This item was submitted to [Loughborough's Research Repository](#) by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

Undergraduate mathematics teaching in first year lectures: can it be responsive to student learning needs?

PLEASE CITE THE PUBLISHED VERSION

<https://doi.org/10.1007/s40753-020-00111-y>

PUBLISHER

Springer

VERSION

AM (Accepted Manuscript)

PUBLISHER STATEMENT

This is a post-peer-review, pre-copyedit version of an article published in International Journal of Research in Undergraduate Mathematics Education. The final authenticated version is available online at:
<https://doi.org/10.1007/s40753-020-00111-y>.

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Petropoulou, Georgia, Barbara Jaworski, Despina Potari, and Theodossios Zachariades. 2020.
"Undergraduate Mathematics Teaching in First Year Lectures: Can It Be Responsive to Student Learning Needs?". Loughborough University. <https://hdl.handle.net/2134/12631322.v1>.

UNDERGRADUATE MATHEMATICS TEACHING IN FIRST YEAR LECTURES: CAN IT BE RESPONSIVE TO STUDENT LEARNING NEEDS?

Georgia Petropoulou¹, Barbara Jaworski², Despina Potari¹ and Theodossios Zachariades¹

¹ National and Kapodistrian University of Athens,

Abstract

This paper investigates mathematics teaching to first year students of two mathematics departments in the context of large-group lectures. It goes beyond what may be seen as implicit in this context by focussing on the links between two lecturers' thinking about students' learning of advanced mathematics and their practice. Data includes lecture observations and reflective discussions with the lecturers. Three layers of data analysis are drawn on Activity Theory, the Teaching Triad, and, grounded analytical approaches. Findings suggest that the formation of teaching goals is motivated by aspects of sensitivity to student's participation and that the complementarities among these aspects are important qualitative characteristics of teaching in this context.

Key words: teachers' thinking, teaching goals, student learning needs, Teaching Triad

Introduction

Undergraduate mathematics teaching is a foreground research object (Winsløw, Gueudet, Hochmuth & Nardi 2018) as it 'matters' for students' mathematical learning (Hiebert & Grouws 2007). In the case of teaching to large groups of exclusively mathematics students, what is at stake is generations of students' enculturation into advanced mathematical practices (Artigue, Batanero & Kent 2007). During their university studies, these students - some of whom will go on to become mathematics teachers themselves - are enculturated into both the practice of doing mathematics and also of teaching it. For Winsløw & Grønbaek (2014), university institutions must particularly take into account the needs of these students. Arguably, it is important to explore how this enculturation can be achieved.

The predominant mode of large-group teaching is the lecture, which is also the most common form of teaching internationally (Paoletti et al. 2018). Although teaching in a lecture form has received particular criticism (e.g. Biggs 2003), research has shown that lecturing is not a unified teaching paradigm and it therefore needs - and is currently receiving - further exploration (e.g. Weber 2004; Speer, Smith & Horvath 2010; Weinberg, Wiesner & Fukawa-Connelly 2016). What differs in the enactment of teaching in lectures has been discussed and attributed to cultural differences among countries, to differences in teaching experience or to individual variations among teachers, such as different individual views of making mistakes (Artemeva & Fox 2011; Pinto 2013). Although research on lecture teaching and on the factors that frame it is developing and some studies highlight students' alienation from the lecture content (e.g. Lew, Fukawa-Connelly, Mejía-Ramos & Weber 2016; Paoletti et al. 2018), research in how this teaching can take into account the students and their needs is still in its infancy. In our study we attempt to investigate how mathematics teaching

of large groups can be linked to the students who are being addressed. The aim is to identify different characteristics of teaching to large groups of students that is responsive to their learning needs. This is a central issue for mathematics education researchers (Hiebert & Grouws 2007).

Student learning needs have been under scrutiny by mathematics education researchers. For example, cognitive difficulties with advanced mathematical concepts have been reported in the works of Weber (e.g. 2001) and Tall (e.g. 2008) and affective/social needs, such as the need of social belonging in the new academic environment, have been discussed in the works of Hannula (2006) and Freeman, Anderman & Jensen (2007). Nardi, Jaworski & Hegedus wrote in 2005: ‘despite an often subtle recognition and analysis of students’ difficulties, research on the teaching and learning of undergraduate mathematics *only rarely probes into what teachers’ perceptions are of these difficulties*’ (p. 294, our emphasis). Up to now, studies exist in smaller classrooms that do probe into teachers’ engaging with students thinking (e.g., Wagner, Speer, & Rosa 2007; Speer & Wagner 2009) and we, as a research community, begin to gain insights into how teachers engage with students’ thinking in the context of small classrooms. As far as this context is concerned, interaction is the prevailing norm. However, in large group lectures, where teachers cannot interact with each individual student, the ways teachers think about student learning needs can be crucial and warrants additional literature. To this end, we see *teachers’ thinking about student learning needs* as a lens for investigating how large-scale teaching can be related to the students. Thinking about student needs is an inclusive concept which comprises teachers’ views, perceptions and values on teaching/learning situations and on mathematical content *for* students.

This paper is a part of a wider study aimed to characterise the nature of undergraduate mathematics teaching with regard to student learning needs in the context of large-group lectures. It must be clear that this is a study of teaching, not of students. We want to learn more about the qualitative characteristics that teaching in this context can receive in relation to the great number of students to whom it is addressed. Thus here, we focus on the ways two university teachers think and act to meet student learning needs in this context. We seek to uncover and cross-analyse different teaching approaches which take students into account in large-scale teaching. Such a work has the potential to refer to both *actual practice* and *teaching theories*. Increasing awareness of how teaching may provide large groups of students with better learning opportunities speaks to actual practice, whereas interpreting the cross-analytic process itself is relevant to teaching theory. With all the above in mind, the research questions that we address are the following:

1. *What are the characteristics of university mathematics teaching and the teachers’ underlying rationales that seek to meet student learning needs?*
2. *What insights do we gain from an analysis of two teaching cases in considering how teaching to large groups of students can be responsive to students’ needs?*

Associated literature

The functioning of lectures

In the lecture setting, the teacher traditionally presents new material to the students and does most of the “telling”; the students listen and take notes (Viirman 2014). Having no intention to legitimate student passive learning, Lobato, Clarke & Ellis

(2005) pointed out that telling is important in teaching since students cannot be expected to reinvent entire bodies of mathematics unaided. Pritchard (2010) argued that '[t]he lecture as a mode of teaching comes in for particular criticism' (p. 609) but lectures have been remarkably resilient in "hard" disciplines - including mathematics. He noted that the 'overall picture seems to be that in some subjects, including mathematics, there is a distinct student preference for lectures' (p. 613) although he also indicated that student opinion should not be accepted uncritically. However, what Pritchard noted resonates with studies such as Cretchley's (2005), which 'indicated that mathematics attendees regarded lectures as an efficient and companionable way to meet new material and build understanding and confidence' (p. 22). Pritchard also considered that a 'strength of lectures is that they can simultaneously address cognitive and affective factors' (p. 611) and can perform well the functions of communicating information, modelling expert thought processes, and motivating students' appropriate attitudes and enthusiasm.

Beyond the potential of a lecture to promote students' learning, there are examples in the research literature that also indicate its limitations. For example, a lecture can turn students into passive listeners instead of active learners (Biggs 2003); the main mathematical ideas are delivered orally and students are not developing the understandings expected of them (Lew, Fukawa-Connelly, Mejia-Ramos & Weber 2016). Paoletti et al. (2018) examined teachers' questioning in advanced mathematics lectures and they found that in some cases, questioning promoted opportunities for students to participate in the learning process and in others, it did not. These differing views of the potentials of a lecture to take students into account challenged us to scrutinise the ways large-group lectures can be responsive to the many students that attend.

Characterising mathematics teaching in lectures

We draw on studies that characterise teaching approaches at university level. Artemeva and Fox (2011) characterised teaching in lectures by studying a large sample of teachers in seven countries. They found that "chalk talk" (talking while writing in chalk on the board) is a common teaching practice in many cultures and it provides a basis for further discussions between the teacher and the students (e.g. comments for everything that was being written on the board or questions to the students). Viirman (2014) claimed that the discursive practices included in chalk talk can be very different and he pointed out the need for studying these practices. Our study scrutinises our participants' chalk talk teaching and provides deeper characterisations of it in relation to students.

As we investigate teaching in lectures in relation to the students, we draw on studies that characterise the nature of teachers' questions to students. Literature (e.g. Fukawa-Connelly 2012) has shown that this nature plays an important role for students' engagement in lectures. In large-group lectures, one can often observe teachers asking *rhetorical* questions that are not intended to be answered. According to Viirman (2015), teachers can use rhetorical questions to focus attention, give meta-comments and/or indicate how mathematics is actually done. Additionally, teachers may choose to be explicit about the topic at hand by highlighting missing pieces of mathematical information. Another strategy teachers can implement is asking *inviting* questions that encourage students to respond and provide them with a participation opportunity (Jaworski & Didis 2014) and also *control* questions (e.g. "Are there any questions?"). These arise 'when a particularly important or complicated piece of mathematics has been presented' or 'when the teacher is about to move on from one topic to another'

(Viirman 2015, p. 1175). Furthermore, teachers can pose challenging *genuine* questions or *enquiries* which require the students to reflect on the advanced mathematics presented.

