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Wave-power absorption from a finite array of oscillating wave surge converters

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Abstract

Semi-analytical and fully numerical modelling is developed in the framework of the inviscid potential flow theory to investigate the dynamics of a wave farm made by flap-type wave energy converters in the nearshore. The hydrodynamic parameters and the efficiency of the system in typical layouts are calculated with both models. Good agreement is shown between the two approaches. Parametric analysis undertaken with the semi-analytical model allows to identify a near-resonant phenomenon which is responsible for increasing the absorbed power by the single elements of the array. Such result could be used as a preliminary design criterion. The numerical model is then applied to analyse a configuration of practical engineering interest, i.e. an array of two staggered converters. The dynamics arising in this more complex system is explained, showing that non-symmetric layouts can be less effective.

Keywords: Wave energy, wave-structure interaction, arrays, Oscillating Wave Surge Converter, numerical modelling

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1. Introduction

Commercialisation of wave energy systems requires the deployment of wave energy converters (WECs) in large arrays as a fundamental market acceleration strategy. When working together in an array, WECs can interfere in either a constructive or destructive way, depending on the geometric layout and their mutual distance (see [1]–[10]). In this paper we investigate the dynamic interactions arising within an array of large flap-type WECs, namely the Oscillating Wave Surge Converters (OWSCs). Each OWSC is made by a flap hinged on a foundation at the bottom of the ocean and pitching under the action of incident waves in the nearshore [11]. Examples of OWSC at an advanced stage of design are the WaveRollerTM developed by AW Energy (<http://aw-energy.com>) and the Oyster 800TM WEC developed by Aquamarine Power Limited (APL, www.aquamarinepower.com). In order to investigate the behaviour of an array of several OWSCs, four quantities are essential: the characteristic wave amplitude and wavenumber, A and k respectively, the characteristic width of the converters b and their mutual characteristic distance s . Various parameters can be formed from those quantities which are used to identify the regime of the system: A/b , kb , ks . First, in this paper we shall restrict our analysis to monochromatic waves of small-amplitude, for which $A/b \ll 1$. Second, we shall consider large flaps, so that $kb = O(1)$, and intermediate spacing between them, for which $ks = O(1)$. With the assumption $A/b \ll 1$, the behaviour of the system can be described by recurring to the linearised versions of the inviscid-irrotational equations of motion (potential-flow model, see for example [12]). Such hypotheses do not allow to consider either random-sea, vortex-shedding and nonlinear diffraction effects, which are currently being investigated with the aid of different models [13, 14, 15, 16, 17]. Yet the linearised potential-flow model provides an insightful description of the system dynamics which is fundamental for the successful design of such a costly project. Another important parameter to characterise the system regime is the product kb between the wavenumber of the incident wave and the characteristic width of the converters. Several existing analytical models are indeed applicable to the OWSC in the limiting cases $kb \ll 1$ and $kb \gg 1$. The first case corresponds to the so-called “point-absorber” approximation [1, 3], while the second one refers to the “line-absorber” limit [9]. However, considering an average OWSC width $b \simeq 30$ m and a characteristic wavelength $\lambda = 2\pi/k \simeq 100$ m (see for example [11]), yields $kb = O(1)$, which falls outside the limits of applicability of

38 the aforementioned theories. Recently, new models in the regime $kb = O(1)$
 39 have been developed to investigate the behaviour of an OWSC in a channel
 40 [18, 19], an infinite array of OWSCs [20] and a single OWSC in the open
 41 ocean [21, 22, 23]. However, the analysis of a finite array of OWSCs seems
 42 not to have been undertaken yet. Indeed several theoretical models are avail-
 43 able to investigate the interactions in an array of floating bodies which are
 44 also implemented in numerical routines (see for example [5] and [24]–[30]).
 45 Some of these models rely on simplifying assumptions on the parameter ks .
 46 For $ks \ll 1$, the spacing between the elements can be neglected without ap-
 47 preciable consequences, as shown by Mei *et al.* [25] and Adamo & Mei [29]
 48 for an array of closely-spaced storm gates. On the other hand, when $ks \gg 1$
 49 the wide-spacing approximation can be applied, for which radially outgoing
 50 waves are approximated as plane waves [5]. Here we shall consider the inter-
 51 mediate case $ks = O(1)$, where interference effects between the elements of
 52 the array must be appropriately accounted for [12].

53 In this paper, a twofold analytical and numerical approach is undertaken
 54 to investigate the dynamics of a finite array of OWSCs in the open ocean. In
 55 §2 a general mathematical model of the system is introduced and the govern-
 56 ing equations are detailed. Then a new semi-analytical model for an in-line
 57 array of OWSCs in normally incident waves is derived. This model, yet nec-
 58 essarily based on some simplifying assumptions, provides a valuable physical
 59 insight on the system dynamics. In addition to the analytical approach, a
 60 finite-element model is presented to solve numerically the governing equations
 61 of the system for more general layouts. Then in §3 both models are validated.
 62 The dynamics of two and three in-line converters is discussed. For the first
 63 time, a near-resonant phenomenon - already known for arrays of floats - is
 64 shown to occur for an array of OWSCs. Hence the semi-analytical solution
 65 is employed in §3.2 to investigate the parametric behaviour of the system
 66 with respect to the period of the incident wave and the spacing between the
 67 flaps, showing potential for constructive interaction at near-resonant periods.
 68 Finally, in §4 the potential of the numerical model is applied to analyse a
 69 configuration of practical engineering interest, i.e. an array of two staggered
 70 OWSCs.

Figure 1: Geometry of an array made by M OWSCs. (a) plan view, (b) section of the m -th OWSC.

2. Mathematical model

2.1. Governing equations

Referring to figure 1, consider a system made by a finite number M of OWSCs in an ocean of finite depth h' . Let the primes denote physical quantities. Each OWSC is modelled as a rectangular flap of thickness a'_m and width w'_m , hinged on a bottom foundation of height c'_m , $m = 1, 2, \dots, M$. Under the action of monochromatic incident waves of amplitude A'_I and period T' , the flaps are able to perform a pitching motion, from which useful energy is extracted by means of linear power take-off (PTO) mechanisms linked to each device. Given the linearity of the system, all flaps oscillate with the same period T' but with potentially different phases, depending on the geometry. A Cartesian system of reference $O'(x', y', z')$ is set at an arbitrary origin O' on the still water level $z' = 0$, with the centreline of each flap aligned with the y' axis and the z' axis pointing vertically upwards (see again figure 1). Assume that the m th flap is able to perform oscillations of angular amplitude $\theta'_m(t')$ about the y' axis at $z' = -h' + c'_m$; t' is time. Now let $\mathcal{L}'_m$ denote the region occupied by the m th flap at the equilibrium position (i.e. $\theta'_m = 0$) and let $\partial\mathcal{L}'_m$ be its solid boundary. Assume that the fluid is inviscid and incompressible and the flow irrotational. Then there exists a potential Φ' for the velocity field $\mathbf{v}' = \nabla'\Phi'$ which satisfies the Laplace equation

$$\nabla'^2\Phi'(x', y', z', t') = 0 \quad (1)$$

in the fluid domain. In (1), $\nabla' f' = (f'_{,x'}, f'_{,y'}, f'_{,z'})$, where subscripts with commas denote differentiation with respect to the relevant variables. Assuming that the flaps perform small-amplitude oscillations, the set of boundary conditions (b.c.'s) associated to the Laplace equation (1) can be linearised as shown in [19] thus yielding

$$\Phi'_{,t't'} + g\Phi'_{,z'} = 0, \quad z' = 0 \quad (2)$$

for the kinematic-dynamic b.c. on the linearised free-surface,

$$\Phi'_{,z'} = 0, \quad z = -h' \quad (3)$$

for the no-flux condition at the bottom and finally

$$\Phi'_{,\hat{n}} = V'_{m\hat{n}}, \quad \text{on } \partial\mathcal{L}'_m, \quad m = 1, 2, \dots, M \quad (4)$$

for the kinematic b.c. on the flaps. In the latter expression, $V'_{m\hat{n}}$ is the component of the m th-flap velocity along the normal $\hat{n} = (n_{x'}, n_{y'}, n_{z'})$ directed out of the fluid on the boundary $\partial\mathcal{L}'_m$ [12]. Hence for pitching motion about the y' axis, expression (4) yields

$$\Phi'_{,x'} = -\theta'_{m,t'}(t')(z' + h' - c'_m)H(z' + h' - c'_m), \quad m = 1, 2, \dots, M, \quad (5)$$

along the flap sides normal to the x' axis, where the Heaviside step function H assures absence of flux through the bottom foundation, and

$$\Phi'_{,y'} = 0, \quad m = 1, 2, \dots, M, \quad (6)$$

along the flap sides normal to the y' axis.

