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Transition from noise-induced to self-sustained current spiking generated by a NbO_x thin film threshold switch

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A transition to self-sustained current oscillations was investigated in a circuit with a NbO_x thin film device, acting as an artificial neuron. Above a threshold of the applied voltage, the circuit begins to demonstrate persistent but rather random current spiking, which becomes more regular with increasing applied voltage. Experimental measurements reveal two scaling regions of the interspike interval dependence on source voltage: an initial logarithmic dependence which transitions to a power law with exponent $-1/2$. Using a numerical model of the artificial neuron circuit, we observe good agreement with our experimental data. We show that the logarithmic scaling behavior is associated with noise-induced spiking, whilst the power law is attributed to noisy self-sustained oscillations which would exist in the system even with the absence of noise. These findings stress the importance of noise in the dynamics of NbO_x threshold switches and shed light on their spiking response to variation of the input signals. The reported approach to spiking control in NbO_x artificial oscillation neurons can help to develop an optimal design for neuromorphic neural networks and oscillator-based computing systems.

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One of the main obstacles in the field of neuromorphic computing is the limited performance of artificial neural networks using traditional hardware based on Von Neumann architecture. These systems tend to be power hungry and computationally intensive.¹ Metal oxide memristor based neuromorphic devices have become a focus of investigation offering a possible solution to this problem due to their demonstrated potential for low energy consumption and high scalability.²⁻⁴

Spiking neural networks (SNNs) and oscillator-based computing are an area of recent study, which has shown devices with high computational ability and energy efficiency.⁵⁻⁸ SNNs are designed to emulate biological neural systems, which communicate using action potential pulses (spikes).⁹ Hence, a SNN requires a component capable of converting dc or ac signals into spikes. Further, oscillator-based computing can be realized with oscillators made of threshold switches (volatile memristors).¹⁰⁻¹²

One promising type of device is based on NbO_x thin films, which exhibit negative differential resistance (NDR) behavior that coincides with voltage driven threshold switching in NbO_x .^{13,14} When inserted into a series circuit with a load resistance R_L , of a sufficient value to place the load line within the NDR region, and a dc source voltage V_S , the devices exhibit self-sustained oscillation.^{15,16} Such a circuit effectively resembles a Pearson-Anson oscillator¹⁷ as the NbO_x device provides NDR, and also has a parasitic capacitance [Fig. 1 (a)]. Hence, a NbO_x thin film device can convert dc voltage signals into high frequency spiking output,¹⁸⁻²⁰ which is analogous to a biological afferent nerve converting sensory input to action potentials.^{21,22} Artificial neural devices using self-oscillating NbO_x have been demonstrated experimentally,^{23,24} with output spike frequency being shown to increase with increasing input voltage.^{19,23,25}

In a typical oscillating regime, output spiking period T scales with the source voltage V_S in the expected form for a Pearson-Anson circuit.¹⁹

$$T \propto R_L C_D \ln \left(\frac{V_S - V_h}{V_S - V_{th}} \right) \quad (1)$$

where V_h is the hold voltage, V_{th} is the threshold voltage, and C_D is the parasitic capacitance of the NbO_x device. The value V_{th} is the voltage required for the device to switch into its low resistance state, and V_h is the hold voltage below which the device drops back into the high resistance state. It is assumed that the spiking period is dominated by the capacitor charging process, hence this oscillating regime is rather deterministic in its nature.