Teaching in large-group lectures is dynamic, as contextual elements seem to transform it. Studies that compare teaching in different conditions indicate some of these elements. For example, curriculum coverage, goals for student learning, and the role of the teacher (Johnson, Caughman, Fredericks & Gibson 2013) or differences in teaching experience (Pinto, 2013) were found to be of importance in lecture teaching. Other studies (e.g. Wagner & Keene 2014; Gabel & Dreyfus 2017) focussed on interventions and their outcomes. Our study adopts this dynamic view of teaching but it is situated in a context that does not include interventions from our side - interventions that could have an impact on the teaching. We are interested in the daily teaching of our participants and in the contextual elements that form this daily teaching - elements that our participants themselves pointed out in our discussions.

Teachers' thinking and teaching goals in lectures

There are studies in the lecture setting that reveal the role of teachers' thinking behind teaching approaches. For example, Weber (2004) showed that traditional instruction is based on a lecturer's beliefs about students, learning and mathematics. Petropoulou, Potari & Zachariades (2011) focussed on the role of a teacher's reflections on the different sources of experiences, such as research in mathematics and in mathematics education that this university teacher brought in their lecture teaching. Mills (2014) offered insights into the perceptions of teachers when using particular types of examples in class. Other studies interviewed mathematicians about their thinking on a range of issues, such as the role of the advanced mathematical content in university teaching (e.g. Nardi 2008). In our study, we take a more focussed view of teachers' thinking by investigating such thinking about student learning needs in particular. We relate teachers' thinking to the students as our interest is in how the teaching goals and student learning needs are related in order to investigate in depth the link between teaching and learning.

Studies in the lecture setting that report on teaching goals relate goals and practice. Hemmi (2010) interviewed mathematicians about their goals for teaching and how they affect actions. Other studies (e.g. Paterson, Thomas & Taylor 2011); Hannah, Stewart & Thomas 2011; Pinto 2013) drew on the work of Schoenfeld (2011) and related lecturers' goals, resources and orientations to their in-the-moment decisions. For example, Hannah, Stewart & Thomas (2011) found that a lecturer's goals and the methods employed to achieve them, arose from the teacher's beliefs, values and preferences. These studies, however, do not analyse the actual teaching practice but rather base their findings on the teachers' reporting. Lew et al. (2016) noted that there is a gap in the research literature with regard to studies in university teaching that both interview the teacher about their intentions and observe teaching. We attempt to bridge this gap in the particular context of large-group lectures by drawing on both the observation of teaching and the reflective discussions with the two lecturers. We highlight the context here to distinguish this study from current studies (e.g. Dawkins, Oehrtman & Mahavier 2019; Laursen & Rasmussen 2019) which focussed on teaching goals or strategies that enhance student learning and situated in the context of inquiry-based undergraduate teaching to small groups of students. Inquiry-based teaching approaches are important. However, they presuppose students' engagement in the learning process in ways that are difficult to achieve in a lecture hall with more than 200 students experiencing the advanced mathematical concepts for the first time.

Moreover, we are aware of studies at the secondary level (e.g. Stouraitis 2016) that open the scope of research to address the broader social, temporal and cultural dimensions of a teacher's decisions. We follow this line of research since we consider thinking and goals to be inseparable from the social process, as we will explain below.

Theoretical background

In this paper, we consider teaching in Pring's (2000) terms: 'An action might be described as "teaching" if, first, it aims to bring about learning, second, it takes account of where the learner is at, and, third it has regard for the nature of what has to be learnt'. (p. 23)

Taking a sociocultural view, we seek to conceptualise teaching in relation to the learning of mathematics in a university environment. Learning is seen as participation in sociocultural contexts (Vygotsky 1978) and learning mathematics at this level can be seen as enculturation into advanced mathematical practices (Artigue, Batanero & Kent 2007). These practices are distinguished from those at secondary or primary level for reasons related to the mathematical content, the teachers and the students involved (Biza, Jaworski & Hemmi 2014).

The concept of activity is central in the sociocultural perspective referring to 'any motivated and object-oriented human enterprise, having its roots in cultural history, and depending for its actual occurrence on specific goal-oriented actions' (Van Oers 2001, p.71). We use Activity Theory (Leont'ev 1978) to make sense of the complexities of teaching, just as some other studies do (e.g. Jaworski & Potari 2009; Stouraitis 2016). By employing the language of Activity Theory in relation to teaching in terms of the notions of *activity*, *goal* and *action*, we focus this study on the *teaching activity* of lecturing through the *teaching goals* a teacher has for their students' and through goal-directed *teaching actions* they employ in relation to the environment in which action takes place.

Leont'ev (1978) explained that every *activity* has a motive which is an energising force for the activity. This is something that it is not necessarily attained, but rather drives the activity forward. *Goals* are results that can be achieved and they are conscious. If the motive of an activity becomes conscious it becomes a *motive-goal*. *Action* is the behaviour of a human actor which is consciously and purposefully aimed towards a goal. Even if actions are often undertaken by individuals, they are truly social; they are of social origin and they have social consequences relating to real life outcomes (von Cranach & Valach 1984). For example, a teacher's actions can influence students' own activity and their developing world views. Actions are realised by using the available tools and are determined by the prevailing objective conditions that reinforce or restrict what can be done.

These concepts of Activity Theory were central in our recognition of how teaching substantiates thinking. For example, our participants carried very strong views about student learning needs that motivated goal-directed actions. The motive-goal of the teaching activity we observed was made explicit by the lecturers in the reflective discussions we had with them. Their actions were formed in relation to their goals and the underlying thinking and this was an embedding process. This theoretical position is supported by Wertsch (1991). According to Wertsch, thinking is inseparable from any social process; it is more than a mental functioning of the individual, it derives from participation in social life and it also reformulates social life. As regards goals, Leont'ev (1978) pointed out that the important process of goal formation itself usually

remains unexamined. He writes: ‘in pedagogical experiments we always place before the subject a, so to speak, “ready” goal; for this reason, the process of goal formation itself usually escapes investigation’ (p. 64). In the case of university mathematics teaching in large-group lectures, teachers, acting in relation to their students, form goals which are interpreted as actions in this particular context. Studying the goals’ formation and their relation to the teaching actions is a main focus of this study.

Goals and actions are also considered by other frameworks. For example, Alan Schoenfeld's (2011) theoretical framework for analysing teachers’ in-the-moment decision-making attends to actions and goals. Schoenfeld suggests that what people do is a function of their resources (e.g. their knowledge), their goals (conscious or unconscious aims they are trying to achieve), and their orientations (e.g. their beliefs, values, biases, dispositions, etc). Although this framework addresses important dimensions of a teacher’s in-the-moment thinking, it does not take up the broader social, long-term and institutional dimensions of teachers’ thinking. In our study, goals and actions are seen as embedded in the conditions and constraints that are beyond the control of the individual teacher and take on a particular meaning in the case of university institutions (Winsløw, Barquero, De Vleeschouwer & Hardy 2014).

We scrutinised the relationship between teaching activity (expressed through thinking, goals and actions) and student needs by using the theoretical construct of the Teaching Triad (Jaworski 1994). The Teaching Triad is an analytic framework that emerged from an ethnographic study of investigative mathematics teaching at secondary level. Its main goal was to capture essential elements of the complexity of mathematics teaching by analysing classroom interactions. The Triad consists of three “domains” of activity in which teachers engage: *Management of Learning*, *Sensitivity to Students* and *Mathematical Challenge*. “Management of Learning” captures how the teacher organises the classroom learning environment (e.g., groupings, planning of tasks, setting norms). “Sensitivity to Students” captures teacher knowledge of students and attention to their needs and in particular the ways that the teacher interacts with individual students and guides group interactions. Sensitivity to Students has been shown to relate to both *affective* (e.g., offering praise, encouraging students to participate) and *cognitive* (e.g., judging appropriate questions, inviting explanation) domains. In the Jaworski & Potari’s 2009 study the Sensitivity to Students was shown also to include a *social* dimension which describes teachers’ knowledge of what it is possible for the class as a whole, or how much help the class might need to achieve teaching objectives. “Mathematical Challenge” captures the challenges offered to students to engender mathematical thinking and activity. This includes tasks set, questions posed and emphasis on metacognitive processing. The above elements are closely interrelated as Potari & Jaworski’s 2002 study indicated. These researchers claimed that a balance between Sensitivity to Students and Mathematical Challenge is an indicator of effective mathematics teaching in the sense that students can be seen to be involved in rich and meaningful mathematical activity.

