Tunable Frequency Filter Based on Twisted Bilayer Photonic Crystal Slabs (Supplementary Material)

Beicheng Lou[†] and Shanhui Fan[*,‡]

† *Department of Applied Physics, and Ginzton Laboratory, Stanford University, Stanford, California 94305, USA*

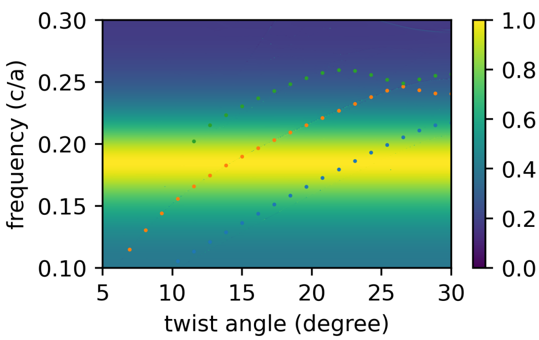
‡*Department of Electrical Engineering, and Ginzton Laboratory, Stanford University, Stanford, California 94305, USA*

E-mail: shanhui@stanford.edu

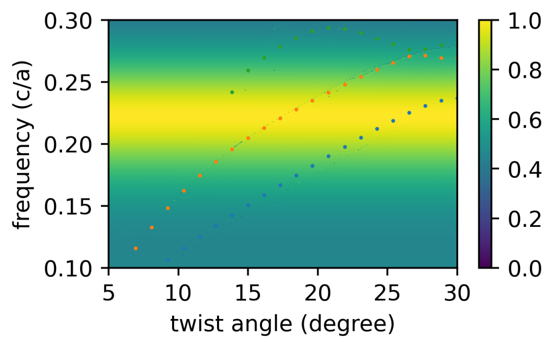
Here we provide further discussion on the effect of homogeneous slabs, the effect of air gap between layers and the effect of finite beam size.

Effect of homogeneous slabs

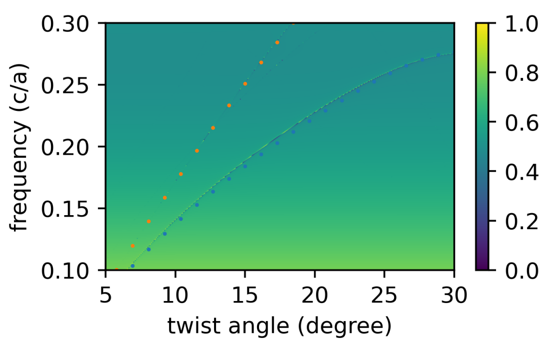
The homogeneous slabs in the design are intended for tuning the background transmission. Ideally, one wants the background transmission to be near unity in the frequency range of interest, so that the transmission dips for guided resonances have the desired lineshape for notch filter application. One could certainly design the dielectric pattern in the photonic crystal slabs to achieve both having a near-unity background transmission and having a guided resonance with desirable lineshape and tunable across the frequency range. However, it is much more efficient if one adjusts the dielectric pattern for the guided resonance and adjusts the homogeneous layer for the transmission background, where the adjustment of the latter only causes a relatively mild shift on the resonant frequency.



(a) Original structure



(b) Homogeneous slabs $\varepsilon_r = 8$



(c) Homogeneous slabs removed

Figure S1: A comparison between transmissions as a function of frequency and twist angle for (a) the original structure for Fig. 4 in the main text, (b) the original structure with the homogeneous slabs having dielectric constant $\varepsilon = 8$ instead of $\varepsilon = 12$, (c) the original structure with the homogeneous slabs removed.

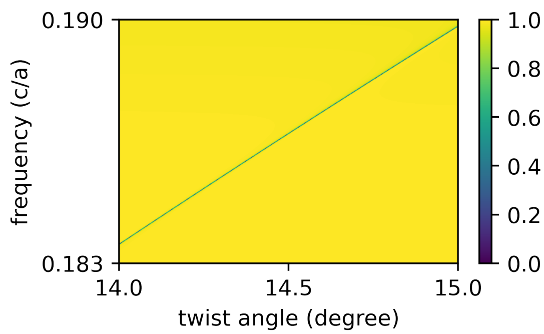
Fig. S1 shows the transmission as a function of frequency and twist angle for the original structure from the main text (Fig. S1a), the original structure with the homogeneous slabs having dielectric constant $\varepsilon_r = 8$ instead of $\varepsilon_r = 12$ (Fig. S1b), and the original structure with the homogeneous slabs removed (Fig. S1c). A comparison between Fig. S1a and S1b illustrates that varying the dielectric constant of the homogeneous layers adjusts the transmission background without significantly shifting frequencies of the guided resonances. With homogeneous slabs having $\varepsilon_r = 8$, the structure could serve as a notch filter in a higher frequency range e.g. between $0.21c/a$ and $0.23c/a$.