Additional noise and stochasticity components present in the spiking process have been shown to provide better adaptation to changes in the network training environment.²⁶ Stochastic SNNs

are also observed to improve network speed and efficiency compared to non-spiking stochastic methods.^{27–29} It is well-known that spiking demonstrated by NbO_x devices becomes more irregular closer to the threshold,^{19,25} hence quantitative analysis of spiking in this region requires consideration of noise in the system, and is needed in order to control and utilize stochastic features in SNN applications or to achieve a controllable transition into dynamical chaos or synchronization for oscillator-based computing applications.³⁰

In this Letter we analyze spiking behavior in circuits with NbO_x thin film devices for voltage bias close to the threshold of generation. To do so, across a range of V_S we calculate the average interspike interval (ISI), $\bar{T}$, as

$$\bar{T} = \frac{1}{S-1} \sum_{i=1}^{S-1} (t_{i+1} - t_i) \quad (2)$$

where t_i is the temporal position of the maximum point of an i -th spike, and S is the total number of spikes at a given voltage value V_S . We observe spiking that transitions from stochastic to more deterministic with increasing V_S , with the dependence of $\bar{T}(V_S)$ transitioning from scaling as a logarithmic law to a power law with exponent $-1/2$. Our numerical simulations suggest that the two different scalings are defined by interplay between noise-induced and self-sustained oscillations. In the former case, the spiking is persistent but rather noise-induced and can not exist without noise in the system. In the latter case, the system undergoes an instability causing the onset of self-sustained oscillations, which are generated by the circuit even in the absence of noise and dominate all noise effects.

The NbO_x devices were fabricated as follows. A Si-doped GaN (5 μm)/AlN (300 nm)/Al₂O₃ (430 μm) wafer, provided by PI-KEM, was used as a substrate for thin film growth.³¹ Doped GaN acts as a bottom electrode for the device and hence enables vertical device characterization. A NbO_x thin film (25 nm) was deposited onto the substrate using reactive DC magnetron sputtering at room temperature using a Nb target and an O₂/Ar gas flow ratio of 3/5. Total chamber pressure during growth was 2.5 mTorr. X-ray photoelectron spectroscopy measurements have confirmed the formation of a NbO_x layer [see Fig. S1 in the supplementary material]. Top electrodes consisting of a 10 nm Ti layer and a 50 nm Pt layer were deposited using a shadow mask to make 100 μm diameter circular contacts. Connection to the top electrodes was made via a tungsten micro-positioner tip (0.35 μm radius) positioned on the surface of the Pt layer. Current-driven dc current-voltage (IV) characteristics of the device were measured using a Keithley 4200A-SCS, and are presented in Fig. S2 in the supplementary material. Current-driven IV sweeps demonstrate NDR

behavior that is necessary for being able to generate current oscillations.

The schematic of the series circuit for the device under test is shown in Fig. 1(a). This circuit consists of the NbO_x device, represented here by a resistance R_D and parasitic capacitance C_D in parallel, a load resistor ($R_L = 10 \text{ k}\Omega$), a monitor resistor ($R_M = 47 \text{ }\Omega$), and a source voltage (V_S). Measurements of circuit current oscillation were taken over the monitor resistor R_M . Source voltage was supplied by an RS Pro RSDG800 Waveform Generator, with source voltage and circuit current being recorded using a Picoscope 5443D.

For numerical simulation of the electrical circuit in Fig. 1(a) we used a dimensionless version of the modified memristor model proposed by Pickett et al.² to simulate a NbO_x memristor/threshold switch and Kirchhoff's current law for the artificial neuron circuit. The switching mechanism in this model is based on a phase transition of a fraction of the material in the conductive channel from insulating to metallic phase. This transition is mainly driven by Joule heating, a thermodynamic mechanism, therefore we believe that the main mechanism of the noise is related to electrical fluctuations in the system. This model has been shown to accurately describe electrical properties of NbO_x devices, including current-voltage characteristics and current spiking.² To take into account the effects of thermal noise in the system, we extended the equations to include a stochastic voltage, with an amplitude dependent upon the device resistance. Hence, the model used in our simulations is given by the set of stochastic differential equations:

$$\frac{ds}{dt} = \frac{V^2/R_D(s) + 1/\ln(s)}{\frac{1-s^2+2s^2\ln s}{s\ln^2 s} + \chi s}, \quad (3a)$$

