Online supplement: Transient staffing at the beginning of work

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This document is an online supplement that provides additional numerical support for our observations.

1 Numerical experiments - effect of the arrival rate

In this first section, we present two tables that confirm the observations made for the effect of the arrival rate.

y	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$	$\lambda = 3.5$	$\lambda = 4$	$\lambda = 4.5$	$\lambda = 5$
1	10.00	5.00	3.33	2.50	2.00	1.67	1.43	1.25	1.11	1.00
2	13.3	6.26	4.12	3.08	2.46	2.05	1.76	1.54	1.37	1.23
3	21.52	7.88	5.02	3.69	2.93	2.43	2.07	1.81	1.61	1.45
4	–	10.18	6.08	4.39	3.44	2.84	2.41	2.10	1.86	1.67
5	–	14.08	7.41	5.18	4.01	3.28	2.77	2.41	2.13	1.91
6	–	25.48	9.15	6.11	4.64	3.76	3.16	2.73	2.40	2.15
7	–	–	11.67	7.23	5.36	4.28	3.57	3.07	2.70	2.40
8	–	–	16.20	8.62	6.17	4.86	4.02	3.44	3.00	2.67
9	–	–	33.76	10.48	7.14	5.50	4.51	3.83	3.33	2.95
10	–	–	–	13.27	8.30	6.24	5.04	4.25	3.67	3.24
11	–	–	–	18.73	9.77	7.08	5.63	4.70	4.04	3.55
12	–	–	–	65.11	11.78	8.08	6.30	5.19	4.44	3.88
13	–	–	–	–	14.94	9.30	7.05	5.74	4.87	4.23
14	–	–	–	–	22.15	10.88	7.93	6.35	5.33	4.61
15	–	–	–	–	–	13.1	8.97	7.03	5.83	5.01
16	–	–	–	–	–	16.88	10.27	7.81	6.39	5.44
17	–	–	–	–	–	30.44	12.00	8.73	7.02	5.92
18	–	–	–	–	–	–	14.60	9.84	7.73	6.44
19	–	–	–	–	–	–	20.09	11.28	8.55	7.01
20	–	–	–	–	–	–	–	13.32	9.55	7.67
21	–	–	–	–	–	–	–	16.97	10.81	8.44
22	–	–	–	–	–	–	–	37.32	12.57	9.36
23	–	–	–	–	–	–	–	–	15.56	10.54
24	–	–	–	–	–	–	–	–	25.56	12.20
25	–	–	–	–	–	–	–	–	–	15.06
26	–	–	–	–	–	–	–	–	–	24.79
S	44.82	68.88	96.74	148.39	105.09	132.67	123.58	160.71	151.43	154.82
s	3	6	9	12	14	17	19	22	24	26

Table 1: Effect of the arrival rate ($\mu = 0.2$, $\gamma = 0$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

2 Numerical experiments - effect of the abandonment rate

We now present two tables that confirm the observations made for the effect of the abandonment rate.

y	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$	$\lambda = 3.5$	$\lambda = 4$	$\lambda = 4.5$	$\lambda = 5$
1	–	–	5.49	3.47	2.55	2.03	1.68	1.44	1.26	1.12
2	–	–	8.06	4.60	3.29	2.58	2.12	1.81	1.57	1.40
3	–	–	13.47	6.03	4.12	3.16	2.57	2.17	1.88	1.66
4	–	–	–	8.02	5.11	3.82	3.07	2.57	2.21	1.95
5	–	–	–	11.38	6.34	4.58	3.62	3.00	2.57	2.25
6	–	–	–	27.10	7.98	5.47	4.23	3.47	2.94	2.56
7	–	–	–	–	10.42	6.55	4.93	3.98	3.35	2.90
8	–	–	–	–	15.36	7.93	5.73	4.55	3.79	3.26
9	–	–	–	–	–	9.84	6.70	5.20	4.28	3.65
10	–	–	–	–	–	12.97	7.89	5.94	4.82	4.07
11	–	–	–	–	–	23.11	9.46	6.81	5.42	4.53
12	–	–	–	–	–	–	11.75	7.85	6.10	5.04
13	–	–	–	–	–	–	16.11	9.18	6.89	5.6
14	–	–	–	–	–	–	–	10.99	7.82	6.23
15	–	–	–	–	–	–	–	13.86	8.97	6.95
16	–	–	–	–	–	–	–	21.33	10.47	7.79
17	–	–	–	–	–	–	–	–	12.61	8.81
18	–	–	–	–	–	–	–	–	16.46	10.08
19	–	–	–	–	–	–	–	–	–	11.80
20	–	–	–	–	–	–	–	–	–	14.45
21	–	–	–	–	–	–	–	–	–	20.56
S	0.00	0.00	27.02	60.60	55.17	82.04	79.86	104.15	103.41	126.66
s	0	0	3	6	8	11	13	16	18	21