2.2. Body motion and wave-power absorption

Once the velocity potential Φ' is known, the flap dynamics can be fully characterised by solving the following system of Newton's laws of motion (see also [22]):

$$I'_m \theta'_{m,t't'}(t') + C'_m \theta'_m(t') + \eta'_m \theta'_{m,t'}(t') = -\rho \int_{\partial\mathcal{L}'_m} \Phi'_{,t'}(x', y', z', t') [n_{x'}(z' + h' - c'_m) + n_{z'} x'] dS', \quad m = 1, \dots, M. \quad (7)$$

The latter impose the equilibrium of the inertial, linear and damping actions (left-hand side) with the hydrodynamic torque (right-hand side), for each

112 flap. In (7), I'_m , C'_m and η'_m are, respectively, the moment of inertia, the
 113 buoyancy torque and the rate of power take-off (PTO) of the m th flap, given
 114 by the system manufacturer, ρ is water density. Once the system of ordinary
 115 differential equations (7) is solved in terms of the pitching angles $\theta'_m(t')$,
 116 $m = 1, 2, \dots, M$, the performance of the array can be analysed by calculating
 117 the amplitude factor of each flap and the total power output. The m th
 118 amplitude factor is defined as the ratio between the maximum horizontal
 119 displacement of the m th flap at the free-surface in a period and the amplitude
 120 of the incident wave, that is

$$A_m^F = \frac{\tan[\max_{T'} \theta'_m] (h' - c'_m)}{A_I}. \quad (8)$$

121 According to (8), the larger the amplitude factor, the larger the amount of
 122 incoming wave energy which is converted into pitching motion by each flap,
 123 for given non-zero PTO damping. Finally, the average power absorbed by
 124 the system in a period is calculated by summing the contribution of each
 125 single flap (see [22]):

$$P' = \frac{1}{T'} \sum_{m=1}^M \int_0^{T'} (\eta'_m \theta'_{m,t'}) \theta'_{m,t'} dt'. \quad (9)$$

126 In the following, the system of equations (1)–(6) will be solved with either
 127 a semi-analytical model or a fully numerical model. Then the dynamics of
 128 the flaps and the performance of the system will be analysed according to
 129 equations (7)–(9). The semi-analytical model is based on the simplifying as-
 130 sumption that all flaps are aligned on the y' axis. Such assumption restricts
 131 the applicability of the model to idealised cases, but allows to gain a funda-
 132 mental insight into the dynamics of the system which could be overlooked
 133 by relying only on numerical calculations. On the other hand, the numer-
 134 ical model is not restricted by such constraints and can be used to further
 135 investigate practical scenarios where the semi-analytical model cannot be ap-
 136 plied. In this sense, the two models complement well and together provide
 137 an essential tool for an in-depth analysis of the system in the linear regime.

138 2.3. Semi-analytical solution

139 Consider an in-line system of flaps aligned along the y' axis. In this case,
 140 the analytical approach of Adamo & Mei [29] for an array of neighbouring

Figure 2: Geometry of an array made by M inline OWSCs. The thin-plate approximation [19, 20, 22] is applied to calculate the velocity potential in normally incident waves.

141 flood gates can be extended and combined with the integral formulation of
 142 Renzi & Dias [22] for a single OWSC in the open ocean. Such procedure
 143 allows to obtain a new semi-analytical solution of the problem. Assuming
 144 that the characteristic length scale of the system b' (e.g. the width of the
 145 largest flap) is much larger than the thickness of each device, i.e. $b' \gg a'_m$,
 146 the latter can be considered immaterial for calculating the potential Φ' (thin-
 147 plate approximation, see also [19, 22, 29]). As a consequence, each flap is
 148 represented in the (x', y') plane as a line lying on the y' axis, as shown in
 149 figure 2. Now let $y_m^{s'}$ and $y_m^{e'}$ denote, respectively, the coordinates of the start
 150 and end points of the m th flap on the y' axis, with $y_m^{s'} < y_m^{e'} < y_{(m+1)}^{s'} <$
 151 $\dots < y_M^{e'}$. Hence the m th-flap width is $w'_m = y_m^{e'} - y_m^{s'} > 0$ and the relevant
 152 border reduces to

$$\partial\mathcal{L}'_m = \{(x', y', z') : x' = \pm 0, y' \in (y_m^{s'}, y_m^{e'}), z' \in (-h', 0)\}, \quad m = 1, 2, \dots, M. \quad (10)$$

153 Then introduce the following nondimensional variables

$$(x, y, z) = (x', y', z')/b', \quad t = \sqrt{g/b'} t', \quad \Phi = \Phi' \left(\sqrt{g b'} A' \right)^{-1}, \quad \theta_m = (b'/A') \theta'_m \quad (11)$$

154 and constants

$$(c_m, y_m^s, y_m^e, w_m, h) = (c'_m, y_m^{s'}, y_m^{e'}, w'_m, h')/b', \quad A_I = A'_I/A'. \quad (12)$$

155 In (11) and (12) A' is the amplitude scale of the incident wave. This must be
 156 much smaller than the length scale of the system to assure that the converters
 157 perform small-amplitude oscillations, i.e. $A' \ll b'$ (see also [19, 22]). Let us

also consider time-harmonic oscillations of radian frequency $\omega = \omega' \sqrt{b/g} = 2\pi/T$ and factor time out of the time-dependent variables, such as

$$\Phi(x, y, z, t) = \Re \{ \phi(x, y, z) e^{-i\omega t} \}, \quad \theta_m(t) = \Re \{ \Theta_m e^{-i\omega t} \}, \quad m = 1, 2, \dots, M, \quad (13)$$

where $\Re$ denotes the real part, ϕ is the complex spatial potential and Θ_m the complex amplitude of rotation of the m th flap. Given the linearity of the system, position (13) assumes that all flaps oscillate with the same frequency ω (this excludes second-order resonant effects, see [25, 31, 32]), but with different phases $\epsilon_m = \arg(\Theta_m)$. Substituting the nondimensional variables (11) together with the factorisation (13) into the governing equations (1)–(6) yields, respectively,

$$\nabla^2 \phi = 0 \quad (14)$$

in the fluid domain for the Laplace equation (1),

$$\phi_{,z} - \omega^2 \phi = 0, \quad z = 0 \quad (15)$$

for the b.c. on the free surface (2),

$$\phi_{,z} = 0, \quad z = -h, \quad (16)$$

for the no-flux condition at the bottom (3) and finally

$$\phi_{,x} = \Omega_m(z + h - c_m)H(z + h - c_m), \quad \text{on } \partial\mathcal{L}_m, \quad m = 1, 2, \dots, M \quad (17)$$

for the kinematic condition on the flaps (5). In expression (17), $\Omega_m = i\omega\Theta_m$ for the sake of brevity. Now assume for ϕ the same decomposition argument as that of Adamo & Mei [29] (see eq. 2.4), for which

$$\phi = \phi^S + \sum_{\alpha=1}^M \Omega_\alpha \phi^{(\alpha)}. \quad (18)$$

In the latter, $\phi^S = \phi^I + \phi^D$ is the scattering potential, solution of the problem in which the flaps are all held fixed in incoming waves. In turn ϕ^S is the sum of the incident wave potential

$$\phi^I = -i \frac{A_I}{\omega} \frac{\cosh k(z + h)}{\cosh kh} e^{-ikx} \quad (19)$$

and the unknown diffraction potential ϕ^D . In (19) k is the wavenumber given by the well-known dispersion relation $\omega^2 = k \tanh kh$. Finally, $\phi^{(\alpha)}$ is

178 the solution of the unit radiation potential, where the α th flap moves with
179 $\Omega_\alpha = 1$ while all the other flaps are at rest. Note that ϕ^I (19) models a
180 plane wave field normally incident on the system. Oblique incidence will
181 not be investigated here because of two main reasons: (a) the system is to
182 be deployed in the nearshore, where wave fronts are almost parallel to the
183 shoreline due to refraction and (b) in case of oblique incidence at an angle
184 $\beta \neq 0$ with the x axis, the power captured by the OWSC would decrease
185 almost as $\cos^2 \beta$, as shown by Whittaker *et al.* [33]. Hence oblique incidence
186 is undesirable and is usually avoided by orienting the flaps along the direction
187 of predominant incidence. This allows to minimise the reduction in power
188 capture due to obliquely incident waves (e.g. about 3% in random seas for
189 the Oyster device, see [33]) which can be neglected without consequences. In
190 order to solve the system (14)–(17), the linearised potentials introduced in
191 the decomposition (18) must satisfy the governing equation

$$\nabla^2 \begin{Bmatrix} \phi^D \\ \phi^{(\alpha)} \end{Bmatrix} = 0 \quad (20)$$

192 in the fluid domain, together with the boundary conditions

$$\begin{Bmatrix} \phi_{,z}^D \\ \phi_{,z}^{(\alpha)} \end{Bmatrix} - \omega^2 \begin{Bmatrix} \phi^D \\ \phi^{(\alpha)} \end{Bmatrix} = 0, \quad z = 0, \quad (21)$$

$$\begin{Bmatrix} \phi_{,z}^D \\ \phi_{,z}^{(\alpha)} \end{Bmatrix} = 0, \quad z = -h, \quad (22)$$

193

$$\begin{Bmatrix} \phi_{,x}^D \\ \phi_{,x}^{(\alpha)} \end{Bmatrix} = \begin{Bmatrix} -\phi_{,x}^I \\ (z + h - c_\alpha)H(z + h - c_\alpha)\delta_{\alpha m} \end{Bmatrix}, \text{ on } \partial\mathcal{L}_m, \quad m = 1, \dots, M. \quad (23)$$

194 In the latter, $\delta_{\alpha m}$ is the Kronecker delta, m indicates any flap and α the
195 moving flap. The first scalar equation of (23) requires the scattering potential
196 $\phi^S = \phi^I + \phi^D$ to satisfy a no-flux condition on all the flaps. Meanwhile, the
197 second scalar equation of (23) requires the radiation potential $\phi^{(\alpha)}$ to satisfy
198 a no-flux condition on all the non-moving flaps, for which $m \neq \alpha$, and a
199 kinematic b.c. on the moving flap $m = \alpha$, for all $\alpha = 1, \dots, M$ in succession.
200 Now separate the vertical dependence of the spatial potentials by using the
201 decomposition

$$\begin{Bmatrix} \phi^D \\ \phi^{(\alpha)} \end{Bmatrix} = \sum_{n=0}^{\infty} \begin{Bmatrix} \varphi_n^D \\ \varphi_n^\alpha \end{Bmatrix} Z_n(z) \quad (24)$$