$$\tau \frac{dV}{dt} = V_S - \left(1 + \frac{R_L}{R_D(s)}\right) V + D\xi. \quad (3b)$$

where s is the state variable representing the fraction of metallic state in the conducting channel linking the two volatile memristor terminals. All components of the equations are dimensionless. Resistance $R_D(s) = 1/(1 + \delta s^2)$, and delta correlated noise ξ (that is $\langle \xi(t)\xi(t+t_0) \rangle = \delta(t_0)$) with intensity $D = \sqrt{D_0 R_D}$ and constant D_0 . The response of the NbO_x artificial neuron to DC or slowly varying voltage is determined by five parameters: χ_s responsible for sharpness of switching between different spiking regimes, τ being 'RC' time of the circuit capacitor determining voltage relaxation in the system, δ controlling the resistance switching ratio, R_L being the load resistance, and D_0 corresponding to the noise intensity in the insulating state. A complete description of the simulation parameters can be seen in the supplementary material.

FIG. 1. (a) Diagram of the artificial neuron circuit used for experimental measurements. $R_L = 10 \text{ k}\Omega$, $R_M = 47 \text{ }\Omega$. The NbO_x device is represented by a memristive resistance R_D and capacitance C_D in parallel. (b) Voltage input to the circuit (red) from 0 to 1.98 V in 0.01 V increments, and resulting current oscillations (black). Inset: magnified view of the same data set, around the threshold.

Initially, we studied the response of our memristive circuit to a stepped increase in V_S . V_S was supplied, starting at 0 V, and increased in 0.01 V steps of 1 ms duration. Fig. 1 (b) presents the measured voltage input (red curve, right axis) and the resulting current oscillation (black curve, left axis) of the NbO_x device circuit. For smaller voltages the device demonstrates only noise current, however as V_S reaches 1.56 V the prominent spiking oscillations abruptly appear in the current response, with an amplitude (around 0.3 mA) which remains fairly consistent across a range of voltages. Typical spiking behavior is illustrated in Fig. 2 (a) for experimental measurements and (b) for numerical simulation using 3.

To characterize the timescale and regularity of spiking, average ISI ($\bar{T}$) was calculated across each voltage step for both the experimental and simulated data using 2. The number of spikes available for averaging during each voltage step ranged from 28 at 1.56 V, to 1899 at 1.98 V in our experimental data. A complete description of spike identification is presented in the supplementary material [see Fig. S5]. Fig. 3 shows the dependence $\bar{T}(V_S)$ for experimental (a) and simulated (b) data [see plots of ISI values vs. source voltage in Figure S6 in the supplementary material]. The error bars represent the standard error. Both theoretical and experimental graphs evidence that after the spikes emerge, further increase of source voltage decreases the ISI. This corresponds to a

FIG. 2. (a) Spikes generated by the NbO_x device at $V_S = 1.65$ V. (b) Spikes simulated numerically. Simulated amplitude and time values are arbitrary. All spikes shown here are within the deterministic spiking regime.

spiking frequency increase reported elsewhere.^{12,19,23} At the same time, decrease of the standard error evidences more deterministic behavior as V_S grows. In addition, above the range of V_S investigated here (not shown), the ISI begins to increase until spiking ceases entirely once a high enough input voltage is reached.

Next, our analysis showed that, both for experiment and theory, the obtained dependence $\bar{T}(V_S)$ in a close vicinity to the threshold of generation V_{th}^I is well approximated by the logarithmic law $\bar{T} = |a \ln [(V_S - V_{th}^I)/V_{th}^I] + c|$. The threshold values V_{th}^I can be found by extrapolation of the corresponding graphs for lower V_S . In Fig. 3 the red solid lines present the best fit, which returns $V_{th}^I = 1.560$ V for experimental data (a) and 0.582 for simulations (b). Device voltage was simulated using dimensionless units, hence a direct comparison to experimental threshold voltage is not possible. The values of other fitting parameters are given in Table S1 in the supplementary material. Note, the fitting parameter c can represent a normalization constant of the intrinsic delay time associated with non-voltage-driven dynamics, as described by Jiang et al.³²