Table 2: Effect of the arrival rate ($\mu = 0.2$, $\gamma = 0.2$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.25$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.9$	$\gamma = 1$
1	2.50	2.88	3.92	–	–	–	–
2	3.08	3.64	5.44	–	–	–	–
3	3.69	4.50	7.58	–	–	–	–
4	4.39	5.53	11.45	–	–	–	–
5	5.18	6.82	–	–	–	–	–
6	6.11	8.53	–	–	–	–	–
7	7.23	11.11	–	–	–	–	–
8	8.62	16.39	–	–	–	–	–
9	10.48	–	–	–	–	–	–
10	13.27	–	–	–	–	–	–
11	18.73	–	–	–	–	–	–
12	65.11	–	–	–	–	–	–
S	148.39	59.40	28.39	0	0	0	0
s	12	8	4	0	0	0	0

Table 3: Effect of the abandonment rate ($\mu = 0.2$, $\lambda = 2$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

$\mathbf{y}$	$\boldsymbol{\gamma = 0}$	$\boldsymbol{\gamma = 0.1}$	$\boldsymbol{\gamma = 0.25}$	$\boldsymbol{\gamma = 0.5}$	$\boldsymbol{\gamma = 0.75}$	$\boldsymbol{\gamma = 0.9}$	$\boldsymbol{\gamma = 1}$
1	0.63	0.65	0.68	0.75	0.84	0.92	0.98
2	0.78	0.81	0.85	0.95	1.09	1.21	1.31
3	0.91	0.94	1.01	1.14	1.33	1.49	1.63
4	1.04	1.09	1.16	1.33	1.58	1.79	1.98
5	1.18	1.23	1.33	1.53	1.84	2.11	2.35
6	1.32	1.38	1.50	1.74	2.11	2.45	2.76
7	1.46	1.54	1.67	1.96	2.41	2.82	3.20
8	1.61	1.70	1.85	2.19	2.72	3.21	3.68
9	1.76	1.86	2.03	2.43	3.05	3.64	4.22
10	1.91	2.03	2.23	2.68	3.40	4.11	4.83
11	2.07	2.20	2.43	2.94	3.79	4.63	5.52
12	2.24	2.38	2.63	3.22	4.20	5.22	6.33
13	2.41	2.57	2.85	3.52	4.65	5.88	7.31
14	2.58	2.76	3.08	3.83	5.15	6.65	8.57
15	2.76	2.96	3.32	4.17	5.70	7.57	10.36
16	2.95	3.17	3.56	4.53	6.33	8.71	15.77
17	3.14	3.38	3.83	4.91	7.04	10.25	–
18	3.34	3.61	4.10	5.33	7.88	12.78	–
19	3.55	3.84	4.40	5.79	8.90	–	–
20	3.76	4.09	4.71	6.30	10.20	–	–
21	3.99	4.35	5.04	6.86	12.02	–	–
22	4.23	4.63	5.39	7.50	15.50	–	–
23	4.48	4.93	5.78	8.23	–	–	–
24	4.76	5.25	6.20	9.10	–	–	–
25	5.05	5.59	6.67	10.16	–	–	–
26	5.38	5.98	7.20	11.55	–	–	–
27	5.76	6.42	7.81	13.67	–	–	–
28	6.20	6.94	8.57	20.63	–	–	–
29	6.73	7.59	9.56	–	–	–	–
30	7.44	8.46	11.09	–	–	–	–
31	8.46	9.82	15.16	–	–	–	–
32	10.34	13.32	–	–	–	–	–
S	114.22	127.47	137.69	148.94	111.73	85.44	80.80
s	32	32	31	28	22	18	16

