hand, lecturers described teaching MM using more open-ended problems, usually to students after their first year. For example, Charles described his Industrial Mathematical Modelling master's course (an elective for fourth year mathematics undergraduates) as a project-based course entirely assessed by coursework, where he gives students problems saying "go away and come back each week and we'll talk through it (...) I

think it's much closer to the feeling of PhD supervision". Ernest recounted how in a second-year course, designed as an alternative to work experience, students were given "an open-ended problem... [that is] sufficiently messy enough. You can't just dig out an algorithm and solve it". It is important to note that the number of students in advanced courses was relatively small compared to first year courses. In the case of Ernest's department, for example, there were at most 40 first-year undergraduates each year, a cohort that by the fourth year decreased to at most 15 students. Charles usually only has between 9 and 12 students in his cohort.

Diana was the only interviewee to describe a different structure. Her department teaches a large first-year MM problem-solving course which Diana categorised as "I pretty much just throw them in the deep end and then say, 'We'll go around and give [them] lots of help'". This first year cohort consists of around 200 students and a group of handpicked teaching assistants that each support 4-5 groups of students as they attempt to solve the problems given to them. According to Diana, some first year students love the module while others hate it. However, Diana's department is able to sustain this course because "at least being a first-year module, this doesn't count to [students'] overall degree class".²

Diana also described her MM course for third year Biology students, which is a more "structured", more "traditional" course. Although the content is modelling-focused, the teaching is geared towards particular models useful in Biology. She explained that "I want [the students], by the end of that hour, [to] have come up with the system of equations that I want them to have come up with, so that they can then do the analysis of that comfortably". When asked about the rationale for her teaching strategy, Diana explained:

² In most UK universities, first year marks do not count towards the average mark that defines the final classification of an undergraduate degree (e.g. First Class, Upper Second Class (2:1), etc.)

“There is a strong contingent of us who think [...] [that] most of our students aren’t going to go on and become research pure mathematicians. What they’re going to go to do, if they’re going to go out into industry, they’re [...] going to have to do teamwork and problem-solving and presenting things. And these are the skills that we need them to know, to come out of their degree with”.

In short, apart from Diana, the other interviewees’ MM teaching practices mostly followed an examples-driven pattern for first year undergraduate students and a project-driven pattern for more advanced undergraduate and postgraduate students.

One further issue we sought to explore was the role of enjoyment in MM teaching [19]. Seven lecturers mentioned enjoyment in their interviews. Charles, Diana and Odin talked about “having fun” in the context of their own learning/teaching experience, and as a way to influence their students’ interest in mathematics. For example, Charles recounted his experience of doing a master’s course where “I worked so hard and given similar kind of lots of projects. I liked the kind of working together on things. [...] What I want to give to the students is an idea that maths is fun”.

Ernest and Fred both related enjoyment to the usefulness of mathematics. For instance, Ernest talked about his former work with “very tough” teenagers where “if we didn’t do anything related to the real world, if we didn’t try to make it fun, it [...] would have been carnage”.

Gunnar also emphasised fun in relation to real world applications of mathematics, which he distinguished from the “pure enjoyment” resulting from students becoming able to see mathematics beyond its applications and as a form of expression, a form of language:

If you’re able to let them play around with the tasks, that actually lifts their level of awareness about mathematics as a form of expression and a form of a language [...] that can be used not only as a procedural way, but also in a more modelling way, and then [...] you relate [MM] to something concrete and something in real life. But if you’re [...] able to let them work with

mathematics as a pure enjoyment thing of math and pure mathematics as well, then you have really accomplished something.

Finally, Linnea spoke about enjoyment as being able to understand the ideas rather than focus only on the links between mathematics and reality:

I like to be honest and say, 'Now, we are playing. Now, try to imagine that we can do this'.

And we all know that this is not true, [...] it could not describe the reality in detail. But still, we can understand the basic ideas. We can see the relations. [...] I think if the students have fun and they learn a lot, [then] I think it's not so important to all the time remind them of how serious this should be.

From actual practices, we now turn to what lecturers said about any changes that they would like to make, or have made, to their teaching and any obstacles that they have found or had to overcome to achieve their ideal practices.