Although the Teaching Triad was not developed within a sociocultural perspective, it can be embedded in this perspective (Jaworski, Potari & Petropoulou 2017) because it presents a framework in which all the aspects of teaching activity can be seen through its three dimensions. For example, teacher’s actions as related to mathematics and to the students can be seen through Mathematical Challenge and Sensitivity to Students; the teacher’s use of artefacts (e.g. tasks and resources) can be seen through Management of Learning. Briefly, the Teaching Triad can zoom in on the goals and actions of a teaching activity to specify qualities of sensitivity and challenge and how

to these are managed within the given context. The Teaching Triad has been used to characterise university mathematics teaching in the tutorial setting (e.g. Jaworski 2002; Nardi, Jaworski & Hegedus 2005) but it has not been used to characterise teaching in a lecture setting so far. Questions that we ask concern, for example, what Sensitivity to Students could mean in teaching to large groups of students or what a Mathematical Challenge looks like in this context. In this study, we reassess the elements of the Teaching Triad and define the potential meaning they gain in the particular context. This draws us to theoretical claims based on our analysis and theory in an attempt to characterise the teaching we observed. The nature of such claiming is critical; the questioning of the data, the conceptualising from patterns of activity and a close attention to issues, tensions and contradictions in the data can be seen as ‘critical’. Our study is thus a case of *critical theorising* which involves a reflexive process: the critical drawing of theory from interpretative analysis while using theory to make sense of the interpretative analysis (Jaworski, Mali & Petropoulou 2016).

Based on the above theoretical foundations, we investigate teaching in large-group lectures. This “instructional activity” is often described as *transmission* teaching since the teacher present the content to be learned and students listen and take notes (Speer et al. 2010). Here, we adopt Speer et al.’s (2010) distinction between “instructional activities” and “teaching practice” at the university level. According to this distinction, teachers’ enactment of, for example, small group problem solving and whole-class discussion are *instructional activities* while *teaching practice* is more holistic of what teachers do when they are planning, teaching and reflecting on their lesson. For Speer et al., (2010) ‘Using these and other instructional activities involves the use of various teaching practices, including, for example, selecting specific content to be discussed, choosing particular examples or tasks and deciding if and how to modify a lesson in the middle of class’ (p. 100). Thus, we see the lecture, the context of our study as an instructional activity and we are interested in the different university teaching practices that this activity may include. In the next section we address methodological issues with regard to our data and analysis.

Methodological issues

This study is focussed on the student-centred nature of teaching in large-group lectures on the part of the teacher. As we have indicated above, we work within a sociocultural frame with the intention of considering the whole sociocultural context in which teaching and learning take place. We thus take a qualitative approach to the data.

The context of the study

The study is rooted in a wider project aimed to investigate the nature of university mathematics teaching in two different mathematics departments (D1 and D2) in Greece. The topic in focus, calculus, is a compulsory first year course that is taught exclusively in a lecture format. Calculus is a subject also taught at high school (age 17). The main difference between calculus taught at school and calculus taught at university is in the emphasis given to abstract concepts and proving. In university mathematics departments, calculus courses have a more theoretical focus (like analysis courses in other countries). At high school level, the emphasis is on computations and methods. According to our study participants, there is also a difference in the emphasis given by the two university departments in first year

studies. In D1 the emphasis is more on the applications of theorems (e.g. on exercises) while in D2, the emphasis is rather on proofs. Each course is taught for 13 weeks, 6 hours per week (4 hours for theory and 2 hours for exercises) and all occur in large-group lectures. Exercises are usually taught by a PhD student and this teaching is also in a lecture form. According to our participants, first year calculus is one of the most difficult courses for the students in both departments and many students fail the final exams. This failure leads students to delay in taking their degree. Especially at D2, students take their degree after 6.5 years of studies on average instead of 4 years, which is the normal duration of studies for a mathematics degree in Greece. This fact, along with the importance of calculus for university studies in numerous fields, was important for our choice of the topic of our focus.

Study participants

For the wider project, participants' availability was a main factor in the selection process. However please note, they were not extreme or unique cases (Yin 2017) of university teachers. For the study presented here, the participants (L1 and L2) are both active research mathematicians and experienced university teachers (in D1 and D2 respectively). These two participants were chosen because both displayed an overall genuine concern for the students and their learning - albeit they considered the students' learning needs differently. For example, L1 was accustomed to arriving early at the lecture hall in order to informally talk with his students about their possible difficulties with the previous lecture. L2 was very interested in students' varying attendance and students' success rate in the end of course final exams. His practice was to keep statistical information relating to these issues and he often shared this information with his department colleagues. Also, he officially proposed the teaching of calculus courses twice a year in both semesters so that students could have a second chance if they had failed the first.

The process of generating data

Data concerning these two teaching cases were generated through lecture observations (25 hours of lectures, audio-recorded), field notes, and reflective discussions with each lecturer immediately after each lecture (9 hours of discussions). In addition, informal short discussions with students were also conducted during the time of collecting observational data. The first author conducted these observations and discussions. Moreover, L1 participated in group discussions among the participants of the overall project and with us (the authors) whereas L2 did not participate in these meetings due to time constraints. The group discussions (five in total, each lasting more than two hours) allowed us to share meanings about more general issues related to teaching and learning at the university and thus, reconsider and interpret the lecturers' words in a reliable way. We made audio-recordings of all lectures and discussions and transcribed most data from the observations.

The data analysis process

Data analysis used three analytical approaches. First, we used Activity Theory as an analytical framework as it drove our attention to specific constructs/themes related to our research questions (motives, teaching goals and actions) and facilitated the synthesis of these constructs. Second, we followed a grounded approach (Charmaz 2006) to code teachers' thinking, goals and teaching actions and to form categories towards a characterisation of teaching and the teachers' underlying rationale. Finally, we used the Teaching Triad to zoom in on the identified goals and actions and link the observed teaching to students' expected needs. These analytical perspectives were

formulated through our discussions on the basis of the initial analysis of the data and our own theoretical perspectives.

We conducted the analysis in three layers. In the first one, analysis was based on the identification of teaching episodes and teaching actions. Episodes were extracts in transcriptions of observations and concerned a mathematical topic related to a theorem or a proposition (e.g. formulation, examples, proof and applications). The grounded analysis of teaching actions in the identified episodes based on open coding followed. Through constant comparisons, these codes were grouped into categories we formulated using concepts from existing research on the characteristics of the teaching practice. These categories were *Selecting*, *Explaining*, *Extending*, *Evaluating*, and *External supporting actions*. Their meanings can be found in Figure 1 below and we discuss them in detail in the results section. Moreover, since teacher-questioning was considered as an action, the type of each question was also analysed and linked to the above categories. In particular, control questions (Viirman 2015) and inviting questions that form a basis for evaluating whether the student body was following what was being presented were related to evaluating actions as in Axiak's 2004 study. Genuine questions or enquiries (Viirman 2015) which challenge students mathematically were related to extending actions. Rhetorical questions (Viirman 2015) with which teachers can be explicit about the enterprise were recognised to be of explanatory nature and thus, were related to explaining actions. An initial formulation of some of the above categories, the relations between them and types of questions can be found in the Mali & Petropoulou's 2017 paper that investigated commonalities between teaching in lectures and teaching in small-group tutorials. In the present study, Figure 1 is indicative of the fine-grained analysis in this layer and details the teaching actions included in the above categories, as well as the related types of teachers' questions.

In the second layer, our analysis investigated the rationale of the teaching with regard to student needs. In particular, we explored the rationale of teaching by analysing our participants' reflections on their teaching. Reflections focussed on participants' meanings about students' learning needs, as well as on their own teaching goals. Initial analysis in this layer characterised these meanings as having cognitive and affective/social element. It is acknowledged that cognition and emotion have inner connections in activity (Roth & Radford 2011) and affect is socially organised (e.g. by the prevailing norms) (Evans 2006). Goals were grounded in the data - we did not place before each particular teacher a 'so to speak, "ready" goal' (Leont'ev 1978). In particular, following each lecture, we discussed with each teacher the observations made in the lecture hall and the teachers' actions. These discussions led to the identification of teaching goals. Searching within into the data for explicit references to student' needs and the underlying teaching goals was the main focus of the grounded analysis in this layer of analysis.

In the third layer, analysis associated the teachers' thinking about student learning needs with teachers' goals and actions. We based the links on our interpretations and the evidence for each link was again grounded in the data. For example, we found (in the first two layers of analysis) that L1 thought that students benefitted from appropriately selected examples; that one of this teacher's goals was to initiate students into how mathematics researchers think; and that, in his teaching, L1 was used to giving easy examples and prompt students to make mathematical conjectures.

In this layer, we associated the above thinking, goal and actions and the evidence lies in L1's words:

One thing I do consciously in my teaching is that I try to draw students into how a mathematics researcher thinks. I do it intensely through appropriately selected examples. Students may not understand that the teacher tries to give examples that reveal some things. An example is maybe another exercise for them, but not for me. I give an example with a specific goal. I deliberately build something.¹

Figure 1. Codes and categories of teaching actions and related types of questions

¹ Please note all citations were originally in Greek and have been translated into English for this paper by the first author.

