Detailed Responses to the Reviewers' Comments on Manuscript TCON-2016-0056

Manuscript ID: TCON-2016-0056

Title: Time-varying state feedback stabilization of stochastic feedforward nonlinear systems with unknown growth rate

First of all, the authors would like to thank the Editor-in-Chief and the anonymous reviewers very much for their valuable comments and suggestions, which have led to significant improvement of the quality and presentation of the manuscript. In the following, the authors will explain how the reviewers' comments have been taken into account in the revision item by item.

Response to Editor-in-Chief

Comment: *The reviewers have recommended publication, but also suggest some major revisions to your manuscript. Therefore, I invite you to respond to the reviewers' comments and revise your manuscript.*

Response: Thank the Editor-in-Chief for handing our paper and for the encouraging comments. Following the given instructions, we have made detailed responses to all the reviewers' comments item by item in the following.

Response to Reviewer #1

Comment: *This paper studies the time-varying state feedback stabilization problem for a class of stochastic feedforward nonlinear systems with unknown growth rate. A LaSalle-type theorem for stochastic time-varying systems is established and a state feedback controller is designed to guarantee that all signals of the closed-loop system converge to zero almost surely. This paper contains some interesting results. However, there are some dubious points in the main results which need to be further clarified.*

Response: Thanks for the reviewer's constructive comments.

Comment 1: *Page 4, line 28, how to choose the constant “M”? To my understanding, “M” is not a independent constant, which is heavily relied on “t” and “r”. Therefore, the conditions of Lemma 2 in Yu & Xie (2010) are not satisfied. Thus the proof for the conclusion i) of Lemma 1 is wrong.*

Response 1: Thank the reviewer for the comment. Regarding the independence of the constant “M”, please allow us to make the following detailed explanation. In order to facilitate the explanation, we recall the Eq.(6) in the paper, which plays a key role here.

- For any $r > 0$, define the stopping time as $\zeta_r = \inf\{t \geq 0 : |x(t)| \geq r\}$ with $\inf \phi = \infty$. By Dynkin's formula, one has

$$\begin{aligned} EV(x(t \wedge \zeta_r), t \wedge \zeta_r) &= V(x(0)) + E \left(\int_0^{t \wedge \zeta_r} \mathcal{L}V(x(s), s) ds \right) \\ &\leq V(x(0)) + E \int_0^{t \wedge \zeta_r} l(s) ds - E \int_0^{t \wedge \zeta_r} W(x(s), s) ds \\ &\leq V(x(0)) + \int_0^\infty l(s) ds - E \int_0^{t \wedge \zeta_r} W(x(s), s) ds. \end{aligned} \quad (6)$$

Please note that

$$E \int_0^{t \wedge \zeta_r} W(x(s), s) ds \geq 0,$$

which is derived from the fact that $W(x(s), s) \geq 0, \forall s \geq 0$. Thus, we can further deduce from Eq.(6) that

$$EV(x(t \wedge \zeta_r), t \wedge \zeta_r) \leq V(x(0)) + \int_0^\infty l(s) ds. \quad (S1)$$

A detailed explanation of the right hand of Eq.(S1) is made in the following.

- It is clear to see that $V(x(0))$ is an independent constant, which only relies on the initial value $x(0)$, rather than on “t” and “r”.
- In this paper, we assume that $l \in L^1(\mathbb{R}_+; \mathbb{R}_+)$, which means that function $l : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfies $\int_0^\infty l(s) ds < \infty$ (please see the Notation in the Introduction Section of the revised version). Obviously, $\int_0^\infty l(s) ds$ is a constant and also does not depend on “t” and “r”.

Based on the above analysis, it is reasonable to say that the constant “M” is an independent constant, which is not dependent on “t” and “r”. On the other hand, it is worth pointing out that there are multiple choices of the suitable “M” to satisfy the conditions of Lemma 2 in Yu & Xie (2010) (i.e., [R1]), for example, $M = V(x(0)) + \int_0^\infty l(s)ds$, $M = V(x(0)) + \int_0^\infty l(s)ds + 1$, and so forth.

Comment 2: *The equation in Page 4, line 40, needs a detailed proof.*

Response 2: We express our gratitude to the reviewer for this valuable comment. In the following, we would like to give a detailed proof of the equation in Page 4, line 40, which is equally to prove the following proposition.

