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Turbulent closure analysis in heated separated and reattached flow using eddy-resolving data

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In this study, we use Large-Eddy Simulations (LES) to provide a platform to investigate the separated and reattached turbulent flow over a heated blunt plate at $Re_H = 21600$. Surface Nusselt number and flow field data show good agreement with published experiments. The turbulence anisotropy resolved by the LES shows that through the recirculation region the anisotropy develops towards an axisymmetric contraction state in the near-wall profile. In the redeveloping region, profiles show progression towards the plane-strain state. Turbulent closures, providing simple models of the unknown turbulent correlations that arise from the Reynolds-averaging of the Navier-Stokes equations, are routinely applied to complex flows, often with little known about their suitability. The eddy-resolved flow field is used to describe deficiencies in Reynolds-Averaged Navier-Stokes (RANS) modelling using an LES informed Turbulence Transport (LiTT) *a priori* analysis. The Explicit Algebraic Reynolds Stress Model (EARSIM) showed improved agreement, capturing the elevated turbulent stresses in the recirculation region. Closures describing the turbulent heat flux are compared and the Higher-Order Generalised GDH (HOGGDH) closure is discovered to show good agreement with those resolved by the LES, capturing the correct ratio of streamwise to normal turbulent heat flux across the redeveloping boundary layer. An Explicit Algebraic Scalar Flux Model (EASFM) is examined and shows good predictions of the turbulent heat flux angle but underpredicts the magnitude across the recirculation region. An optimal coefficient for the HOGGDH is described to reproduce the turbulent heat flux magnitude from the LES, showing a range of optimal values across the flow.

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I. INTRODUCTION

The study of turbulent flow and heat transfer in separated and reattached flow regimes is of practical interest to the engineering community. Combustor liners, heat exchangers and turbine blades are just a few examples of where these flows are present and to understand and predict their heat transfer is vital for product lifetimes and performance. High Reynolds numbers, common within these applications, promote instabilities within the shear layer and the onset of turbulence is close to the separation point. The turbulent shear layer enhances the turbulent mixing through the reattachment and redeveloping regions. The complexities of the flow present challenges to commonly used, efficient and low-cost two-equation Reynolds-Averaged Navier Stokes (RANS) approaches where a desire for improved accuracy is sought for component design. Understanding the RANS closures within these flows allows for improved engineering judgement and may provide a foundation for further closure development.

Separation, reattachment and the resulting heat transfer have been experimentally studied in backwards and forward facing steps, expansion tubes and channels, blunt plates and surface mounted obstructions. Heated blunt plate geometries have been addressed by Ota and colleagues^{1–5}. Time-averaged Nusselt number distributions scale with $Re^{2/3}$ and peak values exist close to the reattachment location¹. Temperature and velocity profiles have been presented across a blunt plate showing the rise in non-dimensional temperature through the recirculation bubble and the slow redevelopment of the boundary layer post-reattachment¹. Ota and Nishiyama² presented a universal correlation of peak Nusselt number to Reynolds number, from a range of separated and reattached flow cases. The nose shape of the blunt plate was investigated by Ota and Kon³, where the reattachment and peak Nusselt number location were shown to move towards the leading edge as the apex angle of the nose increased.

At an unsteady level, hotwire anemometry enabled measurements of turbulent statistical data in these flow regimes. Profiles of turbulent normal and shear stresses had shown elevated values within the turbulent shear layer and remain characteristically larger in the redeveloping flow region when compared to those of a standard turbulent flat plate⁶. Turbulent heat flux was investigated by Ota and Kon⁴, where correlations with the turbulent shear stress enabled predictions of turbulent Prandtl number, Pr_t . The discussed predictions were scattered, however, a trend-line presented a value of unity close to the wall and with values greater than 4 at the edge of the thermal boundary layer. Pulse Doppler Anemometry (PDA) was used by Djilali and Gartshore⁷ to improve turbulence

measurements in regions of high turbulence intensity showing large values of root-mean-square (rms) velocity in the recirculation region. Kiya and Sasaki⁸ investigated the blunt plate at $Re_H = 26000$ and found the frequency of vortex shedding at $f = 0.6 U_\infty / L_R$. Further work by Kiya and Sasaki⁹ extended the low-frequency analysis of the separation bubble, presenting the response of reversed flow about the reattachment location to the periodic shedding.

Numerical studies have been used to predict the surface heat transfer in heated blunt plate cases. A two-dimensional simulation at $Re_H = 1000$ identified a correlation between the large scale vortex shedding and the surface heat transfer, but turbulence quantities were overestimated¹⁰. A three-dimensional simulation was performed by Tafti and Vanka¹¹ which showed the reattachment location at $6.36H$ downstream of the leading edge, which overestimated the experimental value by 27%. Direct Numerical Simulations (DNS) were used by Yanaoka et al¹² where a reattachment length of $4.9H$, for a $Re = 5000$, showed good agreement to experiments. Isosurfaces of the second invariant of velocity gradient showed the instability of the turbulent shear layer, where a two dimensional vortex is shown to followed by the formation of three-dimensional structures downstream¹². Ratios of streamwise to normal components of the turbulent heat flux vector showed a larger contribution from the streamwise component across the flow and close to the wall. Sugawara et al.¹³ used Large-Eddy Simulations (LES) with a Smagorinski Sub-Grid Scale (SGS) model for heat transfer predictions in a symmetric expansion plane channel at $Re = 15000$. Time-averaged results showed a good estimation of the short and long recirculation regions and the general trends in Nusselt number were well captured, however, the magnitude was underpredicted.

Yang and Abdalla¹⁴ studied an isothermal plate with LES using a dynamic sub-grid-scale model to investigate the secondary instability of a transitional separation bubble. Results presented strong evidence that the elliptical instability was solely responsible for the deformation and three-dimensional breakdown of the Kelvin-Helmholtz instabilities. Langari and Yang¹⁵ used LES to study the transitional separated flow over a flat plate with semi-circular leading edge. Their analysis showed a change in the primary instability from the Kelvin-Helmholtz instability to a bypass-like transition at elevated freestream turbulence levels. Li and Yang¹⁶ extended the previous study by assessing the separated transition under a pressure gradient and freestream turbulence. At low freestream turbulence the pressure gradient bypasses the secondary instability in which coherent two-dimensional rolls distort and breakdown into three dimensional structures. Numerical resolution and sub-grid scale turbulent closures for LES were assessed for separated and reattached flows by Cimarelli et al.¹⁷ using DNS data. Zonal Detached Eddy Simulations (ZDES) were used

by Pain et al.¹⁸ for assessing the large scale dynamics of separating and reattaching axisymmetric flow where large scale helical structure and intermediate structures are shown with a Fourier and Dynamic Mode Decomposition (DMD) analysis.

LES has become more practical and accessible within the study of engineering flows with the increase in computational power. Its ability to resolve large-scale eddies and fluctuations, while modelling the small dissipating eddies, has been able to provide accuracy in a range of flows, such as effusion cooling¹⁹ and jet nozzles²⁰. For the foreseeable future, LES is too expensive for design purposes; however, it can unveil a greater understanding of separated and reattached flow and its heat transfer. This can provide practical modelling advice within lower fidelity modelling.