The whole process of analysis resulted in an analytical framework for analysing student-centred mathematics teaching of large groups of students, which is represented diagrammatically in Figure 2. According to this framework, teachers' thinking about student learning needs (represented as T1, T2, etc.) gave rise to their *motive-goal*: students' learning needs should be taken into account in the teaching. This motive-goal was evident in teachers' accounts of how students' needs should be addressed in teaching. Teaching goals (represented as G1, G2, etc.) specified this motive-goal for each teacher. Goals were linked to categories of teaching actions (represented as A1, A2, etc.). Goals and (categories of) actions were linked to teachers' thinking and showed that teachers' thinking and teaching were related in a dialectic way. This is what the double arrows, in Figure 2, represent. This analytical framework helped us to see the characteristic ways each lecturer takes students into account. It also gave structure to our findings. Finally, we characterised our teaching cases (in the analysis of thinking and teaching actions) by using the theoretical construct of the Teaching Triad. The Triad allowed us to gain deeper insights into the nature of the identified actions and rationale. Thus, we gained insights into these teaching approaches that are related to student learning needs. In this way, we also explored potentials of the Triad's elements for characterising teaching in this context.

Figure 2. An analytical framework for analysing student-centered teaching

The validity of the study we present is strengthened by the long period of data collection (four academic semesters), the applied triangulation of sources (observational data and reflective discussions with the teachers), and triangulation of methods (grounded analytical approach to the data in which research literature was embedded, as well as the theory i.e. Activity Theory and the Teaching Triad). Furthermore, during data generation and analysis, we worked closely together to analyse the data and discuss our interpretations. Especially with regard to the grounded coding, we discussed the codes many times until the four of us reached agreement. The reliability of the findings is also backed by the refinement of our

coding through testing our analysis methods on two further cases within the wider project.

Results

In this section, we present two different cases of student-centred, university teaching in the context of large-group lectures. Each teaching case is exemplified through a teaching episode in which the lecturer was teaching a particular mathematical proposition (e.g. if a series converges then the sequence of the series tends to zero). The analytical framework, presented in the methodology section (Figure 2), gives structure to the findings. We end this section with issues that emerged from the cross-cases analysis.

L1's teaching

L1 'projected a strong ethos of enthusiasm' in his lectures, 'cultivating a group dynamic (Pritchard 2010, p. 618) while presenting important mathematical theory to the students. L1 was gesturing, using 'we' in his talk and establishing his role with comments such as 'you [the students] don't have anything to be afraid of' and 'I am here, just to be of help and nothing else'. Some of the students said in the informal discussions with the observer, that 'L1 wants to help and tries his best', but also that 'calculus is a very difficult subject'.

The main results for this case follow. We also present them diagrammatically in Figure 3 at the end of this subsection.

Thinking about students' learning

L1's predominant thinking about student needs included mainly affective and cognitive concerns for students as learners. In particular, L1 considered that '50% of the learning is about the appropriate psychology of the learner' and that students need to 'gain confidence' in their engagement with the advanced mathematical concepts and to 'have their own space' in the lectures by 'contributing their ideas'. We recognised that, in L1's view, students learn better when the climate is supportive and when they have opportunities to contribute their ideas (teacher's thinking - T1).

In the discussions, L1 emphatically and often mentioned that students need 'to learn how to think mathematically'. Thinking mathematically relates, according to L1, to students' enculturation in 'mathematical ideas'; to the 'process of making connections' among mathematical ideas; and to the mathematical 'production' of new ideas and knowledge. However, L1 pointed to the difficulties students confront in developing mathematical thinking (T2).

Finally, in L1's view, students as university newcomers 'have a need for guidance and orientation; they need to see you act in a transparent way' and that structured tasks and appropriately selected examples would be of benefit for them in their struggling with the advanced mathematical concepts (T3).

In terms of the Teaching Triad, taking into account students' difficulties indicated Sensitivity mainly in the cognitive domain. Also, considering the psychology of the students as crucial for their learning indicated Sensitivity mainly in the affective domain. L1's perception that students need 'to learn how to think mathematically' was the springboard for offering Mathematical Challenge and inviting them to enter into the mathematicians' world as we will see further below.

Motive-Goals

Often, L1's thinking of student learning needs was offered together with an expression of a responsibility and an intention, from his side, to address these needs. We recognised that such thinking was reflected in his goals and actions, as we show below, and, in this sense, thinking about students' learning needs motivated teaching (motive-goal).

One of L1's main goals was to engage students in the lecture (teaching goal - G1). In this way students could have the 'space' to contribute their ideas and L1 could gain a sense of 'what they understand and what they don't':

What I try to do, especially in the first year, is to train students to get into a process of asking questions and generally of participating. I want to have a perception of what they understand and what they don't, what is their background, so I can build on it.

Also, L1 intended 'to offer to newcomers the basic elements of [his] own way of seeing see things in mathematics' and orient, in this way, students' mathematical thinking (G2).

Finally, L1 expressed many times the goal to enculturate students into how mathematicians think and produce mathematical knowledge (G3):

What I try to do through the whole process is to teach them to think mathematically. This is my goal. And I have to make them understand how someone thinks and produces knowledge.

L1's goals for teaching were linked to particular teaching actions as we exemplify below.

Teaching actions

L1 mainly employed selecting, explaining, extending, and evaluating actions in his teaching. Typical in this case were actions of: selecting tasks which required students to engage in 'doing mathematics' (Olson & Knott 2013) (e.g. explore relationships); highlighting the key points of a proof to the students; asking students mathematically challenging questions and valuing their contributions.

The teaching episode that follows exemplifies this teaching approach and illustrates the analytical process. The episode is from a lecture in which L1 first defined a convergent series and then offered examples of convergent and divergent series. Consequently, an intense interaction between L1 and some students was taking place while L1 was attempting to facilitate students to arrive at the conjecture 'if a series converges then the sequence of the series tends to zero' (detailed account and analysis of this interaction can be found in Petropoulou, Jaworski, Potari & Zachariades 2015). Then, the proof of the conjecture followed (Table 1):

Table 1: 1st teaching episode

Excerpt from L1's teaching case - Teacher-student interactions with series and convergence	Teaching actions
<p>1. L1: Now, just because the series $\sum_{k=1}^{\infty} a_k$ converges, what do we have?</p> <p>We have that the sequence – a new sequence – the sequence of partial sums, as we call it, $\langle S_n \rangle$ - we sum up the first n-terms – converges. This is what we said in the definition.</p>	<p>Describing heuristics: focussing on the data and the unknown; using definitions</p>

<p>Let's say that $\langle S_n \rangle$ converges to a number a [<i>writing</i> $\lim_{n \rightarrow \infty} S_n = a$]. We'd like to see about the convergence of $\langle a_n \rangle$; what differentiates $\langle a_n \rangle$ from $\langle S_n \rangle$.</p>	Interrelating
<p>2. Let's go! What is S_{n+1}? It is the sum from 1 to $n+1$. Fine, let's write it [<i>he writes</i>]:</p> $S_{n+1} = \sum_{k=1}^{n+1} a_k = a_1 + a_2 + \dots + a_{n+1}.$	Rhetorical question
<p>Namely if the symbol of the series is not familiar to you, what is this sequence [$\langle S_{n+1} \rangle$]? I add a_2 to a_1 and go on till a_{n+1}.</p>	Explaining
<p>3. Is this sum [S_{n+1}] related to S_n? What was S_n? S1: The S_{n+1} is S_n plus a_{n+1}.</p>	Inquiring questions
<p>4. L1: All right? Thank you very much!</p>	Evaluating
<p>5. Now, S_n is to sum up the first n terms namely the a_1, the a_2 and go on to a_n. So, what is the difference between these two sums [S_n and S_{n+1}]? [pause] S2: One of them has one term more...</p>	Inquiring question
<p>6. L1: That's it! One of them has one term more than the other. [<i>writing</i> $S_{n+1} = S_n + a_{n+1}$, $n \in \mathbb{N}$ - relation (R)] We will see that this very simple observation gives us the result! So, now I solve towards a_{n+1} [<i>writing</i> $a_{n+1} = S_{n+1} - S_n$, $n \in \mathbb{N}$].</p>	Drawing on student's idea (Validating)
<p>7. What information do I have? I have information saying that $\langle S_n \rangle$ converges and I want to get information for $\langle a_n \rangle$. That's why we wrote down the relation (R). Now, what does this very simple algebraic thought say to me? That the term a_{n+1} is equal to a difference.</p>	Rhetorical question Using heuristic (data and unknown)
<p>8. And what about the sequence $\langle S_{n+1} \rangle$? [he waits] S1: It converges. L1: It converges. Why? S1: Because the proposition says so! L1: No! We do have information about $\langle S_n \rangle$. But what about $\langle S_{n+1} \rangle$? S1: It is a subsequence of $\langle S_n \rangle$.</p>	Encouraging reflection by comparing Interaction Pointing out
<p>L1: It is a subsequence of $\langle S_n \rangle$. So, it also converges to the same limit. Thus, the sequence $\langle S_{n+1} \rangle$ converges; the sequence $\langle S_n \rangle$ converges and what about $\langle a_{n+1} \rangle$? S1: It converges.</p>	Guiding question
<p>9. L1: [<i>writing</i> "Therefore $\langle a_{n+1} \rangle$ converges as a difference of convergent sequences and indeed ..."] Let's see the algebra about the point of convergence. [<i>writing and telling</i>: $\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} S_{n+1} - \lim_{n \rightarrow \infty} S_n = a - a = 0$].</p>	Presenting more theory
<p>10. We saw that each of $\langle S_n \rangle$, $\langle S_{n+1} \rangle$ converges. We apply the properties of limits. So, we saw that, necessarily, when the series converges, then the sequence is a null sequence.</p>	Summarising
<p>11. Thus, from now on, if we can directly recognise that the sequence is not a null sequence, this automatically means that the series does not converge!</p>	Inferring
<p>12. Are there any questions so far? Alright? [he waits] Fine!</p>	Control question

In the above episode we can see how L1 attempted to achieve his goals of encouraging students' participation, orienting their thinking and of initiating them into how this specific proof is produced.