Table 4: Effect of the abandonment rate ($\mu = 0.2$, $\lambda = 8$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

3 Numerical experiments - effect of the traffic intensity

We now present two tables that confirm the observations made for the effect of the traffic intensity.

y	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$
1	3.33	1.33	0.67	0.44	0.33	0.22	0.17
2	4.07	1.64	0.83	0.56	0.43	0.29	0.23
3	4.83	1.95	0.99	0.67	0.51	0.35	0.26
4	5.65	2.27	1.15	0.77	0.59	0.40	0.31
5	6.53	2.63	1.33	0.89	0.67	0.46	0.35
6	7.49	3.01	1.52	1.02	0.77	0.52	0.40
7	8.54	3.43	1.73	1.16	0.88	0.59	0.45
8	9.69	3.89	1.96	1.31	0.99	0.67	0.51
9	10.99	4.41	2.22	1.49	1.12	0.76	0.57
10	12.45	5.00	2.51	1.68	1.27	0.85	0.65
11	14.14	5.67	2.85	1.91	1.44	0.97	0.73
12	16.14	6.47	3.25	2.17	1.63	1.10	0.83
13	18.58	7.45	3.73	2.50	1.88	1.26	0.95
14	21.73	8.71	4.36	2.92	2.20	1.47	1.11
15	26.17	10.48	5.25	3.51	2.64	1.77	1.33
16	33.73	13.51	6.77	4.53	3.39	2.27	1.71
17	60.87	24.36	12.19	8.13	6.11	4.08	3.07
S	264.93	106.21	53.31	35.69	26.88	18.03	13.63
s	17	17	17	17	17	17	17

Table 5: Effect of the traffic intensity ($a = 15$, $b = 0$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$
1	4.05	1.62	0.81	0.54	0.41	0.27	0.20
2	5.13	2.06	1.04	0.70	0.53	0.36	0.28
3	6.30	2.53	1.28	0.86	0.65	0.44	0.34
4	7.62	3.06	1.54	1.03	0.78	0.53	0.40
5	9.13	3.67	1.84	1.24	0.93	0.63	0.48
6	10.92	4.38	2.20	1.47	1.11	0.75	0.56
7	13.08	5.25	2.63	1.76	1.33	0.89	0.67
8	15.85	6.35	3.18	2.13	1.60	1.07	0.81
9	19.67	7.88	3.95	2.64	1.98	1.33	1.00
10	25.93	10.38	5.20	3.47	2.60	1.74	1.31
11	46.22	18.49	9.25	6.16	4.62	3.08	2.31
S	163.90	65.67	32.92	22.00	16.54	11.09	8.36
s	11	11	11	11	11	11	11

Table 6: Effect of the traffic intensity ($a = 15$, $b = 1$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

4 Numerical experiments - effect of the service level objective

We now present two additional tables that confirm the observations made for the effect of the service level objective.