Reynolds-Averaged Navier-Stokes (RANS) calculations are commonly used within industry for flow and heat transfer problems because of their efficiency and low-cost over complex geometries. Formulation of the RANS transport equations are achieved using the Reynolds averaging procedure. Instantaneous quantities are decomposed into a time-mean and fluctuating components. Decomposed quantities are introduced over the transport equations and time-averaged to provide the steady transport equations. The resulting momentum and energy transport equations, Eq. 1 and Eq. 2 respectively, are solved in addition to the continuity equation. As a result of the Reynolds averaging procedure over the non-linear terms, two correlation terms arise: the Reynolds stresses, $\overline{u'_i u'_j}$, and the turbulent heat flux $\overline{u'_j T'}$, reflecting the turbulent transport of momentum and heat respectively.

$$\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_i U_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\mu S_{ij}^* - \rho \overline{u'_i u'_j} \right) \quad (1)$$

$$\frac{\partial \rho h_0}{\partial t} + \frac{\partial \rho U_j h_0}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{c_p \mu}{Pr} \frac{\partial T}{\partial x_j} + c_p \rho \overline{u'_j T'} \right) + \frac{\partial}{\partial x_j} \left(u_i \left[2\mu S_{ij}^* - \rho \overline{u'_i u'_j} \right] \right) \quad (2)$$

Closure of the Reynolds stresses are commonly achieved using the Boussinesq hypothesis²¹ for two-equation turbulence models. This constitutive relation is used with two-equation turbulence models and is described in its incompressible form in Eq. 3. Formulation of the eddy viscosity, ν_t , is achieved using transport equations to describe some length and time scale of the turbulence. Non-linear constitutive relations aim to improve upon the deficiencies of the Boussinesq hypotheses. Speziale²² formed a non-linear relation, for the Reynolds stresses, and improved numerical predictions of fully developed duct flow, capturing the secondary flow features that are unseen when using the Boussinesq hypothesis. Reynolds Stress Models (RSM) offer a transport equation

for each unique Reynolds stress component and a seventh to describe the dissipation, but still rely on closures to describe more unknown correlations. These models often suffer from bad numerical convergence and stability over complex geometries.

$$\overline{u'_i u'_j} = \frac{2}{3} \delta_{ij} k - 2\nu_t S_{ij} \quad (3)$$

Explicit Algebraic Reynolds Stress Models (EARSMS) form a closure for the Reynolds stresses explicitly from the parameters of the mean flow for two-equation turbulence models. Pope²³ proposed ten tensor groups for this relation and an implicit production to dissipation ratio for two-dimensional mean flows. Wallin and Johansson²⁴ form an EARSMS model with a near-wall treatment using a van-Driest function. For two-dimensional flows the anisotropy of the Reynolds stresses is evaluated using Eq. 4, where the first term of the equation is akin to the anisotropy of the Boussinesq hypothesis.

$$a_{ij} = f_1 \beta_1 S_{ij} + (1 - f_1^2) \frac{3B_2 - 4}{\max(\Pi_S, \Pi_S^{eq})} (S_{ij}^2 - \frac{1}{3} \Pi_S \delta_{ij}) + \left[f_1^2 \beta_4 - (1 - f_1^2) \frac{B_2}{2\max(\Pi_S, \Pi_S^{eq})} \right] (S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj}) \quad (4)$$

The Reynolds-averaged energy equation requires the closure of the turbulent heat flux vector, $\overline{u'_i T'}$. A simple Gradient Diffusion Hypothesis (GDH), expressed by Eq. 5, is commonly implemented in RANS codes. The model aligns the turbulent heat flux vector to the mean temperature gradient of the flow. The turbulent diffusivity, α_t , which controls the additional diffusion of the turbulence, is isotropic, described by the flows eddy viscosity and the turbulent Prandtl number, Pr_t . These assumptions are not true for complex flows where the size of energy-containing eddies are comparable to or greater than the length over which the thermal gradient changes significantly²⁵. Turbulent flat plate correlations in the log-law region provide a turbulent Prandtl, Pr_t , number of 0.85²⁶. The value has been debated for other flows and a spatially varying turbulent Prandtl number, within the boundary layer, can improve the heat transfer results²⁷.

The Generalised GDH (GGDH) is an algebraic closure introduced by Daly and Harlow²⁸. For the turbulent heat flux closure the equation is shown in Eq. 6. Inclusion of the Reynolds-stress terms introduces anisotropy into the turbulent diffusivity, improving the accuracy of the predictions where an accurate description of the Reynolds stress field is provided. A model constant, c_θ , replaces the turbulent Prandtl number of the GDH. A higher order closure can be achieved using the quadratic products of the Reynolds stresses for the High-Order GGDH (HOGGDH) closure²⁹

presented in Eq. 7 and extends upon these diffusion hypothesis methods. A transport equation of the turbulent heat flux vector can also be found, as performed by Le Ribault and Friedrich³⁰, but these increasing complexities are uncommon within commercial CFD codes.

$$\overline{u'_i T'} = -\alpha_t \frac{\partial \bar{T}}{\partial x_i} = -\frac{\nu_t}{Pr_t} \frac{\partial \bar{T}}{\partial x_i} \quad (\text{GDH}) \quad (5)$$

$$\overline{u'_i T'} = -c_\theta \tau_t \overline{u'_i u'_j} \frac{\partial \bar{T}}{\partial x_j} \quad (\text{GGDH}) \quad (6)$$

$$\overline{u'_i T'} = -c_\theta \tau_t \frac{\overline{u'_i u'_k} \overline{u'_k u'_j}}{k} \frac{\partial \bar{T}}{\partial x_j} \quad (\text{HOGGDH}) \quad (7)$$

GDH and HOGGDH closures are assessed by Ling et al.³¹ for the turbulent scalar flux in discrete hole film cooling flow. Although the HOGGDH captures anisotropy in the scalar flux, the magnitude relies upon the model coefficient. Forward propagation to the energy equation showed the effects upon surface cooling, where the best results were seen from using the GDH with an LES derived isotropic turbulent diffusivity. Abe and Suga²⁹ study the turbulent scalar flux models in differing channel flows. Within shear-free flow regions, the GGDH closure produced a reasonable approximation of the turbulent scalar flux. However, close to the wall the GGDH did not provide a correct ratio of normal to streamwise turbulent scalar flux, whereas the HOGGDH method showed improved predictions.

Explicit Algebraic Scalar Flux Models (EASFM) formulate a closure for turbulent scalar flux in an approach that is similar to the EARSM formulation. Wikström et al.³² propose a model which is formed by applying an equilibrium condition in the normalised scalar flux transport equation. The closure requires additional transport models to obtain the half scalar variance and its dissipation rate to close the system of equations, alongside accurate Reynolds stress predictions from either an EARSM or RSM.

In this paper, LES is used to achieve accurate predictions of heat transfer and turbulent flow for the heated blunt plate flow, shown in Fig. 1. An LES informed Turbulent Transport (LiTT) procedure solves the k - ω SST specific dissipation rate transport equation over an LES time-averaged field. The results are then used to assess the deficits within closures of the Reynolds stresses and the turbulent heat flux. Improved predictions are sought for the HOGGDH closure and an optimal coefficient is studied over the separated and reattached flow regime to improve the surface heat transfer predictions.

FIG. 1: Heated blunt plate geometry and computational domain.