In particular, L1 tried to encourage students' participation by selecting an easy way of proving the proposition (he subtracted the pair of the sequences S_{n+1} and S_n instead of the usual pair of S_n and S_{n-1} found in the textbooks which raise issues about the definition of the sequence S_{n-1} when $n = 1$). Furthermore, by explaining the key points of the proving process through summarising (in 10) and by inferring the point this proof process arrived at (in 11). Moreover, offering written and oral clarifications of

the symbols right from the beginning (in 2); asking questions (in 3, 5 and 8); validating students' contributions in a positive affirming manner (in 4); and drawing on their relevant observations (in 6) were actions he employed to encourage students' participation.

L1 tried to orient students' thinking to key mathematical ideas by summarising these ideas (in 10) and by inferring a strategy that was based on the proved proposition and that would be useful to students 'from now on' (in 11). Furthermore, describing heuristics such as focussing on deriving something useful from the data and looking at the unknown (Polya 1971, p.190) (e.g. in 1) and pointing out irrelevant observations (in 8) were actions he employed to orient students' thinking.

Finally, L1 tried to initiate students into the production of this particular proof by prompting them to search for interrelations (in 1, 3 and 5) and by proposing that they use heuristics (in 7) – strategies that research mathematicians use in their practice.

In terms of the Teaching Triad, we see encouraging students' engagement as an expression of L1's Sensitivity to Students possibly in the affective domain within his overall Management of students' Learning; L1 opened up for students' ideas, built on them, and confirmed that these ideas contribute to the learning process. Moreover, bringing research mathematicians' practices into students' considerations was arguably a Mathematical Challenge for the students. However, such challenges were not always easy to deal with. Initial analyses of L1's case showed 'an uneasy balance' between Sensitivity to Students and Mathematical Challenge in their teaching. This was reflected in a tension between his intentions to include students' ideas in the lecture and to present advanced mathematical meanings at the same time (Petropoulou, Jaworski, Potari & Zachariades 2013). The findings for this case can be diagrammatically found below (Figure 3).

Figure 3. Analysing L1's teaching

Briefly, L1 considered that students need to be included in the mathematical thinking, as well as in the learning process and attempted to meet these needs in his teaching by encouraging participation and by trying to extend and orient students' thinking to the way mathematicians work.

L2's teaching

L2 is a very popular lecturer among the students in his department. Several students told the observer during the lecture breaks that they and their classmates consider this lecturer very 'analytic in his explaining' and the notes they keep from the board during the lectures are 'very helpful for their studying'.

The main results for this case follow and we present them diagrammatically in Figure 4 at the end of this subsection.

Thinking about students' learning

L2's prevailing consideration of students' learning needs valued their getting their degree in time so they could go on with their lives (T1):

It is important for the students not to delay in getting their degree. I do know that students get lost in their first year and that the mathematics department is demanding for the majority of them. If you don't pay attention, the average duration of their studies could easily become seven or eight years.

L2 had a particular concern for the portion of students who do not attend the lectures. He recognised that the variability in some students' attendance could be attributed to

several reasons including participating in social associations; socialising after the hard entry examinations; or having to get a job in parallel with their studies for financial reasons. These concerns were followed by considerations that students are better included in a course when it is well-organised, even if they do not attend on a regular basis (T2):

Students want to know what exactly is expected from them. Even these organisational things about the exact day of the intermediate exam or how each part of the final exam will be scored etc... They want to know! This is one need for them: to be organised.

Also, for L2, students learn better when they are first familiarised with advanced mathematical methods (T3): 'I think Von Neumann was the one who said: "you do not understand mathematics, you just get used to it". This is correct!' In his view, students need 'to see some things many times until these things seem normal and they come easily in their mind'.

Finally, L2 emphasised the need for the students 'to get clarifications', e.g. for 'the concepts behind a theorem', 'the roles of a hypothesis' and 'when an argument is considered as a completed proof'. We recognised that, for L2, students benefit from the clarity of presentation of the overall content (T4).

In terms of the Teaching Triad, considering all the difficulties that students may have as university newcomers is an indication of possible Sensitivity to Students. L2's awareness that there are students who do not attend the lectures for many reasons (e.g. they have to work) indicates Sensitivity to students' social backgrounds. Interestingly, analysis also showed that his rare interaction with students indicated an expression of sensitivity to large cohorts of students:

In an audience of 200 students, if you discuss with two to three of them, these probably will be the strongest students and the others will feel bad. And finally, nothing will remain on the board. Here, we talk about masses of students and how to achieve a practical result for them. That's the point!

'Success in the final exam' for 'as many students as possible' was the main 'practical result for masses of students' as we will see below.

Motive - Goals

The actual situation that L2 prioritised in his intentions was that the students who pass the final exam at the end of the course can go on with their studies without consuming time and this should and could be taken into account in the teaching (Motive-goal):

We want as many students as possible to start their studies smoothly. In this department, given the enormous number of students, the general adjusting difficulties of first year students and the difficulty of the subject, the ideal would be the average student to pass all the compulsory courses at least during the first three years of their studies instead of two which is the scheduled. I believe this is feasible. The most important for this aim is to stabilise the rate of success in the final exam at about 300 students per year.

Thus, L2 paid attention to the mathematics content that students were expected to know for the final examination with the intention to make it relevant to them (G1).

Also, considering that 'if a course is well-organised, then students have more possibilities to go on with their studies smoothly', L2 had the goal of organising the course (G2) by employing particular teaching actions as we exemplify below.

Teaching actions

L2 mainly employed selecting, explaining, extending, evaluating actions - the same as L1 did. However, these actions are characterised differently, as we show below. Furthermore, L2 employed external supporting actions in his teaching (e.g. drawing on an official web-site called “e-class”). Typical in this case were actions of: selecting tasks which connected mathematical procedures and ideas (Olson & Knott 2013); splitting a complex process into steps; establishing mathematical connections; commenting on the resources that all students used (textbook, lecture material) and using alternative resources.

The teaching episode that follows exemplifies this teaching approach and illustrates the analytical process. An initial analysis of a part of this episode can be found in Petropoulou, Jaworski, Potari & Zachariades (2016). The episode is from a lecture in which L2 had defined a convergent series and offered examples of convergent and divergent series (and the example of harmonic series was still on the board). After these steps, L2 formulated the proposition: ‘if a series converges then the sequence of the series tends to zero’ (the same proposition L1 proved in the 1st teaching episode provided here). Then L2 wrote on the board: ‘If $\sum_{k=1}^{\infty} a_k$ converges, then $a_k \rightarrow 0$ ’. Then the proof of the proposition followed (Table 2):

Table 2: 2nd teaching episode

Excerpt from L2’s teaching case – Teacher - explaining of series and convergence		Teaching actions
1.	L2: We will deal with this question which is already difficult. I remember, when I was at school, there was a teacher. If you saw him in the morning, he would greet you saying ‘How are you today? Does your series converge or not?’ The convergence was of such great importance to him!”	Characterising Bringing a personal story
2.	Of course, because you are advanced now, you will ask if the inverse relationship holds. If the inverse held, just taking a look at $\langle a_k \rangle$ and seeing that it tends to 0 would be enough for the series converging. If it didn’t, I would say that the series does not converge, and that would be all!	Counter-speculation Connecting and justifying
3.	However, I have already written an example for you, if you look carefully [the harmonic series]. This series tends to infinity.	Focussing attention
4.	Attention here! This point [the non – convergence of the harmonic series] remains up to the final assessment!	Highlighting
5.	[writing] ‘The quick way’ [of proof]. I will also show the slow one. Ok?	Characterising
6.	I repeat again. We should not forget that a_k are the terms of the sequence. S_n is the sum of the first n terms of the sequence. If a series converges to scR , then S_n also converges to s . [writing: ‘The series converges to an scR i.e. by definition $S_n \rightarrow s$ ’].	Repeating
7.	Now, I define a second sequence $\langle t_n \rangle$ as follows – I am going to write down for you the terms of this sequence. First, I set, let’s say t_1 , to be equal to 0. Then I set the 2 nd term of $\langle t_n \rangle$ to be equal to S_1 , the 3 rd to S_2 ... Ok? the 4 th to S_3 , etc. Namely I set t_1 to be 0 - you can set everything you want. Let $\langle t_n \rangle$ to be $\langle S_{n-1} \rangle$; $\langle t_n \rangle$ is $\langle S_{n-1} \rangle$ if $n \geq 2$ [writing ‘We define a second sequence $\langle t_n \rangle$ as follows: $0, S_1, S_2, S_3, \dots$ i.e. $t_1=0$ and $t_n=S_{n-1}$ if $n \geq 2$ ’]	Describing step by step what he is doing
8.	I want to define the sequence clearly. The books write ‘consider the sequence $\langle S_{n-1} \rangle$ ’, but what is the S_{n-1} if $n=1$? Is it S_0 ? It is not defined! Ok?	Pointing out a gap
9.	If you want to define this sequence $\langle S_{n-1} \rangle$, you set a sequence; you set the first term and then you transfer the terms.	Describing