202 where the $\varphi_n^D(x, y)$ and $\varphi_n^\alpha(x, y)$ are, respectively, two-dimensional (2D) diffrac-
 203 tion and radiation potentials in the (x, y) plane. In (24) the Z_n are the
 204 well-known vertical eigenmodes

$$Z_n(z) = \frac{\sqrt{2} \cosh \kappa_n(z + h)}{(h + \omega^{-2} \sinh^2 \kappa_n h)^{1/2}}, \quad (25)$$

205 where $\kappa_0 = k$, $\kappa_n = ik_n$ and the k_n are the real solutions of the dispersion
 206 relation

$$\omega^2 = -k_n \tan k_n h, \quad n = 1, 2, \dots \quad (26)$$

207 Substitution of the series expansion (24) into (20)–(23) and usage of the
 208 orthogonality property

$$\int_{-h}^0 Z_n(z) Z_m(z) dz = \delta_{nm} \quad (27)$$

209 for the vertical eigenfunctions (25) yields the following system of equations
 210 for the diffraction and radiation potentials:

$$(\nabla^2 + \kappa_n^2) \begin{Bmatrix} \varphi_n^D \\ \varphi_n^\alpha \end{Bmatrix} = 0 \quad (28)$$

211 in the 2D fluid domain and

$$\begin{Bmatrix} \varphi_{n,x}^D \\ \varphi_{n,x}^\alpha \end{Bmatrix} = \begin{Bmatrix} A_I d_n \\ f_{n\alpha} \delta_{\alpha m} \end{Bmatrix}, \quad x = \pm 0, y \in (y_m^s, y_m^e) \quad (29)$$

212 on each flap $m = 1, 2, \dots, M$. In expression (29)

$$d_n = \frac{k (h + \omega^{-2} \sinh^2 k h)^{1/2}}{\sqrt{2} \omega \cosh k h} \delta_{n0}, \quad (30)$$

$$f_{n\alpha} = \sqrt{2} \frac{\kappa_n (h - c_\alpha) \sinh \kappa_n h + \cosh \kappa_n c_\alpha - \cosh \kappa_n h}{\kappa_n^2 (h + \omega^{-2} \sinh^2 \kappa_n h)^{1/2}} \quad (31)$$

213 are real values depending on the system parameters. Finally, it is required
 214 that both φ_n^D and φ_n^α be outgoing disturbances as $x, y \rightarrow \infty$. The system
 215 of equations (28)–(31) can be solved by extending to a finite array of flaps
 216 the method developed by Renzi & Dias [22] for a single flap in the open
 217 ocean. The procedure is detailed in Appendix A and briefly summarised

218 here. Application of Green's integral theorem to a large circle enclosing all
 219 the flaps and usage of the b.c.'s (29) yields a hypersingular integral equation
 220 for each of the 2D potentials. The system is hence carefully de-singularised
 221 by recurring to a series expansion in terms of the Chebyshev polynomials
 222 of the second kind (see Appendix A for details). As a result, the spatial
 223 diffraction potential can be written as

$$\begin{aligned} \phi^D(x, y, z) = & \frac{-iA_I}{8} kx Z_0(z) \sum_{\beta=1}^M w_{\beta} \sum_{p=0}^P a_{0\beta p} \int_{-1}^1 (1-u^2)^{1/2} \\ & \times U_p(u) \frac{H_1^{(1)} \left[k \sqrt{x^2 + (y - 1/2(u_{\beta} + w_{\beta}u))^2} \right]}{\sqrt{x^2 + (y - 1/2(u_{\beta} + w_{\beta}u))^2}} du, \end{aligned} \quad (32)$$

224 where $u_{\beta} = y_{\beta}^e + y_{\beta}^s$, $w_{\beta} = y_{\beta}^e - y_{\beta}^s$, $H_1^{(1)}$ is the outgoing Hankel function of the
 225 first kind and first order, U_p is the Chebyshev polynomial of the second kind
 226 and order $p \in \mathbb{N}$ and finally the $a_{0\beta p}$ are complex coefficients which assure that
 227 the diffraction potential satisfies the b.c. on the flaps (29). Such coefficients
 228 are determined by solving a system of linear equations with a numerical
 229 collocation scheme (see again Appendix A). Therefore the solution (32) is
 230 semi-analytical. Note that the diffraction potential ϕ^D (32) depends only on
 231 the 0th-order vertical mode. This is due to a solvability condition on the
 232 coupled radiation-diffraction problem which excludes higher-order vertical
 233 eigenmodes in ϕ^D (for details see Appendix A and [19]). The unit radiation
 234 potential for the α th plate is given by

$$\begin{aligned} \phi^{(\alpha)}(x, y, z) = & \frac{-i}{8} \sum_{n=0}^{\infty} \kappa_n x Z_n(z) \sum_{\beta=1}^M w_{\beta} \sum_{p=0}^P b_{n\alpha\beta p} \int_{-1}^1 (1-u^2)^{1/2} \\ & \times U_p(u) \frac{H_1^{(1)} \left[\kappa_n \sqrt{x^2 + (y - 1/2(u_{\beta} + w_{\beta}u))^2} \right]}{\sqrt{x^2 + (y - 1/2(u_{\beta} + w_{\beta}u))^2}} du. \end{aligned} \quad (33)$$

235 In the latter, the $b_{n\alpha\beta p}$ are complex coefficients which assure that the ra-
 236 diation potential satisfies the boundary condition on the flaps (29). Such
 237 coefficients are again determined by solving a system of linear equations
 238 with a numerical collocation scheme, so that (33) is semi-analytical.

239 In order to solve the problem fully, the pitching angles Θ_m in the de-
 240 composition (13) must now be determined. This can be done by considering
 241 the equations of motion of the system (7), on one flap - say the β th flap
 242 - at a time. Substituting the factorisation (13) for the total potential and
 243 the pitching angles into (7), together with (18), (19), (24), (A.3), (A.5) and
 244 (A.9), yields after some lengthy algebra

$$[-\omega^2(I_\beta + \mu_{\beta\beta}) + C_\beta - i\omega(\nu_{\beta\beta} + \eta_\beta)] \Theta_\beta - \sum_{\alpha=1}^M{}' (\omega^2\mu_{\alpha\beta} + i\omega\nu_{\alpha\beta}) \Theta_\alpha = F_\beta, \quad (34)$$

245 for $\beta = 1, \dots, M$, where the potential inside the surface integral in (7) has
 246 been evaluated by using the thin-plate approximation (10). In (34), $I_\beta =$
 247 $I'_\beta/(\rho b'^5)$, $C_\beta = C'_\beta/(\rho g b'^4)$ and $\eta_\beta = \eta'_\beta/(\rho b'^4 \sqrt{g b'})$ are, respectively, the
 248 nondimensional moment of inertia, buoyancy torque and PTO damping of
 249 the β th flap and $\sum'$ indicates that the summation must exclude the term
 250 $\alpha = \beta$. Furthermore, in (34)

$$F_\beta = -\frac{i\pi}{4} A_I w_\beta \omega f_{0\beta} a_{0\beta 0} \quad (35)$$

251 is the nondimensional exciting torque acting on the β th flap when all the
 252 flaps are fixed in incident waves, while

$$\mu_{\alpha\beta} = \frac{\pi w_\beta}{4} \sum_{n=0}^{\infty} f_{n\beta} \Re \{b_{n\alpha\beta 0}\} \quad (36)$$

253 and

$$\nu_{\alpha\beta} = \frac{\pi w_\beta}{4} \omega \sum_{n=0}^{\infty} f_{n\beta} \Im \{b_{n\alpha\beta 0}\} \quad (37)$$

254 are, respectively, the nondimensional added mass and radiation damping
 255 due to pressure distribution on flap β when flap α is moving (see for example
 256 [8, 29]). Due to the reciprocity relations of Appendix B it is straightforward
 257 to obtain the well-known results [8, 12, 29, 34]

$$\mu_{\alpha\beta} = \mu_{\beta\alpha}, \quad \nu_{\alpha\beta} = \nu_{\beta\alpha}. \quad (38)$$

258 The latter relations are used in this paper to test the accuracy of the nu-
 259 merical computations in the semi-analytical model. The system of equations

(34) allows to obtain the sought complex amplitude of rotation Θ_β for each flap $\beta = 1, \dots, M$. Physically, the first term on the left-hand side of (34) represents the direct dynamic influence of the β th flap on itself, while the other terms represent the influence of all the other flaps on flap β . Finally, note that in (34) the buoyancy restoring torque acting on the β th flap is fully accounted for by the term C_β [19]. In other words, the thin-plate approximation is applied only to the velocity potential and does not affect the buoyancy property of the flaps [29]. By introducing the matrix components

$$\begin{aligned} [\mathbf{\Theta}]_\beta &= \Theta_\beta, [\mathbf{I}]_{\alpha\beta} = I_\alpha \delta_{\alpha\beta}, [\mathbf{M}]_{\alpha\beta} = \mu_{\alpha\beta}, [\mathbf{C}]_{\alpha\beta} = C_\alpha \delta_{\alpha\beta} \\ [\mathbf{N}]_{\alpha\beta} &= \nu_{\alpha\beta}, [\mathbf{H}]_{\alpha\beta} = \eta_\alpha \delta_{\alpha\beta}, [\mathbf{F}]_\beta = F_\beta, \end{aligned} \quad (39)$$

the system (34) can also be rewritten in the matrix form

$$[-\omega^2 (\mathbf{I} + \mathbf{M}) + \mathbf{C} - i\omega (\mathbf{N} + \mathbf{H})]^\top \mathbf{\Theta} = \mathbf{F}, \quad (40)$$

which is more convenient for implementation in numerical routines (see also [8]). Finally, once the Θ_β are known from (34) or (40), the average absorbed power over a cycle is determined by substituting the second of (13) together with the physical scales of (11) into (9), which then yields

$$P' = \frac{\omega'^2}{2} \sum_{\beta=1}^M \eta'_\beta |\Theta'_\beta|^2. \quad (41)$$

2.4. Fully numerical solution

A finite-element model (FEM) based on the software COMSOL Multiphysics is applied to solve the wave-induced flow, coupled together with a numerical solver of the global dynamic equations of motion (7). The latter allows to calculate the movement of the flaps. For problems in fluid-structure interaction, usage of the boundary-element method (BEM) is also a viable option [see for example 26]. However, often BEM codes are significantly less efficient than volume-discretisation methods. This happens since BEM codes usually employ fully populated matrices. As a consequence, the storage requirements and computational time for a BEM code grows according to the square of the problem size. On the other hand, FEM codes employ banded matrices, whose storage requirements usually grow linearly with the problem size.