II. NUMERICAL PROCEDURES

A. Eddy-Resolving Simulations

Simulations were performed using the *rhoPimpleFoam* solver in OpenFOAM Version 5.0³³. OpenFOAM is an open source CFD tool box that rivals common commercial packages with an abundance of turbulence modelling solutions and has been widely used in research communities. It has developed traction in a broad range of fields such as jet flows^{34,35}, combustion flows³⁶ and two-phase flow modelling³⁷, amongst many others.

The conservation equations for continuity, momentum and energy are solved in the following forms respectively:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i}{\partial x_i} = 0 \quad (8)$$

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\mu \tilde{S}_{ij}^* - \bar{\rho} \tau_{ij}^{sgs} \right) \quad (9)$$

$$\frac{\partial \bar{\rho} \tilde{h}_0}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j \tilde{h}_0}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\bar{\rho} \alpha \frac{\partial \tilde{h}}{\partial x_j} - \bar{\rho} q_j^{sgs} \right) \quad (10)$$

$$\tau_{ij} = -2\nu_{sgs} \tilde{S}_{ij}^* \quad (11)$$

$$\tilde{S}_{ij}^* = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \quad (12)$$

Sub-Grid Scale (SGS) stresses are modelled using the Wall-Adaptive Local Eddy-viscosity (WALE) model³⁸. The SGS stresses are described by a modelled SGS eddy viscosity, ν_{sgs} , and the mesh resolved traceless strain rate tensor, $\tilde{S}_{ij}^*$. The model provides correct near-wall scaling of the eddy viscosity with y^3 without the requirement of a dynamic formulation. WALE has been successfully used within conjugate heat transfer simulations of cooled turbine blades³⁹, the flow and heat transfer of two cylinders in tandem⁴⁰ and rib-roughened internal turbine blade cooling channels⁴¹. The model has also proved successful in reproducing transition from laminar instabilities³⁸. However, WALE predictions of the SGS viscosity are dependant on limitations of the model coefficient c_w . Values of 0.5 provide success in homogeneous isotropic decaying turbulence³⁸ while a value of 0.325 has become common practice in complex flows providing suitable diffusion⁴², although the model results remain relatively insensitive to the coefficient^{43,44}.

$$\nu_{sgs} = (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}} \quad (13)$$

$$S_{ij}^d = \frac{1}{2} (\bar{g}_{ij}^2 + \bar{g}_{ji}^2) - \frac{1}{3} \delta_{ij} \bar{g}_{kk}^2 \quad (14)$$

$$\bar{g}_{ij} = \frac{\partial \bar{u}_i}{\partial x_j} \quad (15)$$

The SGS viscosity is obtained from Eq. 13 where, Δ , is the cube root of the local cell volume and the model coefficient, C_w , of 0.325 is selected. The definition of S_{ij}^d is shown in Eq. 14 formings the traceless symmetric component of the square of the velocity gradient tensor, $\bar{g}_{ij}$.

$$q_{j,sgs} = -\alpha_{sgs} \frac{\partial h}{\partial x_j} \quad (16)$$

$$\alpha_{sgs} = \frac{\nu_{sgs}}{Pr_{sgs}} \quad (17)$$

SGS heat fluxes are obtained from Eq. 16 where the SGS turbulent diffusivity is defined by Eq. 17. The SGS Prandtl number, Pr_{sgs} , was set to 0.4, as used in LES simulations by Schindler et al.⁴⁵ and Eidson⁴⁶ and recommended by Grötzbach and Wörner⁴⁷. Although other simulations have been known to use greater values of Pr_{sgs} ^{13,48}. Initial tests investigated the sensitivity of surface heat transfer results to Pr_{sgs} . Values of 0.4 to 1.0, as used in Pallares and Davidson's⁴⁹ channel flow case, showed no significant difference for the mesh used.

TABLE I: Physical parameters to replicate experimental conditions of Ota and Kon¹.

Plate Thickness	$H = 20 \text{ mm}$
Upstream Velocity	$U_{\infty} = 27 \text{ m/s}$
Upstream Temperature	$T_{\infty} = 300 \text{ K}$
Outlet Pressure	$p = 101325 \text{ Pa}$
Wall Heat Flux	$q = 668.28 \text{ W/m}^2$
Reynolds Number	$Re_H = 21600$

Coupling of the continuity and momentum equations are achieved iteratively using the PISO algorithm. Time stepping is discretised using a first-order implicit Euler scheme with a time step providing a maximum CFL of 0.8 to achieve time accurate and numerically stable results. Convective fluxes are discretised using a central scheme with an upwind bias to achieve second-order spatial accuracy. The discretisation of diffusive fluxes is achieved using a fully central scheme.

Boundary conditions are used to replicate the experimental conditions of Ota and Kon¹ and achieve the conditions outlined in Table I. The Reynolds number is characterised by the plate thickness, H . No-slip wall conditions are applied to the top, bottom and leading-edge surfaces of the plate with a wall-normal pressure gradient of zero. A constant heat flux boundary condition applies 668.28 W/m^2 to the top and bottom surfaces replicating the experimental validation case. A velocity inlet is used with $U_{\infty} = 27 \text{ m/s}$ and $T_{\infty} = 300 \text{ K}$ with pressure gradient set to zero. The pressure outlet boundary condition defines a pressure of 101325 Pa . Periodic conditions are applied to the spanwise constrained boundaries, shown in Fig. 1, simulating an infinite span and permitting vortical structures to pass from one side to the other. Slip walls are applied to the top and bottom boundaries to replicate the blockage conditions of the experimental case.

A tetrahedral octree meshing strategy is used to provide shear layer and near-wall refinements without the high-aspect ratio, skewed cells and unnecessary far-field refinements associated with structured techniques. Results are obtained using a 16 million cell mesh. Effects of grid resolution were assessed with a 5, 11 and 16 million cell mesh and no difference was observed in the time-averaged surface heat transfer between the 11 and 16 million cell meshes. Near-wall cell height was selected to achieve a Δy^+ of less than 1 while streamwise and spanwise spacing were selected to achieve Δx^+ and Δy^+ of less than 25. Volumetric refinements were used in the recirculation region (Fig. 2a), the shear layer and near-wall regions (Fig. 2b) to appropriately resolve the

FIG. 2: Tetrahedral octree meshing strategy showing highlighting locally refined regions. (a) Recirculation refinements and (b) shear layer and near-wall refinements.

FIG. 3: Comparison of resolved to sub-grid scale turbulent kinetic energy.

large scale motions and three-dimensional vortical structures. The bottom side of the plate was coarsened to reduce cell quantity while retaining any flow field coupling between top and bottom surfaces. The numerical results shown throughout this paper will feature the top side of the plate only. For the top side of the plate, the mesh resolves more than 90% of the turbulent kinetic energy as shown by the profiles in Figure 3. Aft of the recirculation centre, 95% of the turbulent kinetic energy is resolved. Analysis within this paper will only consider the resolved fluctuations of the flow as the sub-grid scale contributions are shown to be small.