On the other hand, if the homogeneous slabs are removed, as shown in Fig. S1c, while there are still guided resonances with angle-dependent resonant frequencies, no near-unity transmission background could be found across a very large parameter regime. One may still obtain desirable transmission background in a much higher frequency regime, but higher frequency regime typically leads to complexity in guided resonances and significant diffractions, which are undesirable.

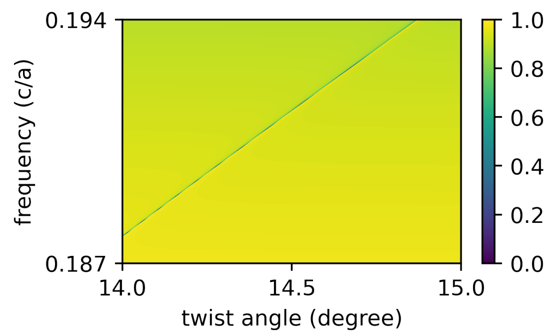
Effect of air gap between layers

While the structure shown in the main text does not have any gap between the layers, low-index gaps are inevitable in practice due to fabrication constraints, especially at the interface between the two photonic crystal layers that are twisted against each other.

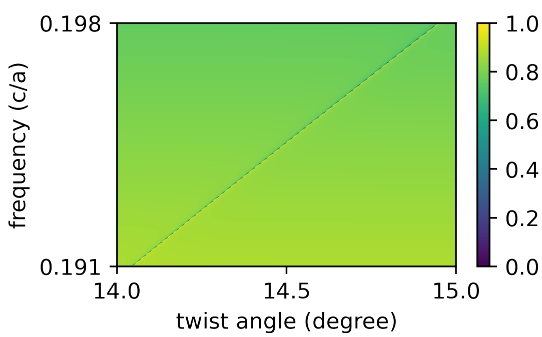
Here we consider an air gap of various sizes between the two photonic crystal layers. Fig. S2 shows a comparison between the transmission spectra for different cases. The frequency of the resonance at each twist angle tends to be higher as the size of air gap increases. The linewidth of the transmission spectrum corresponding to the resonance tends to be smaller as the size of air gap increases. In the limit where the air gap is infinite and the two layers completely decouple, the linewidth will tend towards 0.



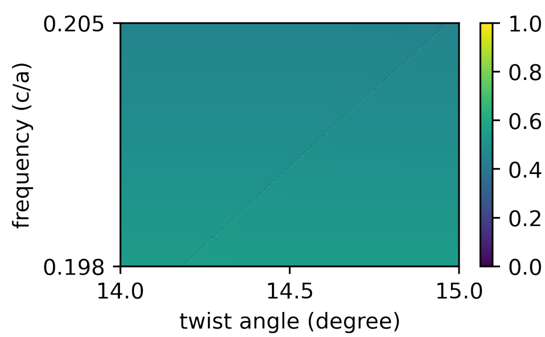
(a) No gap between twisted layers



(b) With a gap of $0.02a$



(c) With a gap of $0.04a$



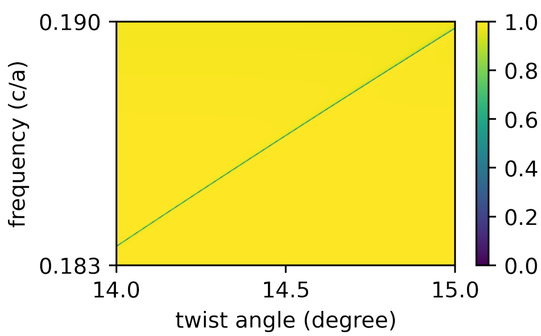
(d) With a gap of $0.1a$

Figure S2: A comparison between transmissions as a function of frequency and twist angle for the structure shown in the main text (a) without any gap between the twisted photonic crystal layers, or with a gap of (b) $0.02a$, (c) $0.04a$, (d) $0.1a$ between the twisted photonic crystal layers. Here a is the lattice constant of the photonic crystal layers.