y	$q^* = 0.1$	$q^* = 0.5$	$q^* = 1$	$q^* = 2$	$q^* = 5$	$q^* = 10$	$q^* = 15$
1	0.02	0.10	0.20	0.40	1.00	2.00	3.00
2	0.13	0.27	0.40	0.62	1.23	2.23	3.23
3	0.23	0.42	0.58	0.82	1.45	2.45	3.45
4	0.35	0.59	0.76	1.03	1.67	2.68	3.68
5	0.48	0.76	0.95	1.24	1.91	2.91	3.91
6	0.62	0.94	1.15	1.46	2.15	3.16	4.16
7	0.77	1.12	1.36	1.69	2.40	3.43	4.43
8	0.93	1.32	1.57	1.92	2.67	3.70	4.71
9	1.10	1.52	1.80	2.17	2.95	4.00	5.00
10	1.27	1.74	2.03	2.43	3.24	4.30	5.31
11	1.46	1.96	2.28	2.71	3.55	4.63	5.65
12	1.65	2.20	2.54	3.00	3.88	4.99	6.01
13	1.85	2.45	2.82	3.30	4.23	5.36	6.40
14	2.07	2.71	3.11	3.63	4.61	5.77	6.83
15	2.29	2.98	3.41	3.97	5.01	6.22	7.32
16	2.53	3.28	3.74	4.34	5.44	6.70	7.89
17	2.78	3.59	4.09	4.73	5.92	7.25	8.57
18	3.04	3.92	4.47	5.16	6.44	7.88	9.44
19	3.32	4.28	4.87	5.63	7.01	8.62	10.64
20	3.62	4.67	5.31	6.15	7.67	9.54	12.63
21	3.94	5.08	5.80	6.72	8.44	10.78	17.71
22	4.28	5.54	6.33	7.36	9.36	12.67	–
23	4.65	6.04	6.93	8.11	10.54	16.62	–
24	5.05	6.60	7.61	8.99	12.20	–	–
25	5.48	7.23	8.40	10.08	15.06	–	–
26	5.96	7.96	9.35	11.54	24.79	–	–
27	6.49	8.81	10.56	13.76	–	–	–
28	7.09	9.87	12.24	18.49	–	–	–
29	7.78	11.25	14.98	–	–	–	–
30	8.59	13.29	22.77	–	–	–	–
31	9.57	17.19	–	–	–	–	–
32	10.84	–	–	–	–	–	–
33	12.68	–	–	–	–	–	–
34	15.95	–	–	–	–	–	–
S	138.86	438.80	507.97	393.73	466.43	287.28	265.24
s	34	31	30	28	26	23	21

Table 7: Effect of the service level objective ($\lambda = 5$, $\mu = 0.2$, $\gamma = 0$, $N = \max(50, 10q^*)$, $\epsilon = 0.001$)

y	$q^* = 0.1$	$q^* = 0.5$	$q^* = 1$	$q^* = 2$	$q^* = 5$	$q^* = 10$	$q^* = 15$
1	0.02	0.11	0.22	0.51	–	–	–
2	0.13	0.29	0.45	0.83	–	–	–
3	0.24	0.46	0.67	1.15	–	–	–
4	0.36	0.65	0.91	1.49	–	–	–
5	0.50	0.85	1.16	1.86	–	–	–
6	0.65	1.06	1.43	2.26	–	–	–
7	0.81	1.29	1.71	2.70	–	–	–
8	0.99	1.53	2.02	3.18	–	–	–
9	1.17	1.78	2.35	3.72	–	–	–
10	1.36	2.05	2.70	4.33	–	–	–
11	1.57	2.34	3.08	5.02	–	–	–
12	1.78	2.65	3.50	5.83	–	–	–
13	2.01	2.99	3.95	6.81	–	–	–
14	2.26	3.35	4.45	8.05	–	–	–
15	2.51	3.73	5.01	9.78	–	–	–
16	2.79	4.16	5.65	13.01	–	–	–
17	3.08	4.62	6.38	–	–	–	–
18	3.39	5.14	7.24	–	–	–	–
19	3.73	5.71	8.29	–	–	–	–
20	4.09	6.36	9.65	–	–	–	–
21	4.48	7.12	11.60	–	–	–	–
22	4.91	8.02	15.48	–	–	–	–
23	5.38	9.12	–	–	–	–	–
24	5.90	10.56	–	–	–	–	–
25	6.49	12.68	–	–	–	–	–
26	7.16	17.02	–	–	–	–	–
27	7.93	–	–	–	–	–	–
28	8.86	–	–	–	–	–	–
29	10.00	–	–	–	–	–	–
30	11.51	–	–	–	–	–	–
31	13.74	–	–	–	–	–	–
32	18.24	–	–		–	–	–
S	138.04	369.01	269.23	166.38	0.00	0.00	0.00
s	32	26	22	16	0	0	0

Table 8: Effect of the service level objective ($\lambda = 5$, $\mu = 0.2$, $\gamma = 1$, $N = \max(50, 10q^*)$, $\epsilon = 0.001$)

5 Numerical experiments - Comparison between the two approximations

In this section, we present additional supporting experiments that illustrate the difference between the two approximations for the estimation of appointment times.