B. LES informed Turbulent Transport (LiTT) Analysis

RANS closures are investigated using the high-fidelity LES data in an *a priori* analysis. Time-averaged results from the LES provides well-resolved Reynolds stress and turbulent heat flux fields. A RANS simulation can be performed and the modelled turbulent transport terms presented; however, the flow fields are inherently different because of the modelling discrepancies and direct comparisons are not possible to experimental or eddy-resolving datasets. Raiesi et al.⁵⁰ pursue an array of solutions for assessing turbulence model behaviour from DNS and LES datasets in an *a priori* analysis and previous work by Parneix et al.⁵¹ used similar *a priori* analysis of second moment closures and the importance of these types of studies are discussed by Durbin⁵². Weatheritt and Sandberg⁵³ propose a solution for extracting a dissipation rate from high-fidelity datasets to aid machine learning model improvements to Reynolds stress field predictions in RANS simulations.

The time-averaged LES field is frozen and the resolved turbulent kinetic energy, k , is calculated. The transport equation for the k - ω SST⁵⁴ specific dissipation rate, ω , is solved iteratively over the frozen LES field (Eq. 18) where the eddy-viscosity, ν_t , is modelled by Eq. 19 and the constants are consistent with those from Menter⁵⁵. The approach enables functional predictions of turbulent dissipation and eddy viscosity that reflect the values that a RANS would calculate if the flow field and turbulent kinetic energy were identical to the LES. With this information, the Reynolds stresses and the turbulent heat flux quantities, modelled by RANS closures, can be assessed. Hence, this *a priori* approach has been coined as an LES informed Turbulent Transport (LiTT) analysis.

$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial \rho u_j \omega}{\partial x_j} = \frac{\gamma}{\nu_t} P_k - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (18)$$

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, \Omega F_2)} \quad (19)$$

The LiTT analysis is firstly used to identify differences in the turbulent anisotropy resolved by the LES to that described by the Boussinesq hypothesis (Eq. 3), where ν_t is taken from the LiTT and the strain-rate, $\bar{S}_{ij}$, is from the time-averaged frozen LES field. Comparisons are furthered by analysing the EARSM formulation with near-wall treatment of Wallin and Johansson²⁴ (Eq. 4) and their enhanced predictions of the Reynolds stress field and anisotropy.

Secondly, the analysis is used to assess functional discrepancies in the turbulent heat flux closures. The GDH (Eq. 5), commonly used within commercial CFD packages, takes the eddy-viscosity described by the LiTT analysis with the commonly used coefficient $Pr_t = 0.85$. The GGDH (Eq. 6) and HOGGDH (Eq. 7) models are assessed with LiTT where the turbulent time scale is defined by $\tau = k/\varepsilon = 1/(\beta^* \omega)$ and the Reynolds stress fields are initially taken from the LES to demonstrate their improvements when a realistic Reynolds stress field is provided. The EASFM of Wikström et al.³² is then investigated with the LiTT for the discussed flow, showing comparisons to the diffusion hypothesis models. The formulation of the EASFM is taken from Wikström et al.³² where the scalar term has been replaced by the temperature and shown in Eq. 20.

$$\overline{u'_i T'} = -(1 - c_{\theta 4}) A_{ij}^{-1} (a_{jk} + \frac{2}{3} \delta_{jk}) \frac{k^2}{\varepsilon} \frac{\partial \overline{T}}{\partial x_i}, \quad (20)$$

where for two-dimensional mean flow,

$$\mathbf{A}^{-1} = \frac{\mathbf{G}\mathbf{I} - (c_S \mathbf{S} + c_\Omega \mathbf{\Omega})}{G^2 - \frac{1}{2} Q_1} - \left(\frac{G}{G^2 - \frac{1}{2} Q_1} - \frac{1}{G} \right) (\mathbf{I} - \mathbf{I}_{2D}), \quad (21)$$

and,

$$Q_1 = c_S^2 \mathcal{I}_S + c_\Omega^2 \mathcal{I}_\Omega, \quad (22)$$

$$G = \frac{1}{2} \left(2c_{\theta 1} - 1 - \frac{1}{r} + \frac{P_k}{\varepsilon} \right). \quad (23)$$

The two-dimensional invariants of the mean strain rate and vorticity are defined as:

$$\mathcal{I}_S = \text{tr} \{ \mathbf{S}^2 \}, \quad \mathcal{I}_\Omega = \text{tr} \{ \mathbf{\Omega}^2 \}. \quad (24)$$

The time scale ratio, r , defines the ratio of scalar to turbulent time scale, where the scalar time scale is equal to the ratio of half-scalar variance to scalar dissipation rate. For the RANS approaches studied, this ratio is unclosed as the scalar variance and dissipation rate are not available. Modifications would require additional transport equations for these two properties or alternatively an explicit relation. For the current analysis, the high fidelity LES dataset is used to describe the

time scale ratio by assuming it is equal to ratio of the integral time scale of the temperature autocorrelation and streamwise velocity autocorrelation across the flow. Finally, Constants within the model are defined as $c_{\theta 4} = 0$, $c_S = c_{\Omega} = 1$ and $c_{\theta 1}$ is set to $1.6(r + 1)/r$. Strain rates, S , and vorticity, Ω , are normalised with the turbulent time scale, τ_t .

C. RANS k - ω SST Analysis

A RANS solution using the two-equation k - ω SST turbulence model is performed for the purpose of comparing first order statistics of the flow. The implementation of the model used is coherent with the LiTT analysis, but the transport equations for continuity, momentum and energy are solved with the turbulent kinetic energy and specific dissipation rate. Closure of the Reynolds stress terms are achieved using the Boussinesq hypothesis constitutive relation and the turbulent heat flux is closed with the GDH.

A two-dimensional structured mesh with $y^+ < 1$ and a clustering of cells focussed around the recirculation region are used to resolve the flow. The meshing strategy involved refinement of the grid until the surface Nusselt number distribution no longer changed with additional refinement. A symmetry plane along the plate centreline was used to replicate the symmetry of the mean flow over the plate.

Other turbulence models were investigated in a preliminary study, but did not provide adequate heat transfer solutions. Thus the selection of the k - ω SST model provides a suitable comparison to the LES and a baseline for further improvements.

TABLE II: Analysis methods and their descriptions.

Method	Description
LES	Pure LES results using the WALE sub-grid scale model.
k - ω SST	Full RANS computation with the k - ω SST turbulence model.
LiTT-BH	LiTT of the Boussinesq hypothesis modelled Reynolds stresses.
LiTT-EARSM	LiTT of the EARSM modelled Reynolds stresses with near-wall enhancement.
LiTT-GDH	LiTT of GDH modelled turbulent heat flux.
LiTT-GGDH	LiTT of GGDH modelled turbulent heat flux.
LiTT-HOGGDH	LiTT of HOGGDH modelled turbulent heat flux.
LiTT-EASFM	LiTT of EASFM modelled turbulent heat flux.

FIG. 4: Instantaneous temperature across (a) spanwise slice at $Z = 0$ and streamwise slices at (b) $X = 2.5H$ and (c) $X = L_r$. "(Multimedia view)"

III. RESULTS AND DISCUSSIONS

Results and discussions will be presented in three parts. Firstly, the success of the LES using WALE SGS modelling will be shown by comparison to experimental results. Secondly, an assessment of the turbulence anisotropy is presented and the deficits of Reynolds stresses modelled by Boussinesq hypothesis in RANS simulations are studied with the LiTT analysis. Finally, the LiTT approach will evaluate RANS closures to the turbulent heat flux vector to identify the best approach for RANS simulations of complex turbulent forced convection flows. Table II presents the method and descriptions that will be used throughout the paper. Legends provided in the figures will remain consistent with the table to aid the reader.