Lecturers' ideal practices and obstacles to implement them

In the interviews, we found a different discourse among the English participants compared to the Norwegian participants. The English lecturers either talked about contemplating small changes to their courses or about changes that they had made already in their courses because they felt these were necessary for what they wanted to achieve. For example, Charles spoke of having already introduced more "formality" in his course by including more content from his research and a more "hands-on approach", such as programming certain procedures with his students. Ernest and Fred talked about offering more choices of project topics, perhaps by bringing other lecturers' research interests into the course. They both expressed that student satisfaction as well as employability data were positive; therefore, they did not feel the need to make further adjustments to their courses. Diana spoke about the obstacles that she and her colleagues had to overcome, such as students' negative attitudes toward MM, some of their colleagues (particularly "pure" mathematicians) thinking that MM "is not proper maths" or

the fact that not all colleagues in the department would want or be prepared to teach a MM course. However, all these obstacles had been dealt with, for example, by having a substantial cohort of colleagues that value MM, by having a first-year MM module that does not count towards the students' degree classification, and by training teaching assistants to help students during group work.

On the other hand, most of the Norwegian interviewees talked at length about their wishes for a better practice. Gunnar, Henrik, Lars and Odin all wanted to include more open-ended problems, or a more inquiry-based teaching approach, which, in the words of Gunnar, "is ideally suited for mathematical modelling". They also addressed the possibility of using digital tools to "spend [...] less time on [...] technical details and [...] methods and moving toward greater extent, towards using a computer algebra system [CAS] and the numerical system to get some time..." (Lars). Odin expressed this in terms of using CAS tools as what he calls "black boxes", which students can use without necessarily knowing the details of how they work:

I think one more aspect, which may open the gate for more creative and interesting things, is the fact that most of our colleagues seem to realise that numerical methods and implementing them is something [the students] should learn. [...] If we allow ourselves to introduce some black boxes... being very clear on them being black, initially, but in the end, they shouldn't remain black or at least should be grey, then there's a huge area of potential for modelling.

Despite being generally satisfied with her MM course, Linnea mentioned how she wished to concentrate on one or two digital tools only, to make it easier for students to master them and use them efficiently for the purposes of MM. She also spoke about assessment of MM skills rather than content, in a way mirroring what her other colleagues said about focusing on "more modelling in the courses rather than technicalities" (Henrik).

Some Norwegian lecturers mentioned potential barriers to their ideal practices such as: inhomogeneous student cohorts, the need for "talkative" students that actively engage in

discussions (Gunnar), teachers not knowing enough engineering to bring in suitable applications (Henrik), and a lack of time to design the courses (mentioned by several interviewees).

In articulating their ideal practices, lecturers made some suggestions that appeared contradictory. For example, talking about the idea of introducing “black boxes” to help students avoid dealing with technical details in order to focus more on the modelling, Odin said: “I’m definitely not a fan of black box pedagogics, I think they’re downright dangerous. (...) This is hard to explain because I don’t agree with myself on the subject”.

We felt that although these lecturers were not satisfied with their current, largely examples-driven practices, they faced obstacles very real to them in developing these practices into their ideal ones. Odin summarised this feeling in the following way:

I’m still struggling to get the creativity into it. I still haven’t cracked that code. And of course, that’s where mathematics really becomes fun, when [students] themselves [...] are trying to be creative instead of connecting the dots which I prepared for them. [...] I’m still making dots for them [...], I’m sad to say.

Discussion and conclusions

Contradictions or tensions, as conceptualised in CHAT, can be used fruitfully to study how social systems can change and develop [22]. In our data, we observe various contradictions within the activity system of teaching MM at university.