y	$\boldsymbol{\lambda = 0.5}$	$\boldsymbol{\lambda = 1}$	$\boldsymbol{\lambda = 1.5}$	$\boldsymbol{\lambda = 2}$	$\boldsymbol{\lambda = 2.5}$	$\boldsymbol{\lambda = 3}$	$\boldsymbol{\lambda = 3.5}$	$\boldsymbol{\lambda = 4}$	$\boldsymbol{\lambda = 4.5}$	$\boldsymbol{\lambda = 5}$
1	–	–	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	–	–	0.16%	0.40%	0.62%	1.01%	0.99%	1.49%	1.24%	2.03%
3	–	–	-0.52%	0.17%	0.37%	0.54%	0.62%	0.75%	0.89%	1.06%
4	–	–	–	-0.34%	0.03%	0.24%	0.51%	0.62%	0.60%	1.12%
5	–	–	–	-1.15%	-0.39%	-0.03%	0.26%	0.36%	0.62%	0.83%
6	–	–	–	–	-0.83%	-0.42%	-0.15%	0.12%	0.04%	0.23%
7	–	–	–	–	-1.71%	-0.89%	-0.44%	-0.31%	-0.17%	0.03%
8	–	–	–	–	-4.56%	-1.46%	-0.99%	-0.69%	-0.54%	-0.29%
9	–	–	–	–	–	-2.33%	-1.33%	-0.94%	-0.72%	-0.54%
10	–	–	–	–	–	-4.21%	-1.95%	-1.33%	-0.97%	-0.85%
11	–	–	–	–	–	–	-2.77%	-1.75%	-1.33%	-1.12%
12	–	–	–	–	–	–	-4.34%	-2.45%	-1.75%	-1.34%
13	–	–	–	–	–	–	-9.38%	-3.22%	-2.18%	-1.71%
14	–	–	–	–	–	–	–	-4.54%	-2.82%	-2.12%
15	–	–	–	–	–	–	–	-7.47%	-3.59%	-2.60%
16	–	–	–	–	–	–	–	–	-4.70%	-3.20%
17	–	–	–	–	–	–	–	–	-6.87%	-3.85%
18	–	–	–	–	–	–	–	–	-13.52%	-4.92%
19	–	–	–	–	–	–	–	–	–	-6.56%
20	–	–	–	–	–	–	–	–	–	-10.22%
21	–	–	–	–	–	–	–	–	–	–

Table 9: Relative difference between the two approximations ($\mu = 0.2$, $\gamma = 0.2$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$	$\lambda = 3.5$	$\lambda = 4$	$\lambda = 4.5$	$\lambda = 5$
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	-0.25%	0.16%	0.43%	0.80%	1.04%	1.29%	1.64%	1.77%	1.96%	1.79%
3	-7.77%	-0.46%	0.17%	0.26%	0.65%	0.90%	0.77%	1.06%	1.41%	1.70%
4	–	-2.27%	-0.70%	-0.11%	0.08%	0.53%	0.46%	0.72%	0.87%	1.03%
5	–	-8.67%	-1.87%	-0.92%	-0.39%	0.01%	0.02%	0.52%	0.74%	1.00%
6	–	–	-4.20%	-1.90%	-1.11%	-0.52%	-0.30%	-0.03%	0.00%	0.42%
7	–	–	-9.43%	-3.32%	-1.86%	-1.27%	-0.95%	-0.58%	-0.11%	-0.18%
8	–	–	-29.21%	-5.74%	-3.15%	-2.02%	-1.48%	-0.94%	-0.85%	-0.45%
9	–	–	–	-10.00%	-4.57%	-3.07%	-2.10%	-1.52%	-1.20%	-0.88%
10	–	–	–	-20.28%	-6.85%	-4.12%	-2.98%	-2.16%	-1.89%	-1.48%
11	–	–	–	–	-10.45%	-5.70%	-3.95%	-2.97%	-2.43%	-1.99%
12	–	–	–	–	-17.30%	-7.74%	-4.99%	-3.88%	-2.96%	-2.49%
13	–	–	–	–	-38.38%	-10.79%	-6.50%	-4.72%	-3.60%	-3.06%
14	–	–	–	–	–	-15.82%	-8.33%	-5.77%	-4.45%	-3.56%
15	–	–	–	–	–	-26.92%	-11.01%	-7.16%	-5.45%	-4.30%
16	–	–	–	–	–	–	-14.98%	-8.89%	-6.48%	-5.14%
17	–	–	–	–	–	–	-22.14%	-11.12%	-7.66%	-5.89%
18	–	–	–	–	–	–	-42.55%	-14.35%	-9.18%	-6.88%
19	–	–	–	–	–	–	–	-19.36%	-11.14%	-8.12%
20	–	–	–	–	–	–	–	-29.85%	-13.58%	-9.37%
21	–	–	–	–	–	–	–	–	-17.17%	-10.81%
22	–	–	–	–	–	–	–	–	-23.28%	-12.63%
23	–	–	–	–	–	–	–	–	-41.03%	-14.86%
24	–	–	–	–	–	–	–	–	–	-18.01%
25	–	–	–	–	–	–	–	–	–	-24.24%
26	–	–	–	–	–	–	–	–	–	–