A. Flow Field

Figure 4 "(Multimedia view)" presents an instantaneous snap shot of the temperature field, showing the influence of turbulent fluctuations and vortex structures upon the thermal state of the flow. The initial two-dimensional roll up of the shear layer is visible in the temperature field with structures consistent with Kelvin-Helmholtz instabilities. The three-dimensional unsteadiness of the turbulent flow is presented in the streamwise slices of Figure 4b "(Multimedia view)" and 4c "(Multimedia view)". At the reattachment location, the presented temperature field shows an enhancement in the turbulent mixing with the presence of three-dimensional vortical structures of varying sizes. The coarsened underside of the plate and its effect upon the thermal field identifies

FIG. 5: Vortex structures within the recirculation and reattachment regions using the λ_2 criterion.
"(Multimedia view)"

FIG. 6: Reattachment location. LES ($\blacktriangle$), $k-\omega$ SST ($\blacksquare$), Ota and Yanaoka¹⁰ ($\blacktriangleleft$), Tafti and Vanka¹¹ ($\blacktriangleright$), Yanaoka et al.¹² ($\blacklozenge$) and Ota et al.⁵⁶ ($\circ$).

the expected under-resolved flow, but presents the large scale disturbances that could drive any coupling between the flow above and below the plate.

The aforementioned turbulent behaviours are presented with the λ_2 criterion in Figure 5 "(Multimedia view)". Initial instabilities in the shear layer show a short two-dimensional spanwise vortex, which breaks down into three dimensional hairpin structures. Hairpin structures elongate as the flow develops across the reattachment location and downstream. Within the recirculation region, very small turbulent structures are unveiled within the large scale recirculating flow.

Figure 6 shows predictions of time-averaged reattachment length, L_r , with Reynolds number, comparing previous numerical studies^{10–12} and the present studies with the experimental correlation of Ota et al.⁵⁶. The time-averaged reattachment length is defined as the distance from the plate leading-edge to the location of zero time-averaged skin friction coefficient. Good agree-

FIG. 7: Comparison of friction coefficient calculated to pulsed wall probe experimental results.

FIG. 8: Time and spanwise-averaged Nusselt number compared to the experimental case of Ota and Kon¹ and two-dimensional $k\text{-}\omega$ SST solutions.

ment is shown the to experimental correlations of Ota et al.⁵⁶. A reattachment length of $4.6H$ is predicted by the present LES. The SST RANS solution provides a reattachment length of $5.7H$ showing an overestimation of 24%.

The time-averaged skin friction coefficient, C_f , is presented in Figure 7. Within the recirculation region, distinguished by the negative value of C_f , the location of minimum C_f is correctly captured. The peak value occurs in the redeveloping region followed by a decreasing value reflecting the development of the boundary layer. Recirculation and redevelopment C_f trends are captured by the SST RANS, but offset by the overestimation of the reattachment length while the magnitude of the wall shear stress is over predicted in the recirculation region.

Time-averaged Nusselt number predictions of the present LES are shown to be in good agreement with the experimental case of Ota and Kon¹. The peak Nusselt number is present at a location just upstream of the reattachment location, consistent with the long separation bubble observations

FIG. 9: Non-dimensional velocity profiles over plate. LES (—), Ota and Kon¹ (◦) and Ota and Kon³ (◻).

FIG. 10: Non-dimensional temperature profiles over plate. Legend see Figure 9.

FIG. 11: RMS streamwise velocity development through the recirculation and reattachment region. LES (—) and PDA⁷ (◦).

of Sparrow et al.⁵⁷. A small underprediction of the Nusselt number is shown in the redeveloping region. Results of the RANS k - ω SST solution are shown at two turbulent Prandtl numbers which

demonstrates the sensitivity of Nusselt number magnitude to the parameter. The RANS $k-\omega$ SST shows an overestimation of the peak Nusselt number location and an overprediction of the magnitude in the redeveloping region. Results from the LES simulation show an improved degree of accuracy in heat transfer predictions over the RANS SST solution.

Non-dimensional velocity profiles are compared in Figure 9 showing the time-averaged flow over the recirculation, reattachment and redeveloping regions. Good agreement is made to the experimental results of Ota and Kon¹ ($Re = 23400$) and Ota and Kon³ ($Re = 21600$). Profile positions are normalised by the reattachment length to compare for the shorter recirculation region observed by Ota and Kon¹. The accurate predictions of the LES show the recirculation bubble height is correctly predicted. Within the redeveloping region, the additional upstream turbulence from the turbulent shear layer, enhances the turbulent fluctuations of the boundary layer flow. The present LES simulation shows good agreement with the influence of this enhancement on the time-averaged profiles observed in the experiment.

Figure 10 compares profiles of non-dimensional time-averaged temperature to the experimental work of Ota and Kon^{1,3} at the Reynolds numbers of 23400 and 25000. General trends, reflecting the increased values within the recirculation region, are well captured. The LES results present a small overprediction in the recirculation and reattachment regions. Unlike the time-averaged velocity field, the temperature field is strongly correlated with the Reynolds number¹². This observation shows that these profiles have decreasing magnitudes with increasing Reynolds number. The present LES work is consistent within this observation although experimental results show comparative magnitudes.

Figure 11 shows the present LES predictions of the root-mean-square values of streamwise velocity compared to the Pulse Doppler Anemometry (PDA) measurements of Djilali and Gartshore⁷ for their non-heated blunt plate study. For these flow regimes, PDA is a well suited measurement technique, where the fluctuating components are a similar or greater value than their mean components. Following the reattachment location, excellent agreement is shown within the profiles and the enhanced turbulent fluctuations in the redeveloping boundary layer are present. Through the recirculation region, LES predictions closely match the experimental data. Small underpredictions of the root-mean-square (rms) value are present within the first $0.25H$ of the profiles of the recirculation region.

The complexity of the turbulent behaviour is observed in the near-wall behaviour of turbulent eddies. Instantaneous streamwise and normal velocities with the flow temperature are presented

FIG. 12: Instantaneous near-wall ($y^+ \approx 10$) contours of (a) streamwise velocity, (b) normal velocity and (c) temperature. "(Multimedia view)"

in Fig. 12 "(Multimedia view)" at a normal plane of $y^+ \approx 10$. A qualitative coupling is observed between the velocity and temperature fields driven by the turbulent eddies. Within the recirculation and reattachment region, normal velocities (Fig. 12b "(Multimedia view)") present eddies in varying orientations at a range of scales. The eddy structures observed in the normal velocities, reflect structures present in the near-wall temperature field (Fig. 12c "(Multimedia view)") which influence the instantaneous surface heat transfer. Heated fluid is lifted from the surface by the positive normal velocity and cooler fluid is deposited on the other side of the eddies where a negative normal velocity is present. Behind the separation point, the near-wall flow is relatively slow presenting a higher flow residence time in which the local surface heat transfer is reduced. Downstream of the reattachment region, the hairpin-like structures present themselves as streamwise velocity streaks and streamwise vorticity in the near-wall region. These structures influence

TABLE III: Description of varying turbulent anisotropy states over the barycentric map (adapted from superquadratic glyphs generated by Jofre et al.⁵⁸).

the thermal field, where hot-streaks of fluid are localised to the streaks of low streamwise velocity and the local surface heat transfer is low.