	Namely the 2 nd term of $\langle t_n \rangle$ is the 1 st term of $\langle S_n \rangle$; the 3 rd is [respectively] the 2 nd etc. Fine. So, you crack the first term and then you get all the other terms of a sequence $\langle S_n \rangle$ which tends to s .	
10.	So, the sequence you get tends to s . So, $\langle t_n \rangle$ goes to s , too. So the difference of the two sequences goes to 0. [<i>writing: 'Then $t_n \rightarrow s$. So, $S_n - t_n \rightarrow s - s = 0$'</i>]	Inferring
11.	But what is the difference of the two sequences? For $n \geq 2$, the difference $S_n - t_n$ is the following: S_n is the sum of the first n terms of $\langle a_k \rangle$ and t_n is the sum of the first $n-1$ terms. Ok? Because [<i>writing</i>] $t_n = S_{n-1}$, so $t_n = \alpha_1 + \alpha_2 + \dots + \alpha_{n-1}$. Thus, the difference $S_n - t_n$ is equal to a_n ; it is the only term left.	Rhetorical question Describing
12.	I have done all these analytically because the book says 'the difference $S_n - S_{n-1}$ is equal to a_n ' and nothing more. This is how we calculate this difference. So this is the proof.	Characterising
13.	Now, if you don't like this way, you can prove the proposition using ϵ . Second proof – we will not set the sequence $\langle t_n \rangle$ [<i>writing: 'Second way'</i>]. [<i>going on with the second way...</i>]	Giving an alternative method
14.	So, you prove a very useful proposition: you keep on hoping to add the terms $\langle a_k \rangle$ if they tend to 0. If $\langle a_k \rangle$ does not tend to 0 then you directly say 'it is over'.	Connecting
15.	This proposition is very useful as a non - convergence criterion. Ok?	Characterising Selecting an example to apply
16.	For example: Someone gives you the sequence $a_k = \frac{k-1}{k+1}$ and asks you if the series of a_k converges or not. Then, they do not ask you anything!	
17.	The first thing you have to do is to look at a_k . You say to them that a_k tends to 1 and not to 0, so the series doesn't converge. Ok? I.e. the first thing you look at is if the k -term inside the series tends to 0.	Providing a solution method
18.	Therefore, the only interesting question about a series can be formulated in the case that the sequence of the series tends to 0. All the other series do not converge!	Refining

In the above episode we can see how L2 attempted to achieve the goal of making the advanced content relevant to the students while proving the same proposition that L1 had proved before (in Table 1). In particular, we can see L2's explaining mainly in describing a process step by step formally (in 7 and 11) or informally (in 9); in focussing students' attention (in 3); in highlighting an important point (in 4); and in repeating what students are expected to know (in 6). In doing so, L2 was writing everything in detail on the board. His use of chalk-talk teaching (Artemeva & Fox 2011) realised the goal of making the content relevant to the students; students do focus on the ideas which are written on the board and not on those that are delivered only verbally (Lew, et al. 2016). Moreover, we can see that, by extending students' mathematical thinking to an alternative solution method, L2 also served this goal of relevance, as he offered students a way of proving without the (complex) sequence t_n . Finally, we can see attempts for relevance in L2's evaluation of the importance of the concept of convergence. His evaluation of the importance of mathematical content was an indication of enhancing relevance (Frymier 2002). Further L2 actions that helped achieve relevance were: Bringing a personal story to students (in 1); characterising approaches to the proof (in 5); and pointing out gaps found in the textbook that all students use (in 12).

L2 tried to organise the course by employing external supporting actions which, in his case, referred mainly to e-class management. E-class was accessible to all students through a web-site consisting of: detailed information about the course; L2's personal notes that included the basic theory of the course and a collection of carefully chosen

exercises with indicated solutions and that also provided questions from past exams. As L2 put it, this site ‘has been very much of a help’ in organising the course and he enriched it on a regular basis. For example, to support further students who could not attend the lectures, L2 assigned a student to keep notes from the lectures. L2 then corrected these notes once a week and uploaded the notes to the e-class website. In informal discussions with the observer, students seemed to appreciate that e-class was well-organised, was up to date and informed by L2. Moreover, L2 actually taught the hours that focussed on exercises himself. (Normal practice is that a PhD student would do this.) L2 took this on in order ‘to teach more exercises’ overall. L2 also organised extra lessons to support the students further. Students were informed about these extra lessons via e-class and, although the lessons often took place on Saturdays, many students attended them.

In terms of the Teaching Triad, in this episode we mainly see L2’s Management of students’ Learning which, however, stems from his Sensitivity to Students. In particular, L2 had in mind that the large portion of students whom he tried to prepare for the final exam included students who could not attend the lectures. This showed Sensitivity in the social domain. Thus, L2 made the content relevant to as many students as he could by ‘showing and telling’ and by organising the course via a website (Management of Learning) where all the students, even those who could not attend the lectures, could have access. L2 also accommodated students’ possible difficulties with gaps found in the textbooks, which is an expression of cognitive Sensitivity to Students. This Sensitivity was instantiated, for example, by providing analytic explanations to bridge that gap (Management of Learning). Finally, the concern about students’ feelings in the case of interaction with others is judged as an expression of his affective Sensitivity, which resulted in a univocal teaching approach (Management of Learning). In this teaching, Mathematical Challenge was not very central in this and often it was overlapped by the Sensitivity to Students. The findings for this case can be diagrammatically found in Figure 4.

Figure 4. Analysing L2's teaching

Briefly, L2 considered that students need to be included in the new academic environment and to obtain their degree in time, so he tried to offer them a well-organised course and to make the mathematical content more relevant to them mainly by providing detailed explanations and drawing on external recourses.

We presented two teaching cases of two teachers, L1 and L2, who taught the same proposition in very different ways based on different considerations about student needs and on different teaching goals. However, their teaching was motivated by the same 'energising force', i.e. that student learning needs should be taken into account in the teaching. This was realised by teaching actions which were comparable, yet different in nature. Linking their actions to their goals and to the thinking behind goals allowed us to gain insights into broader issues regarding university mathematics teaching to large groups of students. We come now to these issues.

Emerging issues

Perspectives of effective inclusion of students in a lecture

Both teachers in this study considered that students need to be included in the new academic environment, which is a very challenging aim in large-group lectures. However, they considered the cohort of students, not as individuals, but as a whole in two different ways, and also attributed different meanings to students' inclusion. For L1, students were included in a lecture first by having the opportunities to interact

(e.g. to contribute their own ideas and receive feedback on them) and second (which includes the first) to be enculturated into the way mathematicians think and produce mathematics. This enculturation into research mathematicians' ways of thinking was, for L1, context related. The students were exclusively mathematics undergraduates (e.g. not engineers or economics students) and their initiation into advanced mathematical practices was expected by the mathematical community at the university. We saw, for example, how L1 focussed on the students who participated in the discussions and how they tried to draw on their ideas. However, in the given context with the large number of students, focussing on the feedback received from some individual students created tension for L1 – he was not aware of the ideas of the many students who remained silent.

For L2, students' effective inclusion in the new academic environment occurred when they had the opportunity to progress smoothly with their studies. By this L2 meant that students need to be supported to avoid any failure in the final examination of the course and to go on with their lives without the interruptions of re-taking courses. This support in the case of L2 was also context related: Since the number of students was very large, support was mainly based on external resources. We saw, for example, how L2 used an electronic environment to provide for the needs of the students who did not attend the lectures or how he tried to make the advanced mathematical content relevant to all university newcomers, e.g. by writing in detail everything on the board. In doing so, L2 offered students opportunities to participate in the community of learners. In this teaching case, the opportunities for participation in lectures did not include direct interaction inside the lecture hall, but instead, could be traced in the availability of resources.