We start with lecturers’ stated aims for teaching MM. Our questionnaire data showed that many lecturers in both England and Norway adhered to a realistic view on the aims of MM in tertiary education; however, more than half declared a mixture of views relating to these aims. This was confirmed by our interview data. A variety of aims, often competing ones, creates tensions. For example, should students be taught MM to develop problem-solving skills or to illustrate theory? Should lecturers’ ultimate aim be “pure enjoyment” or to

prepare students for employment? Should lecturers/departments be satisfied with a practice only because student satisfaction rates are acceptable? Someone might say that the aim should combine all those mentioned above. However, it is this multitude of aims that brings complexity into the activity. Lecturers have to decide what to prioritise given limited time, resources or other considerations. Complexity also translates into learning: we might ask too much of students if they must learn the process of modelling while simultaneously trying to understand a mathematical concept. This might explain why many students find MM difficult [3,23]. A current debate exists [24] between supporters of Cognitive Load Theory (CLT) [25] and those who think that an inquiry-based learning (IBL) approach should be included in mathematics instruction. CLT poses that mathematics learning is problematic when students' working memory is heavily loaded with the process of problem-solving, and therefore direct instruction may be more desirable. Regardless of whether direct instruction or a mixture of IBL teaching with direct instruction is preferable, we concur with our interviewee Linnea who advocated a simple, fun way of learning without placing too much importance on, for example, how "real" a problem is.

In relation to the aims of MM from lecturers' perspectives, we see an inherent primary contradiction between the use value of mathematics – enjoyment and learning for the sake of it – versus the exchange value of mathematics – mathematics for employment or to fulfil institutional commitments. These tensions are part of the history of higher education (mass education, globalisation and competitiveness in world markets) and how universities have evolved (lecturing as the main instructional practice, demands of a 21st century degree education), and therefore, resolving them would require changes at the level of socioeconomic systems, for example, a new national educational policy.

We focus now on secondary contradictions, that is, those that arise within constituents of the activity system. The main tension that we see in our data is that of first-year MM

practices: Should first-year students be taught through examples-driven practices or through open-ended projects? Here the main tension is between the aim of the activity and the division of labour. If the aim is to enculturate students into a modelling frame of mind, then the main labour should come from the students themselves through practice, struggle and reflection. But if the main aim is to show how mathematics is used in professional activities, then the main labour should come from the lecturer who might want to take time, for instance, to develop some mathematical models from scratch or invite a guest lecturer for one of the classes.

There are also other tensions involved. For example, between the subject (lecturer) and the aims of the activity, or between the subject and the rules of the community. Many interviewees related to their professional identity when they suggested that they wanted to show their enthusiasm for mathematics in their teaching, and how much they enjoyed and valued MM. Nevertheless, it seems that most of this “fun” (messy problems and collaborative learning) is reserved for the few “elite” students taking final year modelling courses or entering postgraduate studies. We argue for a wider access to this activity which provides all students with the opportunity to experience MM.

There are also tensions between the subject and the tools used in the practice of teaching MM. Odin recommended the use of “black boxes”, while at the same time believing that such conceptual devices are “downright dangerous” as pedagogical tools. He is still reflecting on how to introduce creativity into his practice, a code that he “has not cracked yet”. It is worth noticing that some researchers argue in favour of pedagogical practices that make use of digital tools to let students make “black boxes” transparent [26].

Every obstacle to the implementation of their ideal practice is a tension yet to be resolved for the lecturers. How can lecturers reconcile their need for time to innovate and design pedagogically well-grounded teaching practices while having to deal with more and

more responsibilities (bringing in funding, publishing in top international journals, taking administrative roles, engaging with stakeholders in producing evidence of impact from research, etc.)?

We believe that, in addition to those discussed in this paper, there are other contradictions in the MM teaching activity systems of university lecturers. Our contribution to knowledge is a conceptualisation of university MM teaching practices that makes contradictions and tensions evident. This research may encourage lecturers to reflect about ways to resolve their own contradictions and tensions, and to promote the discussion of how they could overcome obstacles in their own cultural-historical contexts.

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References

1. Blum W, Niss M. Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. *Educational Studies in Mathematics*. 1991;22:37-68.
2. Blum W, Borromeo Ferri R. Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*. 2009;1:45-58.
3. Jankvist UT, Niss M. Upper secondary school students' difficulties with mathematical modelling. *International Journal of Mathematical Education in Science and Technology*. 2019:1-30.
4. Vorhölter K, Kaiser G, Borromeo Ferri R. Modelling in Mathematics Classroom Instruction: An Innovative Approach for Transforming Mathematics Education. *Transforming Mathematics Instruction. Advances in Mathematics Education*. 2014. p.21-36.
5. Schukajlow S, Kaiser G, Stillman G. Empirical research on teaching and learning of mathematical modelling: a survey on the current state-of-the-art. *ZDM*. 2018;50:5-18.