B. Turbulence Anisotropy

The preceding subsection presented the good predictive capability of LES for complex turbulent flow regimes and their respective surface heat transfer. The flowfield predicted by the RANS $k-\omega$ SST simulation overpredicted reattachment length and Nusselt number distribution. The reattachment length and recirculation region are functions of the flow momentum and their predictions are dependant on the Reynolds stress field and their influence within the momentum transport equations. This section presents the turbulent anisotropy of the Reynolds stress field resolved by the LES. A comparison is then made to the Reynolds stresses calculated from the Boussinesq hypothesis, used in two-equation RANS model approaches, such as the $k-\omega$ SST, and the EARSM model of Wallin and Johansson²⁴ with near-wall treatment. The Reynolds stress field described by these closures are taken from the LiTT analysis allowing direct comparisons to those resolved by the LES.

Turbulence anisotropy represents the deviation in the Reynolds stress tensor to its isotropic state where $\overline{u'u'} = \overline{v'v'} = \overline{w'w'} = 2k/3$. A tensor describing the turbulence anisotropy, a_{ij} , is calculated using Eq. 25 with the resolved LES Reynolds stresses and turbulent kinetic energy.

$$a_{ij} = \frac{\overline{u'_i u'_j}}{k} - \frac{2}{3} \delta_{ij} \quad (25)$$

Predictions of the turbulence anisotropy are made with the present LES data using Eq. 25. Visualisation of the anisotropy is achieved using Barycentric triangles. Unlike the classic invariance or Lumley maps, there is no visual bias to a particular turbulent state. The three corners represent the bounding behaviour of the anisotropy. One component turbulence presents turbulence with a single fluctuating component. Two component turbulence is a state at which two fluctuating components are present and are of equal value. Finally, three component turbulence represents an isotropic state. Each point within the triangle is a realizable state of the turbulence anisotropy. Table III presents a visual aid for the descriptions of turbulence where the dimensions of the shape reflect the relative magnitudes of the normal components. Formulation of the barycentric triangles is achieved by mapping the eigenvalues of a_{ij} on to the triangle.

Figure 13 presents the barycentric triangles for the anisotropy resolved by the LES at six profiles across the recirculation and redeveloping regions. At the wall, all profiles express the two-component limit with near-equal components of streamwise and spanwise normal Reynolds stress. Away from the wall, the turbulence anisotropy progresses to the middle right section of the triangle. Through the recirculation region (Fig. 13a - 13b), the turbulence anisotropy progresses towards the axisymmetric contraction limit for the near-wall region ($Y/H < 0.5$). This condition represents the near-equal components of streamwise and spanwise normal Reynolds stress and an increasing normal component as the profile ascends from the wall. Following the departure from the axisymmetric limit, all profiles show the turbulence anisotropy develops towards the axisymmetric expansion limit prior to their descent back towards the two-component limit away from the wall. Within the redeveloping boundary layer, at $X/L_r = 2.00$, the turbulence anisotropy, for a large section of the profile, is in close proximity to the plane-strain condition. The near-wall region is dictated by a similar curve to $X/L_r = 1.50$, close to the two-component limit, but with a greater divergence from the axisymmetric contraction condition.

Clear deficits are present in the modelled anisotropy from the Boussinesq hypothesis. The formulation ensures the anisotropy is linear with the strain-rate, shown in Eq. 25, which is not true for most engineering flows. The time-averaged state of this flow regime is statistically two-dimensional so $\overline{\partial/\partial z} = 0$. It can then be shown that $\overline{w'w'} = \frac{2}{3}k$, while $\overline{u'u'} = \frac{2}{3}k - \nu_t \frac{\partial \overline{U}}{\partial x}$ and $\overline{v'v'} = \frac{2}{3}k + \nu_t \frac{\partial \overline{U}}{\partial x}$, when we consider the incompressible nature of this flow.

$$a_{ij}^{BH} = -\frac{\nu_t}{k} S_{ij} \quad (26)$$

FIG. 13: Turbulent anisotropy resolved by the LES across the recirculation, reattachment and redevelopment regions. (a) $X/L_r = 0.5$, (b) $X/L_r = 1.0$ and (c) $X/L_r = 2.0$.

FIG. 14: Turbulence anisotropy of the Boussinesq hypothesis across the recirculation, reattachment and redevelopment regions. (a) $X/L_r = 0.5$, (b) $X/L_r = 1.0$ and (c) $X/L_r = 2.0$.

FIG. 15: Turbulence anisotropy of the EARSIM across the recirculation, reattachment and redevelopment regions. (a) $X/L_r = 0.5$, (b) $X/L_r = 1.0$ and (c) $X/L_r = 2.0$.

Figure 14 presents the anisotropic state of the Reynolds stresses modelled from the Boussinesq hypothesis for a profile within the recirculation and redeveloping region. As expected, the plain-strain anisotropy is observed and comparisons to the resolved LES anisotropy of Figure 13 presents large differences in the anisotropic state. Alongside the plain-strain behaviour, the near-wall state of turbulence protrudes from an isotropic rather than a two-component state as seen by the wall-restricted normal fluctuations present in the LES solution.

The anisotropic state of the EARSM is shown in Figure 15. For two dimensional flows can also be analytically shown to follow this plane-strain state away from the wall, where $f_1 = 1$. However, the plain-strain behaviour is enhanced with the inclusion of the β_4 term, $S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}$. Like the Boussinesq hypothesis, the spanwise component $\overline{w'w'} = 2/3k$ and does not deviate from this state like the LES. The near-wall treatment enables a deviation for the plain-strain behaviour, which provides the two-component limit at the wall as shown in Figure 15. Although, the anisotropic behaviour is improved, deviation to the resolved LES is still observed.

Figures 16, 17 and 18 compare the Reynolds stress profiles of $\overline{u'u'}$, $\overline{v'v'}$ and $\overline{u'v'}$ respectively. Comparisons of the streamwise component, Figure 16, show the EARSM captures the peak values across the recirculation region where the turbulent fluctuations are enhanced. In the redeveloping boundary layer, the profiles remain in close agreement with those from the LES. Close to the wall, a small underprediction is shown following the expected peak value in turbulent boundary layer flows. Predictions of $\overline{u'u'}$ by the Boussinesq hypothesis show a considerable deviation for the LES, where the elevated stress in the recirculation region is underestimated. Figure 17 shows the normal Reynolds stress component, where the EARSM shows better agreement to the LES than the Boussinesq hypothesis. However, the rear of the recirculation region and downstream, the peak value occurs closer to the wall in the profile than shown by the LES. Comparisons of the turbulent shear stress, Figure 18, show that the Boussinesq hypothesis, with the LiTT description of eddy viscosity, is in very close agreement to the resolved LES stress. The EARSM predictions of $\overline{u'v'}$ repeat the behaviour shown in the normal stress component, $\overline{v'v'}$, where the peak location is present closer to the wall.