Unlike the semi-analytical model of §2.3, the numerical model is able to reproduce virtually any array layout, for arbitrary angle of incidence of the

Figure 3: Left panel: sketch of the computational domain of the numerical model for an array of two staggered OWSCs (see §4 for parameters). Right panel: same model with the tetrahedral mesh elements represented.

incoming waves, indeed at larger computational cost (see §2.5). The finite-element method is used which ensures an excellent reproduction of domains with complex geometries. Tetrahedral elements are used to model three-dimensional bodies, as shown in figure 3. Let us define a reference frame by considering x' the longitudinal horizontal direction, z' the vertical direction, pointing upwards, and y' the remaining horizontal direction originating from the center of the numerical domain. Similar to the semi-analytical model, the numerical model is based on the assumptions of irrotational flow, inviscid fluid, small-amplitude waves and small movement of the flaps with respect to the wavelength and to the water depth. As a consequence, the flow field is governed again by the Laplace equation (1), the kinematic-dynamic boundary condition on the free surface (2), the no-flux condition at the bottom (3) and the kinematic condition on the wet boundary of the flaps (4), conveniently re-written in the form (see [35])

$$\hat{n} \cdot \nabla \Phi' = n_{x'} r' \theta'_{,t'}.$$

In the latter, $n_{x'}$ represents the x' component of the normal to the flap surfaces at rest and r' is the distance between any point on the flap sides and the hinge axis, for each flap. The flaps are initially considered at rest position: $\theta'_m(0) = 0$, $m = 1, \dots, M$. An approximate radiation condition including a source term for generating the desired incoming waves is imposed at the open

generation boundary (left of the domain in figure 3), which also allows the waves reflected back by the device to freely leave the computational domain:

$$\hat{n} \cdot \nabla \Phi' = -\frac{\cos \alpha_n}{c'} \Phi'_{,t'} + \frac{2A_I' g}{\omega'} k' \frac{\cosh k'(z' + h')}{\cosh k' z'} \cos \omega' t'.$$

In the latter, $c' = \omega'/k'$ is the linear phase speed, k' is the wavenumber solution of $\omega'^2 = gk' \tanh k'h'$ and α_n is the angle formed by the incoming wave direction with the outgoing normal to the considered boundary. At the remaining lateral boundaries, a sink term is imposed, obtained by using a mathematical formulation [36, 37] which allows the waves propagating outwards to freely exit the computational domain:

$$\hat{n} \cdot \nabla \Phi' = -\frac{\cos \alpha_n}{c'} \Phi'_{,t'}.$$

286 The solution of the flow field is then used to fully determine the flap kine-
 287 matics by solving the system of equations of motion (7). Once the pitching
 288 angle θ'_m is determined for each flap $m = 1, \dots, M$ at each time step, the
 289 performance of each OWSC is evaluated by calculating the amplitude factor
 290 (8) and the absorbed power (9). A description of the numerical procedure
 291 adopted in this paper is offered in Appendix C.

292 2.5. Computational aspects

293 The semi-analytical model of §2.3 has been implemented on a parallelised
 294 code on Mathematica[®] 8 (see for example [38]), able to evaluate the exciting
 295 torque (35), the added inertia torque (36), the radiation damping (37), the
 296 amplitude factor (8) and the total extracted power (41) by each flap in the
 297 array at given periods of the incident wave. For the 3-flap array examined
 298 in §3, the first three vertical eigenmodes and six Chebyshev polynomials -
 299 i.e. a (3, 6) configuration - were sufficient to achieve a relative error $O(10^{-3})$
 300 with respect to a (4, 8) configuration whose computational time was almost
 301 two and a half times larger. For a typical array of three flaps, an average
 302 computational time of 28 s was sufficient to determine the abovementioned
 303 parameters for given period of the incident wave on a high-speed computer
 304 equipped with an i7 3.40 GHz CPU and 16 GB RAM.

305 For the numerical model of §2.4, possible symmetries occurring in the
 306 array configuration should always be looked for and exploited in order to
 307 reduce the computational time. As an example, for in-line arrays like the ones
 308 discussed in §3, symmetry of the problem about $y' = 0$ allows computations

309 to be conveniently undertaken for half the physical domain only. Such a
310 procedure is indeed not applicable to more complicated layouts, like the
311 staggered array modelled in §4 (see also figure 3), where the model must be
312 solved for the whole domain. For the most demanding simulation undertaken
313 in this paper, the number of elements and degrees of freedom used were
314 about 550,000 and 120,000 respectively. Typically, long wave lengths require
315 a larger computational time but a coarse mesh, while short waves require a
316 smaller domain but a fine mesh. In all cases, the flume length should be at
317 least 3 times the wave length to assure that there is enough space for waves
318 to develop and then leave the domain. On average, the computational time
319 for 500 s of real-time simulation was about 1 hour on a high-speed computer
320 equipped with an i7 2.67 GHz CPU and 12GB RAM.

321 **3. Discussion**

322 In this section, the semi-analytical model of §2.3 and the numerical model
323 of §2.4 will be used to investigate the dynamics of a finite array of OWSCs.
324 Results will be validated showing good agreement between the two differ-
325 ent approaches in describing the hydrodynamic behaviour of the system.
326 Furthermore, the semi-analytical model will be used to investigate the de-
327 pendence of the system on its main parameters, i.e. the number of flaps and
328 the flap spacing. For the first time, a near-resonant phenomenon is shown to
329 occur for an in-line array of OWSCs.

330 *3.1. Validation*

331 The general analytical and numerical approaches presented in this paper
332 have been already validated with experimental data for a single flap in a
333 channel [19] and with other numerical results for a single flap in the open
334 ocean [22, 23]. In this section, results of the semi-analytical and numerical
335 models for a finite array of OWSCs developed, respectively, in §2.3 and §2.4
336 are compared and validated for two different systems made by two and three
337 in-line flaps.

338 *3.1.1. 2-flap system*

339 First, consider a system of two identical in-line flaps (1–2), each of width
340 $w' = 26$ m hinged at $c' = 4$ m on an ocean of depth $h' = 13$ m. The thickness
341 of the flaps is immaterial for calculating the velocity potential (but not the
342 buoyancy torque, which is fully accounted for) in the semi-analytical model

Figure 4: (a) Exciting torque magnitude in physical variables $|F'| = \rho g A' b'^3 |F|$ and (b) amplitude factor versus period of the incident wave calculated with the semi-analytical model (line) and the numerical model (ticks) for a two-flap system. In the semi-analytical model, $|F|$ is given by (35). In the numerical model, $|F'|$ has been calculated directly by integrating the pressure field acting on the flaps held fixed in incident waves. The legend at the top right corner of (a) shows a sketch of the configuration (not in scale). Parameters are $w'_1 = w'_2 = 26$ m, $c'_1 = c'_2 = 4$ m, $s' = 30$ m and $h' = 13$ m. Waves of amplitude $A'_I = 1$ m are normally incident on the system. Due to linearity and symmetry of the system, $F'_1 = F'_2$ and $A_1^F = A_2^F$.

and is $a' = 2$ m in the numerical model. Such dimensions resemble those of the Oyster800TMOWSC manufactured by APL [11]. The gap between the flaps is $s' = 30$ m, which gives $s = s'/w' = O(1)$. The PTO coefficient is the same for both flaps, i.e. $\eta'_1 = \eta'_2$, and is chosen in order to maximise the total power output (41) in the semi-analytical model, for each given period of the incident wave. The inertia and buoyancy torque coefficients, respectively I and C , are equal for all flaps and have been obtained by private communication with APL. Consider a monochromatic wave of amplitude $A'_I = 1$ m normally incident on the farm. Due to the linearity of the system and the symmetry of the layout, the flaps are expected to oscillate at unison. Hence knowing the behaviour of only one flap is sufficient to characterise the behaviour of the whole system. Figure 4 shows a sketch of the system layout together with the plots of the exciting torque magnitude $|F'_1| = |F'_2|$ in physical variables and the amplitude factor $A_1^F = A_2^F$ for both the semi-analytical model of §2.3 and the numerical model of §2.4. The agreement between the data sets is good at short periods and satisfactory at larger

359 periods, where the influence of the thickness determines the numerical values
 360 to be slightly larger than the semi-analytical ones. With longer waves, in
 361 fact, the jump in potential across a thick flap is larger than that across a
 362 thin flap, to which a long wave passes almost as a uniform swelling [19]. This
 363 determines the exciting torque to be larger for a thick flap and justifies the
 364 slight difference between the semi-analytical and numerical results in figure
 365 4. Figure 4(a) shows that the system attains the maximum exciting torque
 366 at small periods of the incident wave. This is a peculiar characteristic of
 367 the OWSC which is already known to occur for a single device in the open
 368 ocean [18, 22] and for an infinite array of OWSCs [20]. Results of figure 4(a)
 369 reveal that the OWSC maintains this fundamental feature also when working
 370 in a finite array. Furthermore, figure 4(b) shows that the amplitude factor
 371 of either flap increases monotonically with the period of the incident waves.
 372 Already for $T' \geq 7$ s, A^F is larger than 1, which reveals that the system
 373 effectively converts the incident wave motion into pitching motion.