Consider the following stochastic nonlinear system

$$dx(t) = f(x(t), t)dt + g^T(x(t), t)dB(t), \quad x(0) = x_0 \in \mathbb{R}^n, \quad (\text{S2})$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, x_0 is a deterministic initial value, $B(t)$ is an m -dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ with Ω being a sample space, $\mathcal{F}$ being a σ -field, $\{\mathcal{F}_t\}_{t \geq 0}$ being a filtration, and P being a probability measure. Functions $f(\cdot) \in \mathbb{R}^n$ and $g(\cdot) \in \mathbb{R}^{m \times n}$ satisfy the following assumption.

Assumption A1 The generalized local Lipschitz condition. *For all $K > 0$, there exists a continuous function $L(K, t) > 0$ such that, $\forall t \geq 0$ and $\forall |x| \vee |y| \leq K$,*

$$|f(x, t) - f(y, t)| \vee |g(x, t) - g(y, t)| \leq L(K, t)|x - y|.$$

Proposition P1 *Assume that there are a function $\rho \in \Psi(\mathbb{R}_+; \mathbb{R}_+)$, and a continuous nonnegative function $\omega : \mathbb{R}^n \rightarrow \mathbb{R}_+$ such that*

$$E \int_0^\infty \rho(s) \omega(x(s)) ds \leq V(x(0)) + \int_0^\infty l(s) ds < \infty. \quad (\text{S3})$$

Then

$$P\left\{\lim_{t \rightarrow \infty} \omega(x(t)) = 0\right\} = 1,$$

where $x(t)$ is a bounded almost surely stochastic process satisfying (S2), and $\Psi(\mathbb{R}_+; \mathbb{R}_+)$ represents the family of function $\sigma(t)$ satisfying that $\sum_{k=1}^\infty \int_{t_k}^{t_k+\delta} \sigma(s) ds = \infty$ for any $\delta > 0$ and any increasing sequence $\{t_k\}_{k \geq 1}$ ($t_k \rightarrow \infty$ as $k \rightarrow \infty$).

Proof. The proof is divided into the following three steps.

Step 1. We claim that

$$P\left\{\liminf_{t \rightarrow \infty} \omega(x(t)) = 0\right\} = 1. \quad (\text{S4})$$

If this does not hold, then

$$P\left\{\liminf_{t \rightarrow \infty} \omega(x(t)) > 0\right\} > 0.$$

Hence, there is a (fixed) number ε_1 such that

$$P\left\{\liminf_{t \rightarrow \infty} \omega(x(t)) > 2\varepsilon_1\right\} \geq \varepsilon_1.$$

Let $\Omega_1 = \{\liminf_{t \rightarrow \infty} \omega(x(t)) > 2\varepsilon_1\}$. Then, for any $\theta \in \Omega_1$, there exists a random variable $t_1(\theta, \varepsilon_1) > 0$, such that

$$\omega(x(t)) \geq \varepsilon_1, \text{ whence } t > t_1.$$

It follows from $\rho \in \Psi(\mathbb{R}_+; \mathbb{R}_+)$ that for any (fixed) number $\alpha > 0$, there exist two constants $\rho > 0$ and $t_2 = t_2(\alpha, \rho)$ such that

$$\int_t^{t+\alpha} \rho(s) ds \geq \rho, \text{ whence } t \geq t_2.$$

From the above analysis and (S3), it follows that

$$\begin{aligned} \infty &> E \int_0^\infty \rho(s) \omega(x(s)) ds \\ &\geq \varepsilon_1 E \left[I_{\Omega_1} \int_{t_1+t_2}^\infty \rho(s) ds \right] \\ &= \varepsilon_1 E \left[I_{\Omega_1} \sum_{k=0}^\infty \int_{t_1+t_2+k\alpha}^{t_1+t_2+(k+1)\alpha} \rho(s) ds \right] \\ &= \infty, \end{aligned}$$

where I_A is the indicator function of a set A . Obviously, this is a contradiction. So (S4) must hold.

Step 2. We now claim that

$$P\left\{\limsup_{t \rightarrow \infty} \omega(x(t)) = 0\right\} = 1. \quad (\text{S5})$$

If this does not hold, then

$$P\left\{\limsup_{t \rightarrow \infty} \omega(x(t)) > 0\right\} > 0.$$

Hence, there is a (fixed) number ε_2 such that

$$P(\Omega_2) \geq 3\varepsilon_2, \quad (\text{S6})$$

where $\Omega_2 = \{\limsup_{t \rightarrow \infty} \omega(x(t)) > 2\varepsilon_2\}$. On the other hand, recalling the almost sure boundedness of $x(t)$, then for the given $\varepsilon_2 > 0$, there is a $h > 0$ such that

$$P(\Omega_3) \geq 1 - \varepsilon_2, \quad (\text{S7})$$

where $\Omega_3 = \{\sup_{t \geq 0} |x(t)| < h\}$.

