Online Supplement for

“Understanding and managing product line complexity: Applying sensitivity analysis to a large-scale MILP model to price and schedule new customer orders”

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Approximation of the capacity loss function and derivation of probe constraints

We first discuss how to represent the capacity loss due to probe movements in the detailed scheduling problem. The capacity loss due to probe movements is a step function of $p_t - p_{t-1}$ or $q_t - q_{t-1}$, which are distances moved by the probes on Lanes A or B, respectively. Applying the transformation by Schoomer (1964) to the step function, we represent capacity loss as a function of step size and add appropriate constraints. However, we need S binary variables to represent the step function in each slot (S represents the number of steps of the step function). The number of such binary variables can be very large. For instance, we need at least 40×3 binary variables in a scheduling problem of 40 products and three probe movement ranges. To reduce the problem size (the number of decision variables and the number of constraints), we use a piece-wise linear function to approximate the capacity losses due to probe movements:

$$\theta(\Delta p) = \max_{s=1, \dots, S} \{\psi_s + \lambda_s |\Delta p|\}, \quad (\text{A.1})$$

where S is the number of knots of the piece-wise linear function. This function is convex in the probe movement size Δp . The capacity losses on Lanes A and B are given by $\theta(|p_t - p_{t-1}|)$ and $\theta(|q_t - q_{t-1}|)$, respectively. Since $\theta(|p_t - p_{t-1}|)$ and $\theta(|q_t - q_{t-1}|)$ are piece-wise linear functions in $|p_t - p_{t-1}|$ and $|q_t - q_{t-1}|$, respectively, Eq. (A.1) is equivalent to:

$$\psi_s + \lambda_s |p_t - p_{t-1}| \leq v_t, \quad s = 1, \dots, S, \text{ for } t = 1, \dots, N, \text{ and}$$

$$\psi_s + \lambda_s |q_t - q_{t-1}| \leq v_t, \quad s = 1, \dots, S, \text{ for } t = 1, \dots, N.$$

The above two constraints can be rewritten as,

$$-(v_t - \psi_s) \leq \lambda_s (p_t - p_{t-1}) \leq v_t - \psi_s, \quad s = 1, \dots, S, \text{ for } t = 1, \dots, N, \text{ and} \quad (\text{A.2})$$

$$-(v_t - \psi_s) \leq \lambda_s (q_t - q_{t-1}) \leq v_t - \psi_s, \quad s = 1, \dots, S, \text{ for } t = 1, \dots, N. \quad (\text{A.3})$$

We next derive constraints on the probe movements. If the products to be produced in slot t have a wider color band than those produced in slot $t - 1$, the required color band widths are obtained by extending the probes. The distance between the minimum position of the left-side knife and the position of the left probe must be greater than the width of color band:

$$\sum_{k=1}^{N_p} CB_k^A z_{kt} \leq p_t - E, \text{ for } t = 1, \dots, N. \quad (\text{A.4})$$

Similarly, for the right side:

$$\sum_{k=1}^{N_p} CB_k^B z_{kt} \leq q_t - E, \text{ for } t = 1, \dots, N. \quad (\text{A.5})$$

If the required color band width of a product is smaller than that of the product produced in the previous slot, the required width is obtained by trimming the edge of gradient band. The probe position, knife position, and gradient band width must satisfy the following constraints,

$$\sum_{k=1}^{N_p} CB_k^A z_{kt} + l_t = p_t, \text{ for } t = 1, \dots, N, \text{ and} \quad (\text{A.6})$$

$$\sum_{k=1}^{N_p} CB_k^B z_{kt} + r_t = q_t, \text{ for } t = 1, \dots, N. \quad (\text{A.7})$$

The positions of the probes on Lanes A and B must be within a certain range, which are restricted by,

$$\underline{G} \leq p_t \leq \bar{G}, \text{ for } t = 1, \dots, N, \text{ and} \quad (\text{A.8})$$

$$\underline{G} \leq q_t \leq \bar{G}, \text{ for } t = 1, \dots, N. \quad (\text{A.9})$$

The gradient band width can also be obtained by retracting the probes if the required band width is much smaller than that produced in the previous slot. However, the distance between the probe and the closest edge cannot be F cm greater than the required width of the gradient band due to transmission issues. The probe position must also satisfy the following constraints:

$$p_t \leq \sum_{k=1}^{N_p} (CB_k^A + F) z_{kt}, \text{ for } t = 1, \dots, N, \text{ and} \quad (\text{A.10})$$

$$q_t \leq \sum_{k=1}^{N_p} (CB_k^B + F) z_{kt}, \text{ for } t = 1, \dots, N. \quad (\text{A.11})$$

If cutting pattern k is produced in slot t , $z_{kt} = 1$, and the right-hand side of (A.10) equals $F + CB_k^A$, which is the gradient band width plus the maximum allowable distance that the probe can go beyond the gradient band width. Hence, the right-hand side provides the bound of the probe position on Lane A. If no product is assigned on Lane A in cutting pattern k , we set $CB_k^A = \bar{G} - F$. Substituting for CB_k^A and setting $z_{kt} = 1$ in (A.10), we have $p_t \leq \bar{G}$. Hence, the right-hand side is an upper bound on the distance that the probe on Lane A can travel. Constraint (A.11) can be similarly explained.

Additional Reference

Schooner, B.A. (1964) The incorporation of step functions and ramp functions into a linear program model. *Operations Research*, 12(5), 773 – 774.