Unification of Reaction Metrics for Green Chemistry: Applications to Reaction Analysis

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Supplementary Material

Derivation of equation (7)

$$\begin{aligned}
 E_m &= \frac{qy + w}{py} \\
 &= \frac{qy + ax + bx - qy - py}{py}, \text{ since } w = ax + bx - qy - py \\
 &= \frac{ax + bx}{py} - 1 \\
 &= \frac{a + b}{p\varepsilon} - 1, \text{ since } \varepsilon = \frac{y}{x} \\
 &= \frac{1}{\varepsilon(AE)} - 1, \text{ since } AE = \frac{p}{a + b} \\
 &= \frac{1}{RME} - 1, \text{ since } RME = \varepsilon(AE)
 \end{aligned}$$

Derivation of equation (17)

Given a general reaction

including masses of reaction solvent, s , catalyst, c , and post-reaction solvents and other materials, ω , we have the law of conservation of mass given by

$$ax + bx + c + s + \omega = qy + py'' + w + (py - py'') + c + s + \omega$$

where x is the number of moles of reagents A and B, y is the number of moles of products P and Q formed in the reaction, y'' is the number of moles of product P collected after extraction and purification procedures, and a , b , q , and p are the molecular weights of chemical species A, B, Q, and P. The parameter w is the mass of unreacted starting materials.

Let $\epsilon = \frac{y}{x}$ be the yield of reaction before work-up and $\epsilon' = \frac{y''}{y}$ be the yield of

collected product P after work-up and purification. Then, the overall yield of reaction

product P after all operations is $\langle \epsilon \rangle = \left(\frac{y}{x} \right) \left(\frac{y''}{y} \right) = \frac{y''}{x}$.

The environmental impact factor based on mass assuming that the waste is composed of mass of byproduct Q, mass of unreacted starting materials, mass of product lost during the post-reaction operations, and masses of catalyst, reaction solvent, and all post-reaction materials used is given by

$$E_m = \frac{qy + w + (py - py'') + c + s + \omega}{py''}$$

From the law of conservation of mass,

$$w = ax + bx - qy - py$$

Substitution leads to

$$\begin{aligned} E_m &= \frac{qy + ax + bx - qy - py + (py - py'') + c + s + \omega}{py''} = \frac{ax + bx - py'' + c + s + \omega}{py''} \\ &= \frac{ax + bx + c + s + \omega}{py''} - 1 \end{aligned}$$

Since $\langle \epsilon \rangle = \frac{y''}{x}$ we have

$$E_m = \frac{a + b + \left(\frac{c + s + \omega}{x} \right)}{p \langle \epsilon \rangle} - 1$$

Since the atom economy for the reaction is $AE = \frac{p}{a + b}$ then

$$\begin{aligned} E_m &= \frac{1 + \left(\frac{c + s + \omega}{x} \right) \left(\frac{1}{a + b} \right)}{AE \langle \epsilon \rangle} - 1 = \frac{1}{\frac{AE \langle \epsilon \rangle}{1 + \left(\frac{c + s + \omega}{x} \right) \left(\frac{1}{a + b} \right)}} - 1 \\ &= \frac{1}{\frac{AE \langle \epsilon \rangle}{1 + \left(\frac{c + s + \omega}{y''} \right) \left(\frac{\langle \epsilon \rangle}{a + b} \right)}} - 1 = \frac{1}{\frac{AE \langle \epsilon \rangle}{1 + \left(\frac{c + s + \omega}{y''} \right) \left(\frac{\langle \epsilon \rangle AE}{p} \right)}} - 1 \\ &= \frac{1}{\frac{AE \langle \epsilon \rangle p y''}{p y'' + (c + s + \omega) \langle \epsilon \rangle AE}} - 1 \end{aligned}$$

By analogy with equation (7) the reaction mass efficiency is then

$$RME = \langle \epsilon \rangle (AE) \left[\frac{p y''}{p y'' + \langle \epsilon \rangle (AE) (c + s + \omega)} \right] = \frac{1}{1 + E_m}$$

Derivation of equation (31)