Table 10: Relative difference between the two approximations ($\mu = 0.2$, $\gamma = 0$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.25$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.9$	$\gamma = 1$
1	0.00%	0.00%	0.00%	–	–	–	–
2	0.80%	0.61%	0.33%	–	–	–	–
3	0.26%	0.27%	0.03%	–	–	–	–
4	-0.11%	-0.21%	-0.45%	–	–	–	–
5	-0.92%	-0.83%	–	–	–	–	–
6	-1.90%	-1.96%	–	–	–	–	–
7	-3.32%	-4.03%	–	–	–	–	–
8	-5.74%	-11.45%	–	–	–	–	–
9	-10.00%	–	–	–	–	–	–
10	-20.28%	–	–	–	–	–	–

Table 11: Relative difference between the two approximations ($\mu = 0.2$, $\lambda = 2$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.25$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.9$	$\gamma = 1$
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	3.56%	3.65%	2.48%	2.18%	1.88%	1.98%	1.85%
3	2.85%	2.03%	2.65%	2.08%	1.74%	1.38%	1.14%
4	1.97%	2.33%	1.46%	1.44%	1.52%	1.13%	1.02%
5	1.83%	1.37%	1.69%	1.11%	1.17%	0.97%	0.66%
6	1.41%	1.02%	1.48%	0.93%	0.69%	0.77%	0.73%
7	0.78%	1.08%	0.94%	0.82%	0.89%	0.78%	0.57%
8	0.61%	0.82%	0.70%	0.73%	0.74%	0.48%	0.34%
9	0.20%	0.31%	0.16%	0.61%	0.58%	0.40%	0.32%
10	-0.41%	0.09%	0.28%	0.42%	0.34%	0.30%	0.36%
11	-0.69%	-0.37%	0.04%	0.14%	0.47%	0.22%	0.26%
12	-0.75%	-0.63%	-0.49%	0.06%	0.29%	0.30%	0.21%
13	-1.04%	-0.75%	-0.57%	0.08%	0.18%	0.20%	0.17%
14	-1.55%	-1.13%	-0.66%	-0.14%	0.17%	0.17%	0.23%
15	-1.88%	-1.40%	-0.77%	-0.12%	0.05%	0.17%	0.17%
16	-2.09%	-1.61%	-1.22%	-0.21%	0.10%	0.10%	–
17	-2.52%	-2.08%	-1.19%	-0.47%	-0.03%	0.08%	–
18	-2.87%	-2.24%	-1.54%	-0.57%	-0.07%	-0.16%	–
19	-3.16%	-2.68%	-1.56%	-0.64%	-0.10%	–	–
20	-3.69%	-2.91%	-1.77%	-0.68%	-0.16%	–	–
21	-3.95%	-3.21%	-2.00%	-0.85%	-0.37%	–	–
22	-4.24%	-3.39%	-2.30%	-0.92%	-1.66%	–	–
23	-4.58%	-3.53%	-2.39%	-1.07%	–	–	–
24	-4.59%	-3.66%	-2.54%	-1.11%	–	–	–
25	-4.75%	-3.83%	-2.54%	-1.17%	–	–	–
26	-4.52%	-3.63%	-2.41%	-1.00%	–	–	–
27	-3.87%	-3.18%	-2.08%	0.31%	–	–	–
28	-2.77%	-2.20%	-0.99%	19.86%	–	–	–
29	-0.93%	-0.25%	1.24%	–	–	–	–
30	2.65%	3.42%	6.87%	–	–	–	–
31	9.19%	11.26%	31.51%	–	–	–	–
32	24.53%	39.23%	–	–	–	–	–