From (S6) and (S7), it is easy to see that

$$P(\Omega_2 \cap \Omega_3) \geq 1 - \varepsilon_2, \quad (\text{S8})$$

For any (fixed) number $\bar{\eta} > 0$, let us now define a sequence of stopping times

$$\begin{aligned}\tau_h &= \inf\{t \geq 0 : |x(t)| \geq h\}, \\ \tau_1 &= \inf\{t \geq 0 : \omega(x(t)) \geq 2\varepsilon_2\}, \\ \tau_2 &= \inf\{t \geq \tau_1 : \omega(x(t)) \leq \varepsilon_2\}, \\ \tau_{2i+1} &= \inf\{t \geq \tau_{2i} + \bar{\eta} : \omega(x(t)) \geq 2\varepsilon_2\}, \\ \tau_{2i+2} &= \inf\{t \geq \tau_{2i+1} : \omega(x(t)) \leq \varepsilon_2\}, \quad i = 1, 2, \dots\end{aligned}$$

where throughout this paper we set $\inf \emptyset = \infty$. Note from (S4) and the definition of Ω_2 and Ω_3 that

$$\tau_h(\theta) = \infty, \quad \tau_k < \infty, \quad \forall k \geq 1 \quad \text{whenever} \quad \theta \in \Omega_2 \cap \Omega_3. \quad (\text{S9})$$

From (S3) and $\rho \in \Psi(\mathbb{R}_+; \mathbb{R}_+)$, we arrive at

$$\begin{aligned}\infty &> E \int_0^\infty \rho(s) \omega(x(s)) ds \\ &\geq E \left[\sum_{k=1}^\infty I_{\{\tau_{2k-1} < \infty, \tau_{2k} < \infty, \tau_h = \infty\}} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s) \omega(x(s)) ds \right] \\ &\geq \varepsilon_1 E \left[\sum_{k=1}^\infty I_{\{\tau_{2k-1} < \infty, \tau_{2k} < \infty, \tau_h = \infty\}} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s) ds \right] \\ &\geq \varepsilon_1 E \left[\sum_{k=1}^\infty I_{\Omega_2 \cap \Omega_3} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s) ds \right].\end{aligned} \quad (\text{S10})$$

If we can show that

$$E \left[\sum_{k=1}^\infty I_{\Omega_2 \cap \Omega_3} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s) ds \right] = \infty, \quad (\text{S11})$$

then we derive from (S10) that

$$\infty > \varepsilon_1 E \left[\sum_{k=1}^\infty I_{\Omega_2 \cap \Omega_3} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s) ds \right] = \infty, \quad (\text{S12})$$

which is a contradiction. So (S5) must hold.

Step 3. Next we prove (S11). By Assumption A1, Hölder's inequality and Doob's martingale inequality, we compute

$$\begin{aligned}&E \left[I_{\{\tau_{2k-1} \wedge \tau_h < \infty\}} \sup_{0 \leq t \leq T} \left| x((\tau_{2k-1} + t) \wedge \tau_h) - x(\tau_{2k-1} \wedge \tau_h) \right|^2 \right] \\ &\leq 2E \left[I_{\{\tau_{2k-1} \wedge \tau_h < \infty\}} \sup_{0 \leq t \leq T} \left| \int_{\tau_{2k-1} \wedge \tau_h}^{(\tau_{2k-1} + t) \wedge \tau_h} f(x(s), s) ds \right|^2 \right] \\ &\quad + 2E \left[I_{\{\tau_{2k-1} \wedge \tau_h < \infty\}} \sup_{0 \leq t \leq T} \left| \int_{\tau_{2k-1} \wedge \tau_h}^{(\tau_{2k-1} + t) \wedge \tau_h} g(x(s), s) dB(s) \right|^2 \right] \\ &\leq 2TE \left[I_{\{\tau_{2k-1} \wedge \tau_h < \infty\}} \int_{\tau_{2k-1} \wedge \tau_h}^{(\tau_{2k-1} + T) \wedge \tau_h} |f(x(s), s)|^2 ds \right] \\ &\quad + 8E \left[I_{\{\tau_{2k-1} \wedge \tau_h < \infty\}} \int_{\tau_{2k-1} \wedge \tau_h}^{(\tau_{2k-1} + T) \wedge \tau_h} |g(x(s), s)|^2 ds \right] \\ &\leq 2(T+4)TL^2(h, T)h^2.\end{aligned} \quad (\text{S13})$$