We start from the fundamental definition of environmental impact factor based on mass for the entire linear sequence

$$E_{overall} = \frac{\sum_{j=1}^n \overline{w}_j}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} = \frac{\sum_{j=1}^n (w_j + q_j y_j + c_j + s_j + \omega_j)}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)},$$

and for each step j in the linear sequence we define the law of conservation of mass

$$a_j x_j + b_j x_j + c_j + s_j + \omega_j = q_j y_j + p_j y_j + w_j + c_j + s_j + \omega_j,$$

and the atom economy $(AE)_j = \frac{p_j}{a_j + b_j}$. The reaction yield for the first reaction

$$\epsilon_1 = \frac{y_1}{x} \quad (x_1 = x) \text{ and that for subsequent reactions is } \epsilon_j = \frac{y_j}{y_{j-1}} \quad (j = 2, 3, \dots, n). \text{ All}$$

variables are defined by analogy with the derivations of equations (7) and (17) and n is the total number of steps. Then, the waste w_j is $w_j = a_j x_j + b_j x_j - q_j y_j - p_j y_j$ where $x_1 = x$, $x_2 = y_1$, $x_2 = y_3$, etc. Substitution in $E_{overall}$ and following the same sequence of manipulations as for the derivations of equations (7) and (17) we have

$$\begin{aligned} E_{overall} &= \frac{\sum_{j=1}^n (a_j x_j + b_j x_j - q_j y_j - p_j y_j + q_j y_j + c_j + s_j + \omega_j)}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} \\ &= \frac{\sum_{j=1}^n (x_j (a_j + b_j) - p_j x_j \epsilon_j + c_j + s_j + \omega_j)}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} \end{aligned}$$

$$= \frac{\sum_{j=1}^n \left(x_j p_j \left(\frac{1}{(AE)_j} - \epsilon_j \right) + c_j + s_j + \omega_j \right)}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)}$$

The first term in the sum is

$$x_1 p_1 \left(\frac{1}{(AE)_1} - \epsilon_1 \right) + c_1 + s_1 + \omega_1 = x p_1 \left(\frac{1}{(AE)_1} - \epsilon_1 \right) + c_1 + s_1 + \omega_1$$

and the second term is

$$\begin{aligned} x_2 p_2 \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + c_2 + s_2 + \omega_2 &= y_1 p_2 \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + c_2 + s_2 + \omega_2 \\ &= x \epsilon_1 p_2 \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + c_2 + s_2 + \omega_2 \end{aligned}$$

The last term is

$$\begin{aligned} x_n p_n \left(\frac{1}{(AE)_n} - \epsilon_n \right) + c_n + s_n + \omega_n &= y_{n-1} p_n \left(\frac{1}{(AE)_n} - \epsilon_n \right) + c_n + s_n + \omega_n \\ &= x \epsilon_1 \epsilon_2 \dots \epsilon_{n-1} p_n \left(\frac{1}{(AE)_n} - \epsilon_n \right) + c_n + s_n + \omega_n \end{aligned}$$

Generally, for the jth term (j = 2, 3, ..., n)

$$x \epsilon_1 \epsilon_2 \dots \epsilon_{j-1} p_j \left(\frac{1}{(AE)_j} - \epsilon_j \right) + c_j + s_j + \omega_j$$

Dividing all terms by $p_n x \left(\prod_{j=1}^n \epsilon_j \right)$ and summing yields

$$E_{overall} =$$

$$\frac{1}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} \sum_{j=1}^n (c_j + s_j + \omega_j) +$$

$$\frac{p_1}{p_n \left(\prod_{j=1}^n \epsilon_j \right)} \left(\frac{1}{(AE)_1} - \epsilon_1 \right) + \frac{\epsilon_1 p_2}{p_n \left(\prod_{j=1}^n \epsilon_j \right)} \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + \frac{\epsilon_1 \epsilon_2 p_3}{p_n \left(\prod_{j=1}^n \epsilon_j \right)} \left(\frac{1}{(AE)_3} - \epsilon_3 \right)$$

$$+ \dots + \left(\frac{1}{\epsilon_n} \right) \left(\frac{1}{(AE)_n} - \epsilon_n \right)$$

$$= \frac{1}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} \sum_{j=1}^n (c_j + s_j + \omega_j) +$$

$$\frac{p_1}{p_n \left(\prod_{j=1}^n \epsilon_j \right)} \left(\frac{1}{(AE)_1} - \epsilon_1 \right) + \frac{p_2}{p_n \left(\prod_{j=2}^n \epsilon_j \right)} \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + \frac{p_3}{p_n \left(\prod_{j=3}^n \epsilon_j \right)} \left(\frac{1}{(AE)_3} - \epsilon_3 \right)$$