With increase of V_S the character of the scaling both in experimental and theoretical dependencies visibly changes, and is accompanied by appearance of a kink on the corresponding graph of $\bar{T}(V_S)$. This kink can also be seen in ISI standard deviation plots [see Fig. S7 in the supplementary material] suggesting a change in the regularity of spiking as V_S increases. The scaling transition

FIG. 3. Dependence of average ISI on V_S with fits of logarithmic (red lines), and power law scaling (blue dashed lines), respectively: (a) measured experimentally; (b) simulated numerically using (3). Simulated $\bar{T}$ values have been re-scaled to match the first experimental data point.

becomes even more obvious in log-lin graphs [see Fig. 4 (a),(b)], which shows a deviation of the graph points from the straight line at $(V_S - V_{th}^I)/V_{th}^I \approx 0.04$ for experiment (a) and at ≈ 0.35 for simulations (b).

Further, we observed that after a short crossover region a second $\bar{T}(V_S)$ scaling is present, which is quite accurately approximated by an inverse power law of the form $\bar{T} = a [(V_S - V_{th}^I)/V_{th}^I]^{-1/2} + c$. The corresponding best fit is shown in Fig. 3 by blue dashed lines. The values V_{th}^I representing the second threshold were evaluated as 1.620 V for experiment and 0.750 for theory [see values a and c in Table S1 in the supplementary material]. The related log-log graphs confirming the inverse power-law (dashed blue straight lines) in the dependence $\bar{T}(V_S)$ are given in Fig. 4 (c) for experiment and in (d) for simulations.

We also attempted measurements without Pt/Ti top electrodes, instead just using a tungsten micro-positioner tip on the surface of the NbO_x layer as a top connection. This effectively provided a much smaller top electrode area. The results confirmed a transition from logarithmic to power law scaling of ISI was present even with greatly reduced top electrode size [see Figs. S8 and S9 in the supplementary material]. It has previously been shown that pressure exerted by micro-positioner tips can affect the observed electrical properties of threshold switching devices,³³ however this is concerned with variation in measurement of exact values across individual current-voltage sweeps. In this work we observe a qualitative scaling of ISI across numerous spikes (effec-

tively multiple current-voltage sweeps), hence this scaling is unlikely to be affected by the probe tip pressure.

The transition of the ISI-scaling hints at an instability, which could develop in the circuit with variation of V_S . To check this hypothesis, we numerically analyzed the evolution of dynamical regimes of the model (3), which provides a correct qualitative description of the observed phenomena (as shown above).

FIG. 4. Experimental (left) and simulated (right) results displaying logarithmic and power law scaling with control parameter V_S . $V_{th}^{I/II}$ represent the first and second thresholds. Fits of logarithmic scaling indicated by solid red lines on lin-log axes, (a) experimental data with $V_{th}^I = 1.560$ V, (b) numerical simulation with $V_{th}^I = 0.582$. Fits of power law scaling indicated by blue dashed lines on log-log axes, (c) experimental data with $V_{th}^{II} = 1.620$ V (points in red are within the transition region and do not contribute to the fit), (d) numerical simulation with $V_{th}^{II} = 0.750$; only data after respective V_{th}^{II} values is shown.

The calculations show that for small applied voltages, the system stays close to a fixed point (stable focus), and is unable to generate deterministic self-sustained oscillations. In this case, the spiking behavior is induced merely by noise, which eventually switches the device into the low resistance state once the random fluctuation causes the applied voltage to overshoot a certain threshold. This results in a single current spike and a return to the initial high resistance state.