Table 12: Relative difference between the two approximations ($\mu = 0.2$, $\lambda = 8$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.55%	1.29%	2.53%	3.76%	6.24%	7.47%	13.65%
3	0.27%	1.21%	2.76%	4.32%	5.88%	8.99%	7.95%
4	0.00%	0.44%	1.77%	2.21%	4.42%	6.19%	9.73%
5	-0.45%	0.24%	1.38%	1.76%	2.15%	5.19%	6.72%
6	-0.92%	-0.45%	0.54%	1.20%	1.86%	3.18%	5.83%
7	-1.50%	-1.10%	-0.24%	0.34%	1.49%	2.07%	3.80%
8	-2.32%	-1.97%	-1.21%	-0.96%	-0.21%	1.31%	2.82%
9	-3.16%	-2.85%	-2.19%	-1.53%	-1.31%	0.45%	0.45%
10	-4.35%	-3.96%	-3.58%	-3.19%	-2.42%	-2.04%	-0.12%
11	-5.83%	-5.60%	-5.10%	-4.60%	-4.10%	-3.10%	-2.77%
12	-7.85%	-7.65%	-7.23%	-7.08%	-6.94%	-5.80%	-5.23%
13	-10.88%	-10.67%	-10.55%	-10.07%	-9.83%	-9.35%	-8.87%
14	-15.93%	-15.76%	-15.66%	-15.28%	-14.89%	-14.70%	-14.12%
15	-27.00%	-26.92%	-26.78%	-26.57%	-26.36%	-25.94%	-25.80%

Table 13: Relative difference between the two approximations ($a = 15$, $b = 0$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.43%	0.82%	1.80%	2.77%	3.75%	5.71%	9.63%
3	0.22%	0.62%	1.81%	2.61%	3.40%	4.99%	8.18%
4	-0.02%	0.38%	1.03%	1.36%	2.34%	4.31%	4.97%
5	-0.36%	0.13%	0.40%	1.50%	1.50%	3.13%	4.77%
6	-0.60%	-0.33%	0.13%	0.35%	1.04%	2.40%	1.95%
7	-1.04%	-0.70%	-0.51%	-0.13%	0.62%	1.00%	1.38%
8	-1.52%	-1.36%	-1.21%	-0.74%	-0.59%	-0.28%	0.66%
9	-2.38%	-2.23%	-1.98%	-1.73%	-1.73%	-0.99%	-0.74%
10	-4.25%	-4.17%	-3.99%	-3.90%	-3.99%	-3.62%	-3.25%

Table 14: Relative difference between the two approximations ($a = 15$, $b = 1$, $q^* = 5$, $N = 50$, $\epsilon = 0.001$)