6. Villa-Ochoa JA, López CMJ. Sense of Reality Through Mathematical Modelling. Trends in Teaching and Learning of Mathematical Modelling. International Perspectives on the Teaching and Learning of Mathematical Modelling. 2011. p.701-11.
7. Doerr HM, Ärleback JB, Misfeldt M. Representations of Modelling in Mathematics Education. Mathematical Modelling and Applications. International Perspectives on the Teaching and Learning of Mathematical Modelling. 2017. p.71-81.
8. Fredriksen H, Hadjerrouit S. An activity theory perspective on contradictions in flipped mathematics classrooms at the university level. International Journal of Mathematical Education in Science and Technology. 2019;1-22.
9. Campos DF, Pinto MMF. Mathematics teachers' conceptions and constraints for changing teaching practices in Brazilian higher education: an analysis through activity theory. International Journal of Mathematical Education in Science and Technology. 2016;47:1179-205.
10. Fitzsimons GE. Technology mediated post-compulsory mathematics: An activity theory approach. International Journal of Mathematical Education in Science and Technology. 2005;36:769-77.
11. Engeström Y. Learning by expanding: An Activity-Theoretical approach to developmental research. Helsinki: Orienta-Konsultit Oy; 1987.
12. Engeström Y. Expansive learning at work: Toward an activity theoretical reconceptualisation. Journal of Education and Work. 2001;14:133-56.
13. Jaworski B, Robinson C, Matthews J, Croft T. An activity theory analysis of teaching goals versus student epistemological positions. The International Journal for Technology in Mathematics Education. 2012;19:147-52.
14. Macfarlane B. Student performativity in higher education: converting learning as a private space into a public performance. Higher Education Research & Development. 2014;34:338-50.
15. Apple MW. Education, markets, and an audit culture. Critical Quarterly. 2005;47:11-29.

16. Williams J. Audit and evaluation of pedagogy: Towards a Cultural-Historical perspective. In: Rowland T, Ruthven K, editors. *Mathematical Knowledge in Teaching*. Dordrecht: Netherlands: Springer; 2011. p.161-78.
17. Treffert-Thomas S, Viirman O, Hernandez-Martinez P, Rogovchenko Y. Mathematics lecturers' views on the teaching of mathematical modelling. *Nordic Studies in Mathematics Education*. 2017;22:121-45.
18. Kaiser G, Sriraman B. A global survey of international perspectives on modelling in mathematics education. *ZDM*. 2006;38:302-10.
19. Blum W, Leiß D. "Filling up" – the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In: Bosch M, editor. *Proceedings of the 4th European Congress of Mathematics Education*, St. Feliu de Guixols, Spain: CERME; 2006.
20. Rogovchenko Y, Viirman O, Treffert-Thomas S. Joy of mathematical modelling: A forgotten perspective? In: Stillman G, Kaiser G, Lampen E, editors. *Mathematical Modelling Education and Sense Making*. Cham: Switzerland: Springer; In press.
21. Kvale S, Brinkmann S. *InterViews: Learning the craft of qualitative interview*. Los Angeles: CA: SAGE; 2009.
22. Engeström Y, Sannino A. Discursive manifestations of contradictions in organizational change efforts. *Journal of Organizational Change Management*. 2011;24:368-87.
23. Frejd P, Ärleback JB. First Results from a Study Investigating Swedish Upper Secondary Students' Mathematical Modelling Competencies. In: Kaiser G, Blum W, Borromeo Ferri R, Stillman G, editors. *Trends in Teaching and Learning of Mathematical Modelling. International Perspectives on the Teaching and Learning of Mathematical Modelling*. 2011. p.407-16.
24. Treffers A. Direct instruction and problem-solving: Critical examination of Cognitive Load Theory from the perspective of mathematics education. *The Mathematics Enthusiast*. 2019;16:607-20.
25. Sweller J. Story of a research program. *Education Review*. 2016;23:1-18.

26. Sangwin, C. The mathematical apprentice: an organising principle for teaching calculus in the 21st century. In: Proceedings of the Conference on Calculus in Upper Secondary and Beginning University Mathematics, University of Agder, Norway: MatRIC; 2019.

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