Such a mechanism, previously found in diffusive memristors,³² predefines the log-scaling in ISIs for low V_S . With increase of V_S (at $V_S = V_S^{SN} \approx 0.88722$ for $D_0 = 0$) the dynamical system (3) undergoes the saddle-node (fold) bifurcation for limit cycles,³⁴ which gives rise to stable self-sustained periodic spiking of finite amplitude (bifurcation analysis was performed using software package XPPAUT³⁵). This instability is followed by a sub-critical Andronov-Hopf bifurcation³⁴ (at $V_S \approx 0.88724$ for $D_0 = 0$), with the stable fixed point losing its stability as a result. The emerged deterministic oscillations can exist even without voltage noise in the system (i.e for $D_0=0$), and the random fluctuations only slightly affect their shape and coherence. In the vicinity of the bifurcation point V_S^{SN} the random fluctuation of voltage can drive the system into the regime of self-oscillation. The closer V_S is to V_S^{SN} , the more time the system will spend in this regime due to fluctuations. Such noise-driven transitions change the character of the ISI-scaling in the dependence of $\bar{T}(V_S)$. The onset of self-sustained periodic oscillations dramatically improves the coherence of the generated spiking, which is evidenced in a significant decrease of standard deviation of ISI [see error bars in Figs. 3, 4 and Fig. S7 in the supplementary material], and in a much narrower probability density distribution as V_S increases past V_{th}'' [see Figs. S10 and S11 in the supplementary material].

The transient behavior of $\bar{T}$ in the vicinity of saddle-node bifurcation for limit cycles demonstrates some analogy with the dynamics known in the literature as the type-I intermittency,³⁶ which is characterized by alternating “laminar” and “turbulent” epochs in time-realization of state variables. It has been shown that in the vicinity of saddle-node bifurcation, and in the presence of fluctuations, the laminar phase duration is scaled by the power-law ε^β , where ε is some parameter characterizing proximity to the bifurcation, and β is some constant exponent.^{37,38} From this point of view, we can consider a spike as a “turbulent burst” and the transient between two spikes as a “laminar” epoch.

We recognize that there are several different possible noise sources that may affect the system, particularly internal voltage noise previously described as a nanobattery effect by Valov et al.³⁹ Hence, we demonstrated with additional simulations incorporating an emf voltage into the circuit (see the supplementary material, Figs. S3 and S4) that the noise properties influence mostly the average ISI and have little observed effect on the ISI scaling discussed here.

In conclusion, both in experiment and theory we revealed that the onset of self-sustained spiking in NbO_x oscillation neurons can be accompanied by a change of scaling in the dependence of the averaged ISIs $\bar{T}$ on the applied voltage V_S . Our analysis shows that this phenomenon is possi-

ble because of an eminent role of inevitable electric noise present in the device, and an instability that develops in the circuit at larger voltages. For low voltages, far enough from the instability, the stochastic noise-induced spiking dominates in the system providing log-scaling in $\bar{T}(V_S)$ -dependence. As V_S approaches a critical value corresponding to the onset of self-oscillations, the noise drives the device into the deterministic regime of self-oscillation more often, and above a certain voltage the self-oscillations start to dominate in the system. This transition causes a switch of the scaling $\bar{T}(V_S)$ to a power law with the exponent of $-1/2$. Our findings demonstrate that the interplay between random fluctuations and evolving instabilities plays a key role in forming temporal characteristics of spiking in artificial oscillation neurons. Our analysis of the stochastic region of spiking provides valuable information for future control and utilization of stochastic features in neuromorphic systems based on oscillations. Understanding the effects of the observed instabilities on the temporal patterns in spiking of such circuits is crucial for the design of devices with desired characteristics, and therefore is an important task for future study.

See the supplementary material for X-ray photoelectron spectroscopy data, current-voltage (IV) measurements, details of further simulations and experimental data, details of interspike interval analysis, ISI standard deviation data and probability density plots, and tables of logarithmic and power law fit parameters.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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