y	$q^* = 0.1$	$q^* = 0.5$	$q^* = 1$	$q^* = 2$	$q^* = 5$	$q^* = 10$	$q^* = 15$
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	-43.07%	-12.43%	-2.04%	1.92%	1.79%	0.98%	0.68%
3	-48.40%	-20.11%	-7.31%	-0.69%	1.70%	1.00%	0.71%
4	-47.99%	-21.65%	-10.90%	-2.18%	1.03%	1.02%	0.74%
5	-47.32%	-23.32%	-12.93%	-3.96%	1.00%	0.65%	0.49%
6	-46.60%	-24.26%	-14.25%	-5.26%	0.42%	0.60%	0.46%
7	-45.94%	-25.54%	-15.23%	-6.33%	-0.18%	0.76%	0.58%
8	-45.36%	-25.93%	-16.58%	-7.78%	-0.45%	0.49%	0.60%
9	-44.89%	-26.79%	-17.28%	-8.68%	-0.88%	0.60%	0.48%
10	-44.99%	-27.16%	-18.43%	-9.62%	-1.48%	0.26%	0.40%
11	-44.74%	-27.99%	-19.21%	-10.32%	-1.99%	0.17%	0.50%
12	-44.98%	-28.55%	-20.10%	-11.22%	-2.49%	0.22%	0.52%
13	-45.33%	-29.27%	-20.87%	-12.32%	-3.06%	-0.07%	0.57%
14	-45.53%	-30.16%	-21.87%	-13.17%	-3.56%	-0.18%	0.73%
15	-46.18%	-31.26%	-23.11%	-14.35%	-4.30%	-0.24%	1.18%
16	-46.79%	-32.16%	-24.21%	-15.48%	-5.14%	-0.52%	2.00%
17	-47.65%	-33.40%	-25.51%	-16.88%	-5.89%	-0.56%	3.37%
18	-48.78%	-34.84%	-26.91%	-18.30%	-6.88%	-0.45%	5.88%
19	-50.07%	-36.40%	-28.69%	-19.91%	-8.12%	-0.11%	10.49%
20	-51.62%	-38.25%	-30.71%	-21.79%	-9.37%	0.81%	20.71%
21	-53.55%	-40.68%	-33.05%	-24.18%	-10.81%	3.03%	–
22	-56.03%	-43.54%	-36.15%	-27.22%	-12.63%	8.17%	–
23	-59.21%	-47.39%	-40.15%	-31.15%	-14.86%	24.22%	–
24	-63.67%	-52.79%	-45.95%	-37.04%	-18.01%	–	–
25	-71.00%	-61.91%	-55.97%	-47.72%	-24.24%	–	–

Table 15: Relative difference between the two approximations ($\lambda = 5$, $\mu = 0.2$, $\gamma = 0$, $N = \max(50, 10q^*)$, $\epsilon = 0.001$)

y	$q^* = 0.1$	$q^* = 0.5$	$q^* = 1$	$q^* = 2$	$q^* = 5$	$q^* = 10$	$q^* = 15$
1	0.00%	0.00%	0.00%	0.00%	–	–	–
2	-39.00%	-7.78%	-1.48%	1.69%	–	–	–
3	-42.00%	-13.65%	-4.54%	0.73%	–	–	–
4	-42.20%	-14.58%	-5.18%	0.00%	–	–	–
5	-40.56%	-15.02%	-5.82%	-0.25%	–	–	–
6	-39.22%	-15.30%	-5.88%	-0.44%	–	–	–
7	-38.10%	-14.92%	-6.29%	-0.44%	–	–	–
8	-36.54%	-14.80%	-6.05%	-0.55%	–	–	–
9	-35.88%	-14.92%	-5.94%	-0.44%	–	–	–
10	-35.37%	-14.84%	-6.03%	-0.29%	–	–	–
11	-34.60%	-14.71%	-6.06%	-0.31%	–	–	–
12	-34.45%	-14.64%	-5.92%	-0.28%	–	–	–
13	-34.14%	-14.43%	-6.09%	-0.26%	–	–	–
14	-33.78%	-14.47%	-6.22%	-0.37%	–	–	–
15	-33.99%	-14.83%	-6.37%	-0.93%	–	–	–
16	-33.97%	-14.94%	-6.52%	–	–	–	–
17	-34.30%	-15.40%	-6.93%	–	–	–	–
18	-34.79%	-15.84%	-7.61%	–	–	–	–
19	-35.35%	-16.70%	-8.79%	–	–	–	–
20	-36.25%	-17.85%	-11.31%	–	–	–	–
21	-37.46%	-19.46%	–	–	–	–	–
22	-39.01%	-22.19%	–	–	–	–	–
23	-41.20%	-28.22%	–	–	–	–	–
24	-44.42%	–	–	–	–	–	–
25	-50.13%	–	–	–	–	–	–

Table 16: Relative difference between the two approximations ($\lambda = 5$, $\mu = 0.2$, $\gamma = 1$, $N = \max(50, 10q^*)$, $\epsilon = 0.001$)