Contextual elements of teaching in lectures through the interplay of actions and goals

In the teaching episodes presented above, the two lecturers employed the same types of selecting, explaining, extending and evaluating actions in order to fulfil their goals. However, the existing similarities between these actions were too coarse to capture their actual teaching. L1 interacted with (some) students dialogically and tried to elicit their mathematical observations, as they considered that the students learnt better when they could contribute their ideas. L2 considered interaction, in this context, as a time consuming and affectively ineffective practice. As we have already shown, the final examinations, the large number of students, the variance in their attendance and the fact that the course was offered by a mathematics department played an important role in teachers' thinking about student needs and in the formation of the goals they set for their teaching. In the case of L1, thinking about student needs focussed on opportunities for individual participation in the community of mathematics as a research discipline. In the case of L2, such thinking focussed on opportunities for collective participation in the community of university (under)graduates. Based on the detailed analysis of our observations, we recognised that the role of the given context was apparent in what the lecturers did and not only in what they said, for example, in their attempting to establish connections between the new academic expectations and students' prior experiences from school.

The quality of teaching in lectures: mathematical challenge and students' needs

An in-depth sense of the interplay of teaching goals and actions with attention to students' needs at this level was gained by using the Teaching Triad. In the setting of a crowded lecture hall, what has been considered as a part of effective mathematics teaching at other educational levels of instruction (e.g. interaction) could simply be unrealistic. Students' participation in the learning of advanced mathematics was attained by the two teachers in this study not only at the level of the university lecture hall, but also at the level of the available out-of-class resources. The latter is important when the number of students is large because this factor changes the Management of students' Learning. This was evident in the case of L2. The social aspect of Sensitivity to Students can be found in L2's addressing students' needs by drawing on out-of-class resources. Notably, the lecture hall was very crowded during L2's lectures, which indicated students' appreciation of this teaching approach. It seems that in teaching at large-scale, social sensitivity attracts the students more than the other dimensions of sensitivity or mathematical challenge. It also seems that the collective management of students' learning dominates that of the individual students. Seeking to address 200 students 'one-on-one' is practically impossible. In the case of L1, **interaction with individual students took place at the classroom level.** In this case, L1's attention was focussed on students having a positive psychology towards their learning of advanced mathematics. The affective aspect of Sensitivity to students can be found in lecturer's addressing such need. Therefore, L1 encouraged cognitive focussing (on mathematics) through his approach to students' psychology. We recognised that Mathematical Challenge was strongly related (and aligned) to these issues of Sensitivity. The social and the affective dimensions of Sensitivity with regard to students' participation in the learning process of advanced mathematics are qualitative characteristics that teaching in this context can receive. We see these characteristics as complementary qualities towards the development of teaching that takes student needs into account. Our complementary perspective is what mainly distinguishes this study from other studies that only present contrasting teaching cases (e.g. Pinto 2013).

Up to this point, we have presented the teaching cases of two lecturers who carried strong views of student learning needs and the subsequent characteristics assigned to their teaching. Based on the above, we have raised issues which we come now to discuss.

Discussion

We see here a theoretical account of the formation of the teachers' goals and actions and their practical realisation reveals issues for the teachers that intervene in their everyday practices. The fine-grained analysis leading to characterisations of teaching allow us to see into the deeper essences of the teaching process which go beyond superficial accounts about traditional teaching. This analysis is robust enough to move forward our understanding of university mathematics teaching and to offer insights into:

- the process of goal formation by highlighting the links to teacher thinking
- how teaching substantiates thinking by offering specific links between thinking and acting

- what caring teaching practices can look like; caring is about knowledge – knowledge of student needs and acts to meet these needs
- the role of the context; caring teaching practices could hardly be the same in front of 20 and in front of 200 students.

Ultimately, it is not the teaching of these two specific teachers that matters, but the insights we gain into professional practice and its mediation through social/institutional impositions and the deeply caring approaches of the teachers – caring for the mathematics on which they focus but importantly, caring for the quality of their students' learning. These insights came to us through the theoretical lenses of Activity Theory and the Teaching Triad. Activity Theory enabled us to investigate the activity of teaching in its entirety (actions, motive-goals) and to take into account the influence of the particular context. The Triad allowed us to gain insights into the character of teaching actions and goals in relation to students' learning needs. Both allowed us to gain a holistic perspective of the observed teaching and the respective teacher's thinking. In doing so, we uncovered key elements of the social practices of teaching for learning mathematics in a university and the concomitant tensions and issues which are central to the analysis of teaching.

In this study, we did not scrutinise students' learning. We focussed on daily, large-scale teaching because it is important to understand how this teaching can be responsive to the many students to whom it appeals. Thus, we interpreted teaching in terms of students' opportunities for participation in the learning processes within this particular context. We put forward the role of resources as an element of social sensitivity to students and the role of experiences the teachers bring to teaching, such as the experience of doing research in mathematics as an institutional particularity at this level of education which can influence teaching. There is certainly a need for more research considering social and institutional factors which intervene in the teaching/learning processes at this level.

The contribution of this study concerns the research, the theory and the actual teaching practice. First, this study offered to the Teaching Triad - in particular to the social and affective dimensions of Sensitivity to Students - the perspective of students' participation in large-scale teaching. This is a context in which the Triad has not been used before. Second, this study proposed the combination of Activity Theory and the Teaching Triad in investigating teaching within a sociocultural perspective. The congruence between these two analytical frameworks has never been tested on research data before. Third, this study proposed complementary qualitative characteristics of mathematics teaching that novice university teachers might consider in developing their own teaching: challenging students mathematically should be based on the knowledge of students' social backgrounds and on the attention to students' appropriate psychology towards the advanced mathematical content. Importantly, this study also proposed links between student needs and the actual teaching practice: In this particular context student needs can be addressed if they are reflected in teachers' thinking and motivate the process of goal formation. These links could contribute to teaching development, as raising teachers' awareness about student needs is likely to provide means to teachers to refine their teaching goals and benefit their students.

Mathematics education research could and should have an important role on teachers' developing awareness of student learning needs in the large lecture setting and on highlighting alternatives to traditional lecturing. The teaching cases we present in this

study highlight alternatives to traditional instruction in this particular context, as exemplified by the teachers' drawing on resources and incorporating the experience of research in mathematics. These alternatives point towards the development of large-group lecturing which has started to constitute a culture in some mathematics departments (Barton, Oates, Paterson & Thomas 2014). The two teaching cases we highlight in this study are not, of course, generalisable. However, we note that after all, changes in departmental teaching practices can begin with a single faculty member (McClendon et al. 2019).

Conflict of interest statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

- Artemeva, N., & Fox, J. (2011). The writing's on the board: The global and the local in teaching undergraduate mathematics through chalk talk. *Written Communication*, 28(4), 345-379.
- Artigue, M., Batanero, C. & Kent, P. (2007). Mathematics thinking and learning at post-secondary level. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: National Council of Teachers of Mathematics* (pp. 1011-1049). Charlotte, NC: Information Age Publishing.
- Axiak, C. (2004). Being Sensitive to Students' Mathematical Needs: What Does It Take? In Hoines, M. J., & Fuglestad, A. B. (Eds), *Proceedings of the 28th International Group for the Psychology of Mathematics Education*, (V2, pp. 79-86). Bergen, Norway: PME.
- Barton, B., Oates, G., Paterson, J., & Thomas, M. (2015). A marriage of continuance: professional development for mathematics lecturers. *Mathematics Education Research Journal*, 27(2), 147-164.
- Biggs, J. (2003). *Teaching for Quality Learning at University*, 2nd ed., Open University Press, Maidenhead.
- Biza, I., Jaworski, B., & Hemmi, K. (2014). Communities in university mathematics. *Research in Mathematics Education*, 16(2), 161-176.
- Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. London: Sage.
- McClendon, D. M., Weller, K., Trouba, J., Brigance, S., & Dekker, M. (2019). Transforming tradition: from one instructor to an entire department. *Problems, Resources, and Issues in Mathematics Undergraduate Studies (PRIMUS)*, 1-29.
- Cretchley, P. (2005). Mathematics and dumping lectures?: Another perspective on the shift towards learner pragmatism. In Bulmer, M., MacGillivray, H. & Varsavsky, C. (Eds.), *Proceedings of the Fifth Southern Hemisphere Conference on Undergraduate Mathematics and Statistics Teaching and Learning*, 42-48. Queensland, Australia.