The impact of the different Reynolds stresses on the flow field can be qualitatively assessed by recalling the RANS momentum equation (Eq. 1). The smaller streamwise Reynolds stress component from the Boussinesq hypothesis would permit a greater streamwise momentum to balance the momentum transport equation. This would provide a longer recirculation region as is present in the full $k-\omega$ SST RANS simulation shown in Figure 7. An accurate prediction of the Reynolds

FIG. 16: Streamwise Reynolds stress ($\overline{u'u'}$) comparison of constitutive relations to resolved LES.
LES ($\circ$), LiTT-BH (---) and LiTT-EARSM (—).

FIG. 17: Normal Reynolds stress ($\overline{v'v'}$) comparison of constitutive relations to resolved LES.

Legend see Figure 16.

FIG. 18: Turbulent shear stress ($\overline{u'v'}$) comparison of constitutive relations to resolved LES.

Legend see Figure 16.

stress field, and the anisotropy in which the stresses are distributed to the momentum equations,

is required to achieve a good estimation of the reattachment length and recirculation region. The EARSIM shows the improvement and is a suitable candidate for separated and reattached flow regimes.

C. Turbulent Heat Flux

The previous section showed the complex turbulent anisotropy in this separated and reattached flow regime resolved by the LES. The LiTT analysis showed the Reynolds stress field provided by the Boussinesq hypothesis does not accurately capture the “true” anisotropy. This section investigates the accuracy of turbulent heat flux closures with the LiTT analysis, where the modelling of the turbulent heat flux is an important component of turbulent heat transfer predictions, alongside the Reynolds stresses.

LES predictions of turbulent heat flux are shown in Figure 19a and 19b for the streamwise ($\overline{u'T'}$) and normal ($\overline{v'T'}$) components respectively. Peak values are observed within the shear layer, where the increased presence of turbulent fluctuations enhance the mixing. The magnitude of the turbulent heat flux decreases in the downstream, redeveloping region, although the presence of the turbulent shear layer continues to be seen in downstream profiles.

TABLE IV: Table of coefficients for the studied diffusivity closures.

	Coefficient	Common Values	Value Used
GDH	Pr_t	0.8 - 1.0	0.85
GGDH	c_θ	0.3	0.3
HOGGDH	c_θ	0.6 ²⁹	0.4

Comparisons of the three RANS diffusion based closures for the turbulent heat flux vector, calculated from the LiTT analysis, are shown in the contours of Figure 19. Formulation of the Gradient Diffusion Hypothesis (GDH), Generalised GDH (GGDH) and Higher-Order GGDH (HOGGDH) are defined in Eq. 5, 6 and 7 respectively. The magnitude of the turbulent heat flux vector is controlled by the coefficients Pr_t and c_θ which are defined in Table IV. The HOGGDH closure, with the commonly used coefficient, achieved a large overprediction, and tests with a value of 0.4 showed a better agreement to those resolved by the LES.

FIG. 19: Contours of streamwise and normal turbulent heat flux ($\overline{u'T'}$ and $\overline{v'T'}$ respectively) for the LES ((a) $\overline{u'T'}$ and (b) $\overline{v'T'}$) and comparisons of GDH ((c) $\overline{u'T'}$ and (d) $\overline{v'T'}$), GGDH ((e) $\overline{u'T'}$ and (f) $\overline{v'T'}$), HOGGDH ((g) $\overline{u'T'}$ and (h) $\overline{v'T'}$) and EASFM ((i) $\overline{u'T'}$ and (j) $\overline{v'T'}$).

The closure of choice within most commercial CFD solvers is the GDH because of its simplicity. The GDH aligns the turbulent heat flux vector to the mean temperature gradient. Complex flows and their respective heat transfer do not comply to such a rule. A lack of streamwise temperature gradient within the current study results in a negligible streamwise turbulent heat flux component shown in Figure 19c. Figure 19d shows limited success in modelling the normal component, with good agreement seen in the shear layer, but an overprediction in the near-wall region where high normal gradients are present from the prescribed wall heat flux.

Inclusion of the Reynolds stresses in the GGDH and HOGGDH closures provide a degree of anisotropy to the turbulent diffusivity, permitting the turbulent heat flux to deviate from the alignment to the mean temperature gradient provided by the GDH closure. The quadratic products

FIG. 20: Comparison of the angle of the turbulent heat flux vector, ϕ , between the resolved LES and RANS diffusion hypothesis closures. LES ($\circ$), LiTT-GDH ($\cdots$), LiTT-GGDH ($---$) and LiTT-HOGGDH ($—$).

of the Reynolds stresses in the HOGGDH shows qualitatively better agreement to the turbulent heat flux contours of the LES. Deviations to the magnitude are present and some overprediction exists in the near-wall region. The results of the EASFM predictions of turbulent heat flux are shown in Figure 19i and 19j for the streamwise and normal components respectively. The two contours show an underprediction across the flow when compared to the LES, although the general trends are captured, showing the elevated turbulent heat flux across the shear layer and downstream.

A useful measure for assessing the suitability of these closure is the angle formed by the modelled turbulent heat flux. The angle, ϕ , is defined by the normal and streamwise turbulent heat flux (Eq. 27).

$$\frac{\overline{v'T'}}{\overline{u'T'}} = \tan \phi \quad (27)$$

Figure 20 presents the turbulent heat flux angles from four profiles across the recirculation and redeveloping region. Within the redeveloping region, the modelled angle described by the HOGGDH closure is in close agreement with the turbulent heat flux resolved by the LES. Angles defined by the GDH and GGDH do not agree with the LES, providing a greater contribution to the normal component. In the near-wall region, a angle difference of $\pi/2$ is present between the LES and the GDH, highlighting the defects of the standard GDH model's ability to provide accurate turbulent diffusion close to the wall. Within the recirculation region ($0.5L_r$), large differences are observed between the angles of the LES results and all closures. The LiTT analysis shows that the

FIG. 21: Comparison of the angle of the turbulent heat flux vector, ϕ , between the resolved LES, HOGGDH and EASFM models. LES ($\circ$), LiTT-HOGGDH (---) and LiTT-EASFM (—).

HOGGDH prediction of the turbulent heat flux angle better describes the turbulent heat flux than the GDH and GGDH closures.

Figure 21 shows the turbulent heat flux angle predicted by the EASFM and compared against the HOGGDH and LES results. The EASFM results show the angle is in better agreement to the LES than the HOGGDH over all profiles. At $0.5L_r$, the EASFM show a small deviation from the LES, but is in better agreement with the profile than the diffusion hypothesis closures. Within the reattachment location and redeveloping regions, the EASFM shows agreement across the full profile where the HOGGDH showed some deviation away from the wall. However, the turbulent heat flux magnitude predicted by the EASFM is underestimated.

With good agreement of the turbulent heat flux shown by the HOGGDH model in Fig. 19 and Fig. 20, an optimal spatial prediction of its coefficient, c_θ , is sought to achieve the correct magnitude of the turbulent heat flux vector. Calibrating the coefficient by a single component of the turbulent heat flux can provide conflicting values (as shown in Figure 22) for regions where ϕ is not accurately captured. This is true within the recirculation region where the angle dictated by the model is shown to deviate from the LES prediction. An optimal coefficient is derived using a weighted approach described by Eq. 28. Milani et al.⁵⁹ used such an approach for calculating an LES isotropic turbulent diffusivity field to improve GDH predictions. The optimal coefficient exists between a calibration based upon $\overline{u'T'}$ and $\overline{v'T'}$. Figure 22 shows that over the shear layer, the optimal coefficient has a peak within the recirculation region followed by a minima at the reattachment location before ascending across the redeveloping region. Over the initial shear layer development a large, optimal c_θ , attempts to obtain the large turbulent heat flux values that are present in the LES turbulent heat flux.