374 3.1.2. 3-flap system

375 A similar analysis is now made for a more complex system of three in-
 376 line flaps (1 – 2 – 3) with same dimensions and spacing as above. The
 377 PTO coefficient is the same for all the flaps, i.e. $\eta'_1 = \eta'_2 = \eta'_3$, and again
 378 is chosen in order to maximise the total power output (41) for each given
 379 period of the incident wave. A monochromatic wave of amplitude $A'_l = 1$ m
 380 normally incident on the system is considered. Due to symmetry, the system
 381 of equations of motion (7) needs to be solved for flaps 1 and 2 only, with flap
 382 3 mirroring flap 1. Figure 5 shows a sketch of the system layout together
 383 with the plots of the exciting torque magnitudes $|F'_1| = |F'_3|$, $|F'_2|$ and the
 384 amplitude factors $A_1^F = A_3^F$, A_2^F versus the period of the incident wave for
 385 both the semi-analytical and the numerical models. Again, the agreement
 386 between the data sets is good at short periods and satisfactory at larger
 387 periods, due to the influence of the flap thickness, for which the numerical
 388 values are slightly larger than the semi-analytical ones. The same qualitative
 389 comments made above for the two-flap array can be repeated in full also for
 390 the three-flap array. Having validated the models, we shall employ the semi-
 391 analytical model to perform the parametric analysis of the system. The latter
 392 will reveal the existence of a near-resonant phenomenon for in-line arrays of
 393 OWSCs.

Figure 5: (a) Exciting torque magnitude in physical variables $|F'| = \rho g A' b'^3 |F|$ and (b) amplitude factor versus period of the incident wave calculated with the semi-analytical model and the numerical model for a three-flap system. The legend at the top right corner of (a) shows a sketch of the configuration (not in scale). Parameters are $w'_1 = w'_2 = w'_3 = 26$ m, $c'_1 = c'_2 = c'_3 = 4$ m and $h' = 13$ m. The gap is $s' = 30$ m between all flaps. Waves of amplitude $A'_I = 1$ m are normally incident on the farm. Due to linearity and symmetry of the system, $F'_1 = F'_3$ and $A^F_1 = A^F_3$.

3.2. Parametric analysis

A strong point of the analytical approach of §2.3 is that it easily allows to highlight the system dependence on its main parameters, i.e. the number of flaps and the flap spacing, at minimum computational cost.

3.2.1. Influence of number of flaps and near resonance

Figure 6 shows the behaviour of the exciting torque versus the period of the incident wave for several systems: a single flap in the open ocean [22], either flap in an array of two in-line devices, the central flap of an array of three in-line devices and any flap in an infinite in-line array [20]. In such layouts $w' = 26$ m, $s' = 30$ m and $c' = 4$ m for all the flaps, $h' = 13$ m and waves of amplitude $A'_I = 1$ m are normally incident. Note from figure 6 that the peak exciting torque for a finite array moves closer to that of an infinite array by increasing the number of elements. For an infinite array of OWSCs oscillating at unison, Renzi & Dias [20] already showed that the maximum exciting torque occurs at one of the resonant periods of the system transverse modes. The latter correspond to the natural sloshing modes of a channel having the same width as the array spatial period [19]. In a finite array, on the

Figure 6: Exciting torque magnitude in physical variables versus period of the incident wave for a single flap (solid line), either flap of a two-flap system (dashed line), the central flap of a three-flap system (dash-dotted line) and any flap in an infinite array (dotted line). In all layouts $w' = 26$ m, $c' = 4$ m, $s' = 30$ m for all flaps and $h' = 13$ m. Waves of amplitude $A'_l = 1$ m are normally incident on the systems. Values are calculated with the semi-analytical model of §2.3 for the finite arrays and with the semi-analytical model of Renzi & Dias [20] for the infinite array.

other hand, the increased energy leakage associated to the radial spreading of waves towards infinity attenuates such resonant behaviour (see also [39] for a similar mechanism in wave-structure interaction). As a result, the maximum torque for a finite array is lower than the corresponding infinite array value - the attenuation being more significant for less populated arrays - and is also shifted towards longer waves (see again figure 6). The occurrence of peaks in the exciting torque acting on the elements of a finite array at periods close to the transverse-mode resonance of an infinite array is a *near-resonant* mechanism. This phenomenon has similar effects - yet different nature [27] - compared with the near-trapping phenomenon observed in previous work [27, 30, 40], for which frequency dependent resonance yields peaks in the hydrodynamic actions on the elements of a stationary array of cylinders. Indeed, for a finite array of OWSCs the near-resonant mechanisms shown in figure 6 were never described before. Since for an OWSC the exciting torque F' is the power-generating action [19], the near-resonant behaviour of F' (see again figure 6) suggests potential for constructive interaction in a finite array, which is now investigated. Generally, given a wave farm of M identical WECs, an interaction factor can be defined as the ratio between the

total power P' extracted by the system and M times the power P'_s generated by a single device in the open ocean, i.e.

$$q = \frac{P'}{M P'_s}. \quad (42)$$

A value of $q < 1$ indicates that the array interactions have an overall negative impact on the absorbing power of the farm and corresponds to destructive interference; conversely $q > 1$ corresponds to constructive interference. Noting that q (42) hides the real amount of absorbed power by each device, Babarit [8] introduced a modified version of the interaction factor for an array of M identical elements:

$$q_m^{mod} = \frac{P'_m - P'_s}{\max_T P'_s}, \quad m = 1, \dots, M, \quad (43)$$

where P'_m is the power output by the m th device and the maximum must be taken over a selected period interval (e.g. 5 – 15 s in this paper). When $q_m^{mod} > 0$, interference effects increase the absorbed power by the m th element with respect to the isolated case. Figure 7(a) shows the behaviour of the interaction factor q (42) versus the period of the incident wave for the two-flap system, the three-flap system and the infinite array described above. As expected [3, 5, 8], the q factor shows regions of constructive ($q > 1$) and destructive ($q < 1$) interaction. Comparison of figure 6 and figure 7(a) shows that the maximum constructive interaction, corresponding to the largest q factor, is attained in all the systems when the relevant exciting torque is maximum, i.e. at near resonance. This implies that near-resonant interactions arising in a finite in-line array are beneficial to the efficiency of the system, when the flap width is comparable to the gap. In figure 7(b) the modified interaction factor (43) is plotted against the period of the incident waves, again for either flap of a two-flap system, the central flap of a three-flap system and any flap of an infinite array. Note from figure 7 that regions where $q > 1$ correspond to $q_m^{mod} > 0$. This means that for a wave farm of in-line OWSCs, constructive interference ($q > 1$) is usually accompanied by an actual increase of absorbed power by the single elements ($q_m^{mod} > 0$). This is a peculiar property of OWSCs and does not yield in general for systems of different converters (e.g. heaving or surging buoys, see [8]), confirming that the interference effects activating in a wave farm of in-line OWSCs at near resonance are actually beneficial to wave power extraction.

Figure 7: (a) Interaction factor (42) versus period of the incident wave for a two-flap system (dashed line), a three-flap system (dash-dotted line) and an infinite array of flaps oscillating at unison (dotted line). (b) Modified interaction factor q_m^{mod} for either flap of a two-flap system ($q_1^{mod} = q_2^{mod}$, dashed line), the central flap of a three-flap system (q_2^{mod} , dash-dotted line) and any flap of an infinite array (dotted line). The geometry is the same as in figure 6. Values are calculated with the semi-analytical model of §2.3 for the finite arrays and with the semi-analytical model of Renzi & Dias [20] for the infinite array.

3.2.2. Influence of flap spacing

In this section we investigate the parametric dependence of an in-line array of OWSCs on the flap spacing, with the semi-analytical model of §2.3. A system of two in-line flaps is considered, with $w'_1 = w'_2 = 26$ m, $c'_1 = c'_2 = 4$ m and $h' = 13$ m. Waves of amplitude $A'_I = 1$ m are normally incident on the flaps. Figure 8 shows the behaviour of the q factor versus the gap width s' for the two-flap system, respectively for $T' = 7$ s and $T' = 10$ s. Not surprisingly (see [3]), the plots of figure 8 show a sequence of constructive and destructive interference regions. Note also that the interference effects get weaker as the gap width s' becomes nearly three times larger than the flap width, i.e. $q \simeq 1$ as $s' \gtrsim 3w'$. Given the period T' of the incident wave, figure 8 also shows that it is possible to identify an optimum gap width $s'_{opt}(T')$ corresponding to the maximum interaction factor, i.e. to the strongest constructive interference. The optimum gap width for the two-flap array under analysis is reported in figure 8(b) together with the optimum gap width for an infinite array made by the same flaps, for both periods of the incident wave [20]. Note that in both cases s'_{opt} for the 2-flap array is smaller than s'_{opt} for the infinite array. This is clearly a consequence of the near-

Figure 8: (a) Interaction factor (42) versus gap width s' for a two-flap system and two different periods of the incident wave, calculated with the semi-analytical model. The system configuration is sketched at the top right corner (not in scale). Parameters are: $w'_1 = w'_2 = 26 \text{ m}$, $c'_1 = c'_2 = 4 \text{ m}$ and $h' = 13 \text{ m}$. Waves of amplitude $A'_I = 1 \text{ m}$ are normally incident on the system. (b) Values (in metres) of the optimum gap width for both periods of the incident wave, for a system of 2 in-line flaps and an infinite array. The latter are calculated with the model of Renzi & Dias [20].