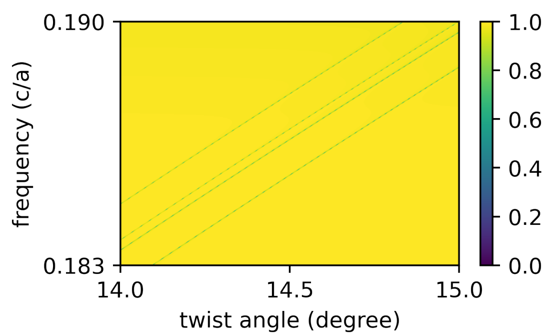
Effect of finite beam size

The structure in the main text is analyzed under the assumption of infinite structure and plane wave incidence, whereas in practice both the structure and the incident beam are finite. While the assumption of infinite structure holds well when the structure is large enough (typically $\sim$ 100s of periods), the finiteness of beam size could lead to more notable deviation from the ideal assumptions, which typically results in broadening of resonant features.

A finite-size beam could be decomposed into a superposition of plane waves with various incident angles. The total transmission can also be calculated as a weighted sum of each component's transmission. For example, for a Gaussian-shaped beam at frequency $0.19c/a$, where c is the speed of light and a is the lattice constant, the incident wave has wave vector of magnitude $k_0 = 0.19 * 2\pi/a$. If the spread of the Gaussian distribution is around $11a$, then the Fourier transform of the Gaussian beam will have a spread of around $0.017 * 2\pi/a$. Such an in-plane wave vector correspond to incident plane wave at an off-normal angle of $\theta = 1°$. Namely, the plane wave decomposition of the Gaussian beam will involve components mostly with off-normal angle less than $1°$.



(a) $\theta = 0$



(b) $\theta = 1°$

Figure S3: Transmission as a function of frequency and twist angle for the structure shown in the main text with incident plane wave at off-normal angle $\theta = 0$ vs $\theta = 1°$

When the incident plane wave is off-normal, a four-fold degeneracy will be lifted in the lowest-order description of twisted bilayer photonic crystal slabs.[1] Fig. S3 shows transmission as a function of frequency and twist angle for the structure shown in the main text under normally incident plane wave vs under a slightly off-normal plane wave. If the incident beam is finite, the total transmission will be a sum of transmissions at various incident angles that interpolate between Fig. S3a and Fig. S3b. The scale of lineshape broadening for finite beam size can thus be calculated.

Without simulating various incident angles, one could predict the scale of broadening from merely the transmission properties under normal incidence as shown in Fig. S3a. The twisted bilayer system has Moire wave vector of magnitude $m(\alpha) = 2\sin(\alpha/2) * 2\pi/a$, where α is the twist angle.[1] If the incident light has wave vector of magnitude k_0 with off-normal angle θ, then the in-plane component of the wave vector has magnitude $k_{\|} = k_0 * \sin(\theta)$. Here we hope to find the sensitivity of resonant frequency f with respect to the incident angle θ:

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial k_{\|}} \frac{\partial k_{\|}}{\partial \theta} \tag{1}$$

While knowing the sensitivity of resonant frequency f with respect to the twist angle α from the transmission properties at normal incidence (Fig. S3a):

$$\frac{\partial f}{\partial \alpha} = \frac{\partial f}{\partial m(\alpha)} \frac{\partial m(\alpha)}{\partial \alpha} \tag{2}$$

A simple substitution of $\frac{\partial f}{\partial m(\alpha)} = \frac{\partial f}{\partial k_{\|}}$ based on the lowest-order theory[1] gives:

$$\begin{aligned} \frac{\partial f}{\partial \theta} &= \frac{\partial f}{\partial \alpha} \left(\frac{\partial m(\alpha)}{\partial \alpha} \right)^{-1} \frac{\partial k_{\|}}{\partial \theta} \\ &= \frac{\partial f}{\partial \alpha} \frac{k_0}{2\pi/a} \frac{\cos(\theta)}{\cos(\alpha/2)} \\ &= \frac{\partial f}{\partial \alpha} \frac{f}{c/a} \frac{\cos(\theta)}{\cos(\alpha/2)} \end{aligned} \tag{3}$$

Therefore, if we use $\Delta_\theta f$ to denote the shift of resonant frequency due to incident angle varying from 0 to 1°, and $\Delta_\alpha f$ to denote the shift of resonant frequency due to twist angle varying from 14° to 15°, then:

$$\Delta_\theta f \approx 0.19 * \Delta_\alpha f \tag{4}$$

which perfectly matches the result shown in Fig. S3b.

References

(1) Lou, B.; Zhao, N.; Minkov, M.; Guo, C.; Orenstein, M.; Fan, S. Theory for Twisted Bilayer Photonic Crystal Slabs. *Phys. Rev. Lett.* **2021**, *126*, 136101.