FIG. 22: An optimal value of c_θ through the shear layer and downstream with comparison to directionally calibrated coefficients.

FIG. 23: Profiles of optimal c_θ across the redeveloping region.

$$c_{\theta,opt} = -\frac{k}{\tau_t} \frac{\overline{u'_i T'} \left(\overline{u'_i u'_k u'_k u'_j} \frac{\partial \overline{T}}{\partial x_j} \right)}{\left(\overline{u'_l u'_m u'_m u'_n} \frac{\partial \overline{T}}{\partial x_n} \right) \left(\overline{u'_l u'_m u'_m u'_n} \frac{\partial \overline{T}}{\partial x_n} \right)} \quad (28)$$

The increasing optimal c_θ across the redeveloping region, seen in Figure 22, is investigated further through profiles presented in Figure 23. Profiles are equally spaced between $X = L_r$ and $X = 2L_r$. Values are blanked where the turbulent heat flux vector is small ($|\overline{u'_i T'}| < 0.1$). Away from the wall, a peak c_θ is shown within the profile, at a location consistent with the shear layer location. The peak is observed to decay downstream where the profile tends towards a linear distribution. As the redevelopment of the boundary layer progresses, the optimal c_θ is shown to increase but does not tend towards an asymptotic value at this early stage of redevelopment. Within the near-wall region, a large value of c_θ is required across all profiles where a considerable streamwise

component is present. The results highlight that a constant c_θ will evidently show deviations, but a range of plausible constants exist from this *a priori* analysis covering the differing regions of this flow. Modelling improvements could be furthered by introduction of a spatially varying c_θ following the results shown.

The investigation of closures and the derivation of an optimal spatial c_θ has been made by use of the Reynolds stresses resolved by the LES. However, common two-equation RANS models will construct the Reynolds stress field from the Boussinesq hypothesis or at best use an EARSM approach as shown earlier. As Reynolds stresses predicted by the Boussinesq hypothesis deviate from those resolved by the LES, the angle and magnitude of the turbulent heat flux vector will deviate from the LiTT analysis. Reynolds stresses modelled by the EARSM were shown to be closely aligned to those predicted by the LES and will provide an improved Reynolds stress field for the HOGGDH closure

IV. CONCLUSIONS

LES of the geometrically simple heated blunt plate showed a variety of complex turbulent behaviours for the moderate Reynolds number of 21,600 studied. Such a configuration holds physical relevance to many engineering flow problems. This paper showed that LES using the WALE sub-grid scale model provided accurate predictions of the surface heat transfer and turbulent flow behaviour. An LES informed Turbulent Transport (LiTT) analysis allowed for closures used in RANS to be directly compared to the well-resolved LES flow field and the main findings of the comparison can be summarised below.

- LES results showed the near-wall anisotropy is characterised by a profile ascending from the two-component limit. Within the recirculation bubble the profiles develop towards the turbulence axisymmetric contraction limit. In the redeveloping region, the turbulence anisotropy develops to a state closer to the plane-strain behaviour exhibited by the Reynolds stresses modelled with the Boussinesq hypothesis and the EARSM away from the wall.
- Results of the EARSM show good agreement to the streamwise and normal component of the Reynolds stress tensor. The anisotropic state with near-wall treatment provides the trend from the two-component limit is provided, but outside of this treatment, the flow still remains in a plane-strain anisotropic state.

- The GDH and GGDH closures fail to reproduce the turbulent heat flux vector. The HOGGDH successfully improves predictions of the turbulent heat flux angle in the redeveloping region but deviates in the complex recirculating flow prior to the reattachment location. The complex EASFM model improved the modelled angle across all regions of the flow, but did not agree with the magnitude shown in the LES.
- A spatially varying coefficient, c_θ , for the HOGGDH turbulent heat flux closure was produced to achieve the magnitude of the LES resolved turbulent heat flux and show ideal values for advancing modelling knowledge. A large value is required in the near-wall region to provide the correct magnitude while the redeveloping region aft of the recirculation region showed values ranging from 0.3 to 0.6. Contours showed the turbulent heat flux field was comparable to the LES.

This paper used well-resolved LES data to demonstrate that the common closures used in RANS simulations do not accurately capture the Reynolds stress and turbulent heat flux fields in turbulent separated and reattached flow. Existing models such as EARSM, HOGGDH and EASFM models show improvements but do not provide a robust solution yet. Our work highlights the need for an advanced turbulent heat flux model that will accommodate improvements to the modelled turbulent heat flux vector within the shear layer and recirculation region, where the turbulent heat flux angle is well predicted by the EASFM but the magnitude is underpredicted. Recent advances to data-driven modelling using high-fidelity datasets demonstrates itself as an encouraging resolution to closure problems of which the current study provides a foundation.

NOMENCLATURE

Roman and Greek

a_{ij}	Turbulence anisotropy tensor
c_θ	Model coefficient
C_f	Skin friction coefficient
H	Plate Thickness (m)
k	Turbulent kinetic energy (m^2/s^2) or Thermal conductivity (W/mK)
L_r	Reattachment length (m)
p	Pressure (Pa)

$\overline{u'_i u'_j}$	Reynolds stress tensor (m^2/s^2)
$\overline{u'_i T'}$	Turbulent heat flux vector (mK/s)
S_{ij}	Strain rate tensor (s^{-1})
T	Temperature (K)
U, V, W	Velocity (m/s)
u', v', w'	Fluctuating velocity (m/s)
α_t	Turbulent diffusivity (m^2/s)
δ_{ij}	Kronecker delta
ν_t	Eddy/Turbulent viscosity (m^2/s)
ϕ	Turbulent heat flux angle
ρ	Density (kg/m^3)
τ_t	Turbulent time scale (s)
ω	Specific dissipation rate ($1/s$)

Subscript

∞	Freestream parameter
res	Resolved parameter
rms	Root-mean-square parameter
sgs	Sub-grid scale parameter
w	Wall value

Non-dimensional Numbers

Nu	Nusselt number
Pr_t	Turbulent Prandtl number
Re	Reynolds number

Acronyms

EARSM	Explicit Algebraic Reynolds Stress Model
EASFM	Explicit Algebraic Scalar Flux Model
GDH	Gradient Diffusion Hypothesis
GGDH	Generalised Gradient Diffusion Hypothesis
HOGGDH	Higher-Order Generalised Gradient Diffusion Hypothesis
LES	Large Eddy Simulation
LiTT	LES informed Turbulent Transport

PDA	Pulse Doppler Anemometry
RANS	Reynolds-Averaged Navier Stokes
SGS	Sub-Grid Scale
WALE	Wall-Adaptive Local Eddy-viscosity

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Inlet
 U_∞, T_∞

(b)

(c)

(a) Slice $Z = 0$

(b) Slice $X = 2.5H$

(c) Slice $X = L_r$

(a) Streamwise velocity

(b) Normal velocity

(c) Temperature

0.00

0.25

0.50

0.75

1.00

Y/H

0.00

0.25

0.50

0.75

1.00

Y/H

0.00

0.25

0.50

0.75

1.00

Y/H

Three Comp.