478 resonant phenomenon discussed in §3.2.1. Since the interactions in a finite
479 array are weaker than in an infinite array, in the finite array configuration
480 the flaps need to be closer than they would need to be in the infinite array
481 in order to interfere constructively. Such considerations should influence the
482 designer's choice on the number of flaps and the optimal flap spacing during
483 the design process of a wave farm of OWSCs. In the following section, we
484 shall apply the numerical model of §2.4 to analyse a configuration of practical
485 engineering interest which cannot be studied with the analytical approach.

486 4. Application

487 Here we apply the numerical model of §2.4 to investigate a staggered array
488 configuration, resembling the one envisaged by APL for a wave farm of Oys-
489 ter WECs [41]. Figure 9 shows a sketch of the layout, where the two OWSCs
490 have the same dimensions as those of §3 and the waves are normally incident.
491 The PTO coefficient is again the same for both converters, i.e. $\eta'_1 = \eta'_2$ and -
492 since the analytical approach cannot be used to optimise the PTO for such
493 configuration - has been tuned in order to achieve the optimum power of an
494 isolated converter (see also [8] for a similar technique). However, this does

Figure 9: Geometry of an array made by 2 staggered OWSCs in normally incident waves. Values are in metres.

495 not imply loss of physical insight, since in this case we are mainly concerned
 496 about investigating the variation of the wave field and the hydrodynamic ac-
 497 tions on the flaps induced by the loss of symmetry in the system. Figure 10
 498 shows a snapshot of the free-surface elevation calculated with the numerical
 499 model for an array of two in-line (left panel) and two staggered (right panel)
 500 flaps, held fixed in incoming waves (scattering problem). When a crest is
 501 about to hit the array in the in-line system, a large difference in free-surface
 502 elevation (and hence in pressure) generates between the sides of the flaps, as
 503 shown in figure 10 (left panel). Here, if allowed to move both flaps would
 504 pitch towards the right pushed by a clockwise net torque. On the contrary, in
 505 a staggered array spatial separation along the direction of propagation of the
 506 waves makes it impossible for a single wave front to impinge on both flaps at
 507 the same time. As a consequence, the flaps are likely to experience different
 508 pressure jumps between their sides. This is clear in figure 10 (right panel),
 509 where the first flap undergoes a jump in pressure between its sides, while
 510 the second flap is all surrounded by a trough, which yields little pressure
 511 difference. If allowed to move, the front flap (1) would now pitch towards
 512 the right while the back flap (2) would remain nearly motionless. This new
 513 dynamics has clear effects on the exciting torque curves. Figure 11(a) shows
 514 the plot of the exciting torque versus the period of the incident waves for
 515 both flaps in the staggered array. Due to the loss of symmetry, the varia-
 516 tion of the exciting torque with respect to the wave period is less predictable
 517 than before. Note that a similar general result has also been obtained by
 518 Chatjigeorgiou [42] for a system of two staggered truncated elliptical cylin-
 519 ders, when moving from a symmetric to a non-symmetric configuration. For
 520 the staggered OWSCs of figure 9, the curve of the front flap peaks at a larger

Figure 10: Snapshot of the free-surface elevation for two flaps in (a) an in-line configuration, (b) the staggered configuration of figure 9. Both flaps are held fixed in incoming waves (scattering problem). In the plot, the thickness of the flaps has been exaggerated for easiness of reading. Waves of amplitude $A_I' = 1$ m and period $T' = 6$ s are normally incident on the system from the left. Values are calculated with the numerical model of §2.4.

Figure 11: (a) Exciting torque and (b) amplitude factor versus period of the incident waves for the staggered array of figure 9. The system configuration is sketched at the top right corner of (a) (not in scale). Waves of amplitude $A_I = 1$ m are normally incident on the system. Values are calculated with the numerical model of §2.4.

period than that of the back flap (see again figure 11a). This determines a reduction in synergy between the two OWSCs, each now working better at a different period than the other. This in turn rules out the concurrent occurrence of efficiency-enhancing near-resonant interactions at the same period for both flaps. Overall, those dynamics are expected to be detrimental for the efficiency of the farm. In order to quantify the reduction in efficiency, the amplification factors of both OWSCs are plotted in figure 11(b) against the period of the incident waves. Even though a certain loss of efficiency was predictable, comparison of figure 11(b) with figure 4(b) shows quite surprisingly that the amplification factors of the staggered system are almost halved with respect to the relevant values of the in-line system. Indeed such a large loss of efficiency is partially due to the usage of non-optimal PTO damping in the staggered configuration. Also, other parameters like the longitudinal and latitudinal spacing between the flaps could be varied, making it possible to increase the wave power output. Such analysis is not undertaken here and could be the topic of a different research project.

5. Conclusions

A three-dimensional potential flow theory has been developed for the hydrodynamic analysis of a finite array of OWSCs. A semi-analytical and a

540 fully numerical approach have been considered in order to solve the gov-
 541 erning equations of the system. Results of both models for systems of two
 542 and three in-line converters resembling the Oyster800TMOWSC developed by
 543 Aquamarine Power Limited are compared, showing good agreement between
 544 the two approaches. Parametric analysis is then undertaken with the semi-
 545 analytical model in order to gain a further insight on the system dynamics
 546 and investigate its dependence on the system main parameters. While in-
 547 vestigating the influence of the number of flaps on the array performance,
 548 a near-resonant phenomenon is identified. Because of near resonance, peaks
 549 in the exciting torque acting on the elements of a finite array manifest at
 550 periods close to the transverse-mode resonance of an infinite array [20]. Such
 551 near-resonant mechanism is responsible for the occurrence of constructive
 552 interference in the array (i.e. $q > 1$), which is accompanied by an effective
 553 increase of absorbed power by the single elements (i.e. $q_m^{mod} > 0$). This is
 554 a distinctive property of an array of OWSCs and does not yield in general
 555 for systems of different converters, like heaving or surging buoys, suggesting
 556 that the near-resonant effects activating in a wave farm of in-line OWSCs
 557 are actually beneficial to wave power extraction. Parametric analysis for an
 558 in-line array also shows that, given the period of the incident wave, there ex-
 559 ists an optimum gap width for which the constructive interaction among the
 560 flaps is maximum. Such result could be used as a preliminary design criterion
 561 for an array of OWSCs. The numerical model is then applied to analyse a
 562 configuration of practical engineering interest, i.e. an array of two staggered
 563 flaps, which cannot be studied with the analytical approach of 2.3. The case
 564 analysed here shows that loss of symmetry in the system can be detrimen-
 565 tal to the array efficiency. Such mechanism could be partially prevented by
 566 using an appropriate control of the PTO system and an optimisation of the
 567 geometric layout.

568 The analysis performed in this paper assumes monochromatic incident
 569 waves. In real seas, the presence of several spectral wave components would
 570 certainly modify the system behaviour. Nevertheless, the fundamental dy-
 571 namics would be still governed by the mechanisms discussed in this paper.
 572 In particular, in random seas the near-resonant behaviour of the array could
 573 be exploited by designing the flap spacing so that the resulting q -factor curve
 574 couples well with the wave spectrum of the incident sea. However, we expect
 575 wave interaction effects to be weaker in random waves than in monochro-
 576 matic waves, due to partial compensation of constructive and destructive
 577 effects over the whole spectrum (see for example [8]). Finally, a separate

analysis must be carried out in order to investigate the existence of motion-resonant modes for an in-line array of OWSCs. Motion resonance is a radiation phenomenon and is different from the near resonance described here, which is instead a scattering phenomenon. Motion resonance could occur for an articulated system of flaps at certain periods of the incident wave, for which the angular oscillations are maximised (perfect trapping for a system of flood gates in a channel has been described for the first time by Mei *et al.* [25]). Investigation is currently ongoing to determine whether motion-resonant modes can occur for a system of OWSCs and will be disclosed in the near future.