- von Cranach, M., & Valach, L. (1984). 14-The social dimension of goal-directed action. In Tajfel, T. (Ed.), *The Social Dimension: European Developments in Social Psychology*, 1, 285- 299.
- Dawkins, P. C., Oehrtman, M., & Mahavier, W. T. (2019). Professor Goals and Student Experiences in Traditional IBL Real Analysis: a Case Study. *International Journal of Research in Undergraduate Mathematics Education*, 5, 315-336.
- Evans, J. (2006). Affect and emotion in mathematical thinking and learning. *New mathematics education research and practice*, 233-255.
- Gabel, M., & Dreyfus, T. (2017). Affecting the flow of a proof by creating presence — a case study in Number Theory. *Educational Studies in Mathematics*, 96, 187-205.
- Hannah, J., Stewart, S., & Thomas, M. (2011). Analysing lecturer practice: The role of orientations and goals. *International Journal of Mathematical Education in Science and Technology*, 42(7), 975–984.
- Hemmi, K. (2010). Three styles characterising mathematicians' pedagogical perspectives on proof. *Educational studies in mathematics*, 75(3), 271-291.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*, 1, 371-404. Greenwich, CT: Information Age.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer.
- Jaworski, B. (2002). Sensitivity and challenge in university mathematics teaching. *Educational Studies in Mathematics* 51, 71-94.
- Jaworski, B. & Didis, M.G. (2014). Relating student meaning-making in mathematics to the aims for and design of teaching in small group tutorials at university level. In P. Liljedahl, S. Oesterle, C. Nicol & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 377-384). Vancouver, Canada: PME.
- Jaworski, B., Mali, A., & Petropoulou, G. (2016). Critical Theorising from Studies of Undergraduate Mathematics Teaching for Students' Meaning Making in Mathematics. *International Journal of Research in Undergraduate Mathematics Education*, 1-30.
- Jaworski, B. & Potari, D. (2009). Bridging the macro- and micro-divide: using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72, 219-236.
- Jaworski, B., Potari, D. & Petropoulou, G. (2017). Theorising university mathematics teaching: The teaching triad within an activity theory perspective. In T. Dooley, & Gueudet, G. (Eds.). *Proceedings of the 10th Congress of the European Society for Research in Mathematics Education* (CERME10). Dublin, Ireland: DCU Institute of Education and ERME.

- Johnson, E. Caughman, J. Fredericks, J. & Gibson, L. (2013). Implementing inquiry - oriented curriculum: From the mathematicians' perspective. *Journal of Mathematical Behavior*, 32(4), 743-760.
- Freeman, T. M., Anderman, L. H., & Jensen, J. M. (2007). Sense of belonging in college freshmen at the classroom and campus levels. *The Journal of Experimental Education*, 75(3), 203-220.
- Frymier, A. B. (2002). Making content relevant to students. *Communication for teachers*, 83-92.
- Fukawa-Conelly, P., T. (2012). A case study of one instructor's lecture-based teaching of proof in abstract algebra: making sense of her pedagogical moves. *Educational Studies in Mathematics*, 81, 325-345.
- Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, 1-18.
- Leont'ev, A. N. (1978). *Activity, Consciousness and Personality*. Englewood Cliffs, NJ: Prentice-Hall.
- Lew, K., Fukawa-Connelly, T. P., Mejía-Ramos, J. P., & Weber, K. (2016). Lectures in advanced mathematics: Why students might not understand what the mathematics professor is trying to convey. *Journal for Research in Mathematics Education*, 47(2), 162-198.
- Lobato, J., Clarke, D. & Ellis, A. B. (2005). Initiating and Eliciting in Teaching: A Reformulation of Telling. *Journal for Research in Mathematics Education*, 36(2), 101-136.
- Mali, A. & Petropoulou, G. (2017). Characterising undergraduate mathematics teaching across settings and countries: an analytical framework. *Nordic Studies in Mathematics Education*, 22 (4), 23-42.
- Mills, M. (2014). A framework for example usage in proof presentations. *The Journal of Mathematical Behavior*, 33, 106-118.
- Nardi, E. (2008). *Amongst mathematicians: Teaching and learning mathematics at the University level*. New York: Springer.
- Nardi, E., Jaworski, B., & Hegedus, S. (2005). A spectrum of pedagogical awareness for undergraduate mathematics: From "tricks" to "techniques". *Journal for Research in Mathematics Education*, 36(4), 284-316.
- Olson, J., C. & Knott, L. (2013). When a problem is more than a teacher's question. *Educational Studies in Mathematics*, 83, 27-36.
- Paoletti, T., Krupnik, V., Papadopoulos, D., Olsen, J., Fukawa-Connelly, T. & Weber, K. (2018). Teacher questioning and invitations to participate in advanced mathematics lectures. *Educational Studies in Mathematics* 98, 1-17.
- Paterson, J., Thomas, M. & Taylor S. (2011). Decisions, decisions, decisions: what determines the path taken in lectures? *International Journal of Mathematical Education in Science and Technology*, 42:7, 985-995.
- Petropoulou, G., Jaworski, B., Potari, D. & Zachariades, T. (2016). Addressing large cohorts of first year mathematics students in lectures. In Nardi, E., Winslow, C. & Hausberger, T. (Eds), *Proceedings of 1st Conference of*

- International Network for Didactic Research in University Mathematics*, pp. 390-399. Montpellier, France: INDRUM.
- Petropoulou, G., Jaworski, B., Potari, D. & Zachariades, T. (2015). How do research mathematicians teach Calculus? In K. Krainer, & N. Vondrová (Eds) *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education*, pp. 2221-2227. Prague, Czech Republic: CERME.
- Petropoulou, G., Jaworski, B., Potari, D. & Zachariades, T. (2013). Mathematical Challenge and Sensitivity to Students in University Lecturing: An uneasy balance. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, 5, 144. Kiel, Germany: PME.
- Petropoulou, G., Potari, D., & Zachariades, T. (2011). Inquiring mathematics teaching at the university level. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, 3, pp. 386-392. Ankara, Turkey: PME.
- Pinto, A. (2013). *Revisiting university mathematics teaching: A tale of two instructors*. Paper presented in the Eighth Congress of European Research in Mathematics Education (CERME 8), Antalya, Turkey.
- Polya, G. (1971). *How To Solve It: A New Aspect of Mathematical Method* (2nd Edition). New Jersey: Princeton University Press.
- Potari, D. & Jaworski, B. (2002). Tackling Complexity in Mathematics Teaching Development: Using the Teaching Triad as a Tool for Reflection and Analysis. *Journal of Mathematics Teacher Education*, 5, 251-280.
- Pring R. (2000). *Philosophy of Educational Research*. London: Continuum.
- Pritchard, D. (2010). Where learning starts? A framework for thinking about lectures in university mathematics. *International Journal of Mathematical Education in Science and Technology*, 41(5), 609–623.
- Roth, WM. & Radford, L. (2011). *A Cultural-Historical Perspective on Mathematics Teaching and Learning*. Rotterdam, The Netherlands: Sense Publishers.
- Schoenfeld, A. H. (2011). *How we think: A theory of goal-oriented decision making and its educational applications*. New York: Routledge.
- Speer, N. M., Smith, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior*, 29, 99-114.
- Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, 40(5), 530–562.
- Stouraitis, K. (2016). Decision making in the context of enacting a new curriculum: an activity theoretical perspective. In Csíkos, C., Rausch, A., & Sztányi, J. (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4*, pp. 235–242. Szeged, Hungary: PME.

- Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal* 20(2), 5-24.
- Van Oers, B. (2001). Educational Forms of Initiation in Mathematical Culture. *Educational Studies in Mathematics*, 46, 59-85.
- Viirman, O. (2014). The functions of function discourse: University mathematics teaching from a commognitive standpoint. *International Journal of Mathematical Education in Science and Technology*, 45(4), 512-527.
- Viirman, O. (2015). Explanation, motivation and question posing routines in university mathematics teachers' pedagogical discourse: a commognitive analysis. *International Journal of Mathematical Education in Science and Technology*, 46(8), 1165-1181.
- Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.
- Wagner, J. F. & Keene, K. A. (2014). Exploring differences in teaching practice when two mathematics instructors enact the same lesson. In T. Fukawa-Connolly, G. Karakok, K. Keene, & M. Zandieh (Eds.), *Proceedings of the Seventeenth Annual Conference on Research in Undergraduate Mathematics Education* (pp. 322-335). Denver, CO.
- Wagner, J., Speer, N., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior*, 26(3), 247-266.
- Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational studies in mathematics*, 48(1), 101-119.
- Weber, K. (2004). Traditional instruction in advanced mathematics courses: a case study of one professor's lectures and proofs in an introductory real analysis course. *The Journal of Mathematical Behavior*, 23, 115-133.
- Weinberg, A., Wiesner, E. & Fukawa-Connolly, T. (2016). Mathematics lectures as narratives: insights from network graph methodology. *Educational Studies in Mathematics*, 91, 203-226.
- Winsløw, C., Gueudet, G., Hochmuth, R., & Nardi, E. (2018). Research on university mathematics education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds), *Developing research in mathematics education - twenty years of communication, cooperation and collaboration in Europe* (s. 60-74). London and New York: Routledge
- Winsløw, C., Barquero, B., De Vleeschouwer, M., & Hardy, N. (2014). An institutional approach to university mathematics education: From dual vector spaces to questioning the world. *Research in Mathematics Education*, 16(2), 95-111.
- Winsløw, C., & Grønbaek, N. (2014). Klein's double discontinuity revisited. *Recherches en Didactique des Mathématiques*, 34(1), 59-86.
- Yin, R. K. (2017). *Case study research and applications: Design and methods*. Sage publications.