$$+ \dots + \left(\frac{1}{\epsilon_n (AE)_n} \right) - 1$$

or

$$E_{overall} = \frac{1}{RME_{overall}} - 1$$

where

$$\frac{1}{RME_{overall}} = \frac{1}{p_n x \left(\prod_{j=1}^n \epsilon_j \right)} \sum_{j=1}^n (c_j + s_j + \omega_j) +$$

$$\frac{p_1}{p_n \left(\prod_{j=1}^n \epsilon_j \right)} \left(\frac{1}{(AE)_1} - \epsilon_1 \right) + \frac{p_2}{p_n \left(\prod_{j=2}^n \epsilon_j \right)} \left(\frac{1}{(AE)_2} - \epsilon_2 \right) + \frac{p_3}{p_n \left(\prod_{j=3}^n \epsilon_j \right)} \left(\frac{1}{(AE)_3} - \epsilon_3 \right)$$

$$+ \dots + \left(\frac{1}{\epsilon_n (AE)_n} \right)$$

Woodward-Rabe Synthesis of Quinine

Reaction 1

Reaction 2

Reaction 3

Reaction 4

Reaction 5

Reaction 6 Reaction 7 Reaction 8 Reaction 9 Reaction 10

Reaction 11

Reaction 12

Reaction 13

Reaction 14

Reaction 15 Reaction 16 Reaction 17 Reaction 18

Reaction 19

Stork Synthesis of Quinine

Reaction 1

Reaction 2

Reaction 3

Reaction 4

Reaction 5

Reaction 6

Reaction 7

Reaction 8

Reaction 9

Reaction 10

Reaction 11

Reaction 12

Reaction 13

Reaction 14

Reaction 15

Reaction 16

Reaction 17

Derivation of Equation (44)

After cycle 1 the RME with respect to the desired (-) alcohol product is

$$RME_1^{(-)} = \frac{172x(1-\lambda_1)\lambda_2}{x[16+27.5\lambda_1+172]+\omega}$$

In cycle 2 we have for the oxidation step:

and for the reduction step:

After cycle 2 the cumulative RME with respect to the (-) alcohol is

$$\begin{aligned}
 RME_2^{(-)} &= \frac{172x(1-\lambda_1)\lambda_2 + 172x\lambda_1(1-\lambda_1)\lambda_2}{x[16 + 27.5\lambda_1 + 172] + x[16\lambda_1 + 27.5\lambda_1^2] + 2\omega} \\
 &= \frac{172x(1-\lambda_1)(1+\lambda_1)\lambda_2}{x[16(1+\lambda_1) + 27.5\lambda_1(1+\lambda_1) + 172] + 2\omega} \\
 &= \frac{172x(1-\lambda_1)(1+\lambda_1)\lambda_2}{x[(16 + 27.5\lambda_1)(1+\lambda_1) + 172] + 2\omega}
 \end{aligned}$$

Similarly, after cycle 3

$$RME_3^{(-)} = \frac{172x(1-\lambda_1)(1+\lambda_1+\lambda_1^2)\lambda_2}{x[(16+27.5\lambda_1)(1+\lambda_1+\lambda_1^2)+172]+3\omega}$$

Generally, after cycle m

$$RME_m^{(-)} = \frac{172x(1-\lambda_1)(1+\lambda_1+\lambda_1^2+\dots\lambda_1^{m-1})\lambda_2}{x[(16+27.5\lambda_1)(1+\lambda_1+\lambda_1^2+\dots\lambda_1^{m-1})+172]+m\omega}$$

Now,

$$1+\lambda_1+\lambda_1^2+\dots\lambda_1^{m-1} = \frac{1-\lambda_1^m}{1-\lambda_1} \text{ for } 0 < \lambda_1 < 1$$

Therefore,

$$\begin{aligned} RME_m^{(-)} &= \frac{172x(1-\lambda_1)\left(\frac{1-\lambda_1^m}{1-\lambda_1}\right)\lambda_2}{x\left[(16+27.5\lambda_1)\left(\frac{1-\lambda_1^m}{1-\lambda_1}\right)+172\right]+m\omega} \\ &= \frac{172x(1-\lambda_1)(1-\lambda_1^m)\lambda_2}{x[(16+27.5\lambda_1)(1-\lambda_1^m)+172(1-\lambda_1)]+m\omega(1-\lambda_1)} \end{aligned}$$

$$RME_m^{(-)} = \frac{172x(1-\lambda_1)(1-\lambda_1^m)\lambda_2}{x[(16+27.5\lambda_1)(1-\lambda_1^m)+172(1-\lambda_1)]+m(1-\lambda_1)\omega}$$

which is equation (44).