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Appendix A. Semi-analytical solution

In this section the system of equations (28)–(29) for the n th-mode 2D spatial potentials φ_n^D and φ_n^α (24) will be solved by extending the semi-analytical approach of Renzi & Dias [22] to a finite system of flaps. First consider the 2D Green function

$$G_n(x, y; \xi, \eta) = \frac{1}{4i} H_0^{(1)}(\kappa_n \rho), \quad (\text{A.1})$$

solution of the system

$$(\nabla^2 + \kappa_n^2) G_n = 0, \quad G_n \rightarrow \frac{\ln \rho}{2\pi} \text{ as } \rho \rightarrow 0,$$

where $\rho = \sqrt{(x - \xi)^2 + (y - \eta)^2}$, $(\xi, \eta) \in \mathbb{R}^2$ [19]. In (A.1), $H_0^{(1)}$ is the Hankel function of the first kind and 0th order, outgoing for large argument, while $\kappa_0 = k$ and the $\kappa_n = ik_n$ are still the solutions of the dispersion relation (26) for $n > 0$. Application of Green’s integral theorem [12] to a 2D surface

606 enclosed by a large circumference centred at the origin O and containing all
 607 flaps yields

$$\begin{Bmatrix} \varphi^D(x, y) \\ \varphi_n^\alpha(x, y) \end{Bmatrix} = \sum_{m=1}^M \int_{y_m^s}^{y_m^e} \begin{Bmatrix} \Delta\varphi_{nm}^D(\eta) \\ \Delta\varphi_{nm}^\alpha(\eta) \end{Bmatrix} G_{n,\xi}(x, y; \xi, \eta)|_{\xi=0} d\eta \quad (\text{A.2})$$

608 for either potential, where the notation is that of §2.3. In (A.2), the subscript
 609 after the vertical bar indicates the point at which the derivative must be
 610 calculated and

$$\begin{Bmatrix} \Delta\varphi_{nm}^D(y) \\ \Delta\varphi_{nm}^\alpha(y) \end{Bmatrix} = \begin{Bmatrix} \varphi_n^D(0^-, y) \\ \varphi_n^\alpha(0^-, y) \end{Bmatrix} - \begin{Bmatrix} \varphi_n^D(0^+, y) \\ \varphi_n^\alpha(0^+, y) \end{Bmatrix}, \quad y \in (y_m^s, y_m^e). \quad (\text{A.3})$$

611 The first line of (A.3) indicates the jump in the n th diffraction potential
 612 across the m th flap, when all the flaps are fixed. Similarly, the second line of
 613 (A.3) indicates the jump in the n -th radiation potential across the m th flap
 614 when the α th flap is moving and the remaining flaps are fixed. Note that if
 615 $M = 1$, (A.2) reduces to expression (A.1) of Renzi & Dias [22] for a single
 616 flap in the open ocean. On the other hand, if $M = \infty$ and assuming that all
 617 flaps move at unison, (A.2) reduces to expression (B7) of Renzi & Dias [19]
 618 for a flap in a channel, equivalent to an infinite array of flaps oscillating at
 619 unison in the open ocean [20]. Back to the finite array, now focus on one flap
 620 at a time, say the β th flap. Application of (29) to the β th flap together with
 621 the integral form (A.2) and (A.1) yields the following hypersingular integral
 622 equations for the jumps in potential across such flap:

$$\sum_{m=1}^M H. \int_{y_m^s}^{y_m^e} \begin{Bmatrix} \Delta\varphi_{nm}^D \\ \Delta\varphi_{nm}^\alpha \end{Bmatrix} (\eta) \frac{H_1^{(1)}(\kappa_n |y - \eta|)}{|y - \eta|} d\eta = \frac{4i}{\kappa_n} \begin{Bmatrix} A_I d_n \\ f_{n\alpha} \delta_{\alpha\beta} \end{Bmatrix}, \quad y \in (y_\beta^s, y_\beta^e). \quad (\text{A.4})$$

623 The latter must then be applied in succession to all flaps, $\beta = 1, \dots, M$ to
 624 ensure that the solution fully satisfies the b.c.'s (29). In (A.4), the symbol $H.$
 625 before the integral sign indicates a Hadamard finite-part integral [19, 20, 22],
 626 whose kernel is singular at $\eta = y$. In order to resolve this singularity, perform
 627 the following change of variables inside the integral of (A.4):

$$u_m = y_m^s + y_m^e, \quad \zeta = \frac{2\eta - u_m}{w_m}, \quad \begin{Bmatrix} \Delta\varphi_{nm}^D(\eta) \\ \Delta\varphi_{nm}^\alpha(\eta) \end{Bmatrix} = \begin{Bmatrix} P_{nm}(\zeta) \\ Q_{n\alpha m}(\zeta) \end{Bmatrix}, \quad (\text{A.5})$$

628 with $\zeta \in (-1, 1)$ and define

$$v_m(y) = \frac{2y - u_m}{w_m} = \frac{2y - y_m^s - y_m^e}{y_m^e - y_m^s}, \quad y \in (y_\beta^s, y_\beta^e), \quad (\text{A.6})$$

629 where m indicates any flap and β refers to the flap on which the b.c. (A.4)
 630 is being applied in succession. In (A.5) P_{nm} indicates the jump in the n th
 631 2D diffraction potential across the generic m th flap in the new system of
 632 variables. Similarly, $Q_{n\alpha m}$ denotes the jump in the n th 2D radiation potential
 633 across the m th flap when the α th flap is moving and all the other flaps are
 634 fixed. Now let $m = \gamma$ denote any but the β th flap, i.e. $\gamma \neq \beta$. Hence
 635 substituting (A.5) into the boundary condition (A.4) on flap β and separating
 636 the term $m = \beta$ in the summation (A.4) from all other terms $m = \gamma \neq \beta$
 637 yields

$$\begin{aligned} & H. \int_{-1}^1 \left\{ \begin{array}{c} P_{n\beta}(\zeta) \\ Q_{n\alpha\beta}(\zeta) \end{array} \right\} \frac{H_1^{(1)} \left(\kappa_n \frac{w_\beta}{2} |v_\beta(y) - \zeta| \right)}{|v_\beta(y) - \zeta|} d\zeta \\ & + \sum_{\gamma=1}^M \int_{-1}^1 \left\{ \begin{array}{c} P_{n\gamma}(\zeta) \\ Q_{n\alpha\gamma}(\zeta) \end{array} \right\} \frac{H_1^{(1)} \left(\kappa_n \frac{w_\gamma}{2} |v_\gamma(y) - \zeta| \right)}{|v_\gamma(y) - \zeta|} d\zeta \\ & = \frac{4i}{\kappa_n} \left\{ \begin{array}{c} A_I d_n \\ f_{n\alpha} \delta_{\alpha\beta} \end{array} \right\}, \quad y \in (y_\beta^s, y_\beta^e), \end{aligned} \quad (\text{A.7})$$

638 where $\sum'$ indicates that the sum must be evaluated for all $\gamma \neq \beta$. This
 639 separation allows to assess the singular behaviour of the kernels in (A.7).
 640 For $m = \beta$, (A.6) reveals that $v_\beta(y) \in (-1, 1)$. Hence the kernel of the
 641 first integral in (A.7) is singular when $\zeta = v_\beta(y) \in (-1, 1)$. On the other
 642 hand, for $m = \gamma \neq \beta$, (A.6) reveals that $v_\gamma(y) \notin (-1, 1)$, so that $\zeta \neq v_\gamma(y)$
 643 and the kernels of the integrals inside the summation of (A.7) are never
 644 singular. Hence the singularity is now confined in the first Hadamard finite-
 645 part integral of (A.7). The latter can be finally solved by following the general
 646 method devised by Renzi & Dias [19]. First expand the Hankel function inside
 647 the first integral of (A.7) according to §8.444 of Gradshteyn & Ryzhik [43]
 648 as

$$H_1^{(1)} \left(\kappa_n \frac{w_\beta}{2} |v_\beta(y) - \zeta| \right) = \frac{4}{i\pi} \frac{1}{\kappa_n w_\beta |v_\beta(y) - \zeta|} + R_n \left(\kappa_n \frac{w_\beta}{2} |v_\beta(y) - \zeta| \right), \quad (\text{A.8})$$

649 where

$$R_n(z) = J_1(z) \left[1 + \frac{2i}{\pi} \left(\ln \frac{z}{2} + \sigma \right) \right] - \frac{i}{\pi} \left[\frac{z}{2} + \sum_{j=2}^{+\infty} \frac{(-1)^{j+1} (z/2)^{2j-1}}{j!(j-1)!} \left(\frac{1}{j} + \sum_{q=1}^{j-1} \frac{2}{q} \right) \right],$$

650 J_1 is the Bessel function of first kind and first order and $\sigma = 0.577\,215\dots$
 651 the Euler constant. Then also expand the unknown jumps in potential as

$$\left\{ \begin{array}{c} P_{nm}(\zeta) \\ Q_{n\alpha m}(\zeta) \end{array} \right\} = (1 - \zeta^2)^{1/2} \sum_{p=0}^{\infty} \left\{ \begin{array}{c} A_I a_{nmp} \\ b_{n\alpha mp} \end{array} \right\} U_p(\zeta), \quad m = 1, \dots, M, \quad (\text{A.9})$$

652 where U_p is the Chebyshev polynomial of the second kind and order $p \in \mathbb{N}$,
 653 while the a_{nmp} and $b_{n\alpha mp}$ are unknown complex coefficients to be determined.
 654 Substituting (A.8) and the series expansions (A.9) into the hypersingular
 655 integral equations (A.7) for each $\beta = 1, \dots, M$ in succession and developing
 656 some lengthy algebra [see 19, for a similar procedure] finally yields

$$\sum_{p=0}^{\infty} \left\{ \left\{ \begin{array}{c} a_{n\beta p} \\ b_{n\alpha\beta p} \end{array} \right\} C_{n\beta p}(v_\beta) + \sum_{\gamma=1}^M \left\{ \begin{array}{c} a_{n\gamma p} \\ b_{n\alpha\gamma p} \end{array} \right\} D_{n\beta\gamma p}(v_\beta) \right\} = -\pi w_\beta \left\{ \begin{array}{c} d_n \\ f_{n\alpha} \delta_{\alpha\beta} \end{array} \right\}, \quad v_\beta \in (-1, 1), \quad \beta = 1, \dots, M, \quad (\text{A.10})$$

657 where the orthogonality property of the Chebyshev polynomials has also been
 658 used [see 19]. In (A.10)

$$C_{n\beta p}(v_\beta) = -\pi(p+1)U_p(v_\beta) + \frac{i\pi\kappa_n w_\beta}{4} \times \int_{-1}^1 \frac{(1 - \zeta^2)^{1/2} U_p(\zeta) R_n\left(\frac{1}{2}\kappa_n w_\beta |v_\beta - \zeta|\right)}{|v_\beta - \zeta|} d\zeta \quad (\text{A.11})$$

659 and

$$D_{n\beta\gamma p}(v_\beta) = \frac{i\pi\kappa_n w_\beta w_\gamma}{4} \int_{-1}^1 (1 - \zeta^2)^{1/2} U_p(\zeta) \times \frac{H_1^{(1)}\left(\frac{1}{2}\kappa_n |v_\beta w_\beta + u_\beta - u_\gamma - w_\gamma \zeta|\right)}{|v_\beta w_\beta + u_\beta - u_\gamma - w_\gamma \zeta|} d\zeta \quad (\text{A.12})$$

are complex coefficients which can be evaluated numerically. Expression (A.10) defines two different systems of linear equations for given n and α , both valid for any $v_\beta \in (-1, 1)$. The unknowns are the complex coefficients a_{nmp} and $b_{n\alpha mp}$ of the series expansions (A.9). Note that in (A.10) the term $C_{n\beta p}$ indicates the n th mode, p th order contribution of the β th-flap on itself, while $D_{n\beta\gamma p}$ indicates the n th mode, p th order contribution of the γ th-flap on the β th-flap. As discussed by Renzi & Dias [19, 20, 22], systems like (A.10) can be solved numerically with a collocation scheme by truncating the external summation to a finite integer $p = P$ and evaluating the coefficients (A.11) and (A.12) at the specific points

$$v_\beta = v_q = \frac{\cos(2q+1)\pi}{2P+2}, \quad q = 0, 1, \dots, P,$$

660 for any β , which correspond to the zeros of the Chebyshev polynomials of
661 the first kind. As a consequence, (A.10) transforms into

$$\begin{aligned} & \sum_{p=0}^P \left\{ \begin{Bmatrix} a_{n\beta p} \\ b_{n\alpha\beta p} \end{Bmatrix} C_{n\beta p}(v_q) + \sum_{\gamma=1}^M \begin{Bmatrix} a_{n\gamma p} \\ b_{n\alpha\gamma p} \end{Bmatrix} D_{n\beta\gamma p}(v_q) \right\} \\ & = -\pi w_\beta \begin{Bmatrix} d_n \\ f_{n\alpha} \delta_{\alpha\beta} \end{Bmatrix}, \quad q = 0, \dots, P; \quad \beta = 1, \dots, M. \end{aligned} \quad (\text{A.13})$$

662 The first system (upper line of A.13) yields the series coefficients a_{nmp} for
663 the jump in potential of the diffraction problem P_{nm} (A.9), for each modal
664 order n and body m . The second system (lower line of A.13) yields the
665 series coefficients $b_{n\alpha mp}$ for the jump in potential of the radiation problem
666 $Q_{n\alpha m}$ (A.9) for each mode n and body m , when the α th-body is moving
667 and the others are at rest. Now note from (30) that $d_n = 0$ for $n > 0$.
668 As a consequence, the diffraction system (upper line of A.13) becomes the
669 homogeneous form of the radiation system (lower line of A.13) for $n > 0$.
670 Hence in order the solution of the radiation problem to be unique, it must
671 be $a_{n\beta p} = 0$, $n > 0$. Now, the vertical eigenmodes with $n > 0$ correspond to
672 imaginary solutions of the dispersion relation $\omega^2 = k \tanh kh$, i.e. such terms
673 represent evanescent modes [12]. Hence the fact that $a_{n\beta p} = 0$ for $n > 0$
674 physically implies that the incident wave is not able to excite evanescent
675 modes when interacting with the system of in-line fixed plates [see also 19,
676 20, 22, for a similar behaviour].

677 Now given the most generic configuration with M bodies and considering
678 N vertical eigenmodes in the radiation problem, the above method requires

the solution of $M \times (P + 1)$ linear equations for the diffraction problem and of $N \times M \times M \times (P + 1)$ linear equations for the radiation problem. However, occurrence of symmetries in the geometric layout of the system can strongly simplify (A.13) and reduce the number of unknowns (this is the case, for example, of a series of in-line identical flaps). Incidentally, consider the case when there is only one body of width w , i.e. $M = 1$. Here there are no mutual contributions and the $D_{n\beta\gamma p} = 0$ in (A.13). As a consequence, the latter strongly simplifies to

$$\sum_{p=0}^{\infty} \left\{ \begin{array}{c} a_{n1p} \\ b_{n11p} \end{array} \right\} C_{n1p}(v_q) = -\pi w \left\{ \begin{array}{c} d_n \\ f_{n1} \end{array} \right\}, \quad q = 0, \dots, P, \quad (\text{A.14})$$

which corresponds exactly to the system (A.9) of Renzi & Dias [22] for a single flap in the open ocean. For a general number of flaps, (A.13) gives the complex coefficients of the series expansions in (A.9), from which the jumps in diffraction and radiation potentials can be calculated according to (A.5). Substitution of the jumps in potential into Green's formula (A.2) and then into the decomposition (24) finally yields the sought complex spatial potentials ϕ^D (32) and $\phi^{(\alpha)}$ (33).

Appendix B. Reciprocity relation

Consider the complex 2D radiation potentials $\varphi_n^{\alpha_1}$ and $\varphi_n^{\alpha_2}$, with $\alpha_1 \neq \alpha_2$. As shown in §2.3, both potentials must satisfy the Helmholtz equation

$$(\nabla^2 + \kappa_n^2) \left\{ \begin{array}{c} \varphi_n^{\alpha_1} \\ \varphi_n^{\alpha_2} \end{array} \right\} = 0, \quad (\text{B.1})$$

the kinematic boundary conditions

$$\left\{ \begin{array}{c} \varphi_{n,x}^{\alpha_1} \\ \varphi_{n,x}^{\alpha_2} \end{array} \right\} = \left\{ \begin{array}{c} f_{n\alpha_1} \delta_{\alpha_1 m} \\ f_{n\alpha_2} \delta_{\alpha_2 m} \end{array} \right\} \quad \text{on } \partial\mathcal{L}_m, \quad m = 1, \dots, M, \quad (\text{B.2})$$

and be outgoing at large distance from the array. Application of Green's integral theorem in a large circular region enclosing all the flaps to $\varphi_n^{\alpha_1}$ and $\varphi_n^{\alpha_2}$ together with (B.1)–(B.2) yields the relation

$$f_{n\alpha_2} \int_{y_{\alpha_2}^s}^{y_{\alpha_2}^e} \Delta \varphi_{n\alpha_2}^{\alpha_1}(y) dy = f_{n\alpha_1} \int_{y_{\alpha_1}^s}^{y_{\alpha_1}^e} \Delta \varphi_{n\alpha_1}^{\alpha_2}(y) dy. \quad (\text{B.3})$$

701 Substituting the change of variables (A.5) and the series expansion (A.9) into
702 (B.3), renaming $\alpha_1 = \alpha$ and $\alpha_2 = \beta$ and developing the algebra finally gives

$$w_\beta f_{n\beta} b_{n\alpha\beta 0} = w_\alpha f_{n\alpha} b_{n\beta\alpha 0}, \quad (\text{B.4})$$

703 which is a reciprocity relation between the 0th-order coefficients of the Cheby-
704 shev expansion (A.9) for the radiation problem.

705 **Appendix C. Numerical procedure**

706 Since the mathematical problem is hyperbolic, it is solved by means of
707 a time-marching numerical scheme. In order to correctly reproduce the
708 wave field, the minimum mesh element size must be at least one tenth of
709 the simulated wave length. It is important for the efficiency of the time-
710 stepping algorithm to assemble the time independent matrices only once.
711 The Generalized- α method is used. This is a one-step implicit method for
712 solving the transient problem, which attempts to increase the amount of
713 numerical damping present without degrading the order of accuracy. Such
714 method allows for high frequency energy dissipation and second order ac-
715 curacy. The method was first developed for the second-order equations in
716 structural mechanics [44, 45]. This linear system solver uses the restarted
717 generalized minimum residual (GMRES) with incomplete LU preconditioner
718 for fast convergence. The latter is an iterative method for general linear sys-
719 tems of the form $Ax = b$. For models with many degrees of freedom (roughly,
720 more than 100,000 to 1,000,000 depending on available computer memory),
721 the direct solvers typically require too much memory. In those cases, the
722 more memory-efficient iterative solvers like GMRES perform better. How-
723 ever, iterative solvers are less stable than direct solvers in that they do not
724 always converge (i.e. arrive at a solution). For the simulations carried out in
725 this paper, the value of the number of iterations is 50. A larger restart value
726 increases the robustness of the interactive procedure, but it also increases
727 memory use and computational time. For large problems, the computational
728 cost is often very large to produce a preconditioner of such a high quality
729 that the termination criteria are fulfilled for a small number of iterations and
730 for a small restart value. For those problems it is often advantageous to set
731 up a preconditioner with a somewhat lesser quality and instead increase the
732 restart value or iterate more steps. Doing so typically increases the condition
733 number for the preconditioned system, so an increase in the error-estimate

734 factor might be needed as well. The incomplete LU preconditioner performs
 735 an incomplete LU factorization of the system matrix A . That is, it drops
 736 small elements during the column-oriented Gaussian elimination. Thus it
 737 saves memory, and the resulting factors L and U are approximate. The
 738 numerical solution is obtained using the software COMSOL Multiphysics.

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