Since ω is a continuous function, it must be uniformly continuous in the closed ball $S = \{x \in \mathbb{R}^n \mid |x| \leq h\}$. We can, therefore, choose $\delta = \delta(\varepsilon_2)$ so small that

$$|\omega(x) - \omega(y)| < \frac{\varepsilon_2}{2} \text{ whenever } |x - y| < \delta, \quad x, y \in S. \quad (\text{S14})$$

We further choose $T = T(\varepsilon_2, h) > 0$ sufficiently small for

$$\frac{2(T+4)TL^2(h, T)h^2}{\delta^2} < \varepsilon_2.$$

It then follows from (S13) that

$$P\{\{\tau_{2k-1} \wedge \tau_h < \infty\} \cap \Omega_k^1\} \leq \frac{2(T+4)TL^2(h, T)}{\delta^2} < \varepsilon_2,$$

where $\Omega_k^1 = \left\{ \sup_{0 \leq t \leq T} \left| x((\tau_{2k-1} + t) \wedge \tau_h) - x(\tau_{2k-1} \wedge \tau_h) \right| \geq \delta \right\}$. Consequently,

$$\begin{aligned} P\{\Omega_2 \cap \Omega_3 \cap \Omega_k^1\} &\leq P\{\{\tau_{2k-1} < \infty, \tau_h = \infty\} \cap \Omega_k^1\} \\ &= P\{\{(\tau_{2k-1} \wedge \tau_h < \infty, \tau_h = \infty\} \cap \Omega_k^1\} \\ &\leq P\{\{(\tau_{2k-1} \wedge \tau_h < \infty\} \cap \Omega_k^1\} \\ &< \varepsilon_2, \end{aligned}$$

from which we further compute

$$\begin{aligned} &P\left\{\Omega_2 \cap \Omega_3 \cap \left\{ \sup_{0 \leq t \leq T} \left| x((\tau_{2k-1} + t) \wedge \tau_h) - x(\tau_{2k-1} \wedge \tau_h) \right| < \delta \right\}\right\} \\ &= P\{\Omega_2 \cap \Omega_3\} - P\{\Omega_2 \cap \Omega_3 \cap \Omega_k^1\} \\ &\geq 2\varepsilon_2 - \varepsilon_2 = \varepsilon_2. \end{aligned}$$

Using (S14), we derive that

$$\begin{aligned} &P\{\Omega_2 \cap \Omega_3 \cap \Omega_k^2\} \\ &\geq P\left\{\Omega_2 \cap \Omega_3 \cap \left\{ \sup_{0 \leq t \leq T} \left| x((\tau_{2k-1} + t) \wedge \tau_h) - x(\tau_{2k-1} \wedge \tau_h) \right| < \delta \right\}\right\} \\ &\geq \varepsilon_2, \end{aligned}$$

where $\Omega_k^2 = \left\{ \sup_{0 \leq t \leq T} \left| \omega(x((\tau_{2k-1} + t))) - \omega(x(\tau_{2k-1})) \right| < \frac{\varepsilon_2}{2} \right\}$.

On the other hand, it follows from $\rho \in \Psi(\mathbb{R}_+; \mathbb{R}_+)$ that for $T > 0$, there exist two constants $\varepsilon_3 = \varepsilon_3(T) > 0$ and $t_3 = t_3(\varepsilon_3, T)$ such that

$$\int_t^{t+T} \rho(s) ds \geq \varepsilon_3, \text{ whence } t \geq t_3.$$

By the definition of τ_k , there exists an integer $N = [t_3/\bar{\eta}]$ (where $[t_3/\bar{\eta}]$ denotes the integer part of $t_3/\bar{\eta}$) such that for any $\theta \in \Omega_2 \cap \Omega_3$,

$$\tau_{2k-1} \geq t_3, \text{ whence } k \geq N + 1.$$

Hence, for any $\theta \in \Omega_2 \cap \Omega_3$, we have

$$\int_{\tau_{2k-1}}^{\tau_{2k-1}+T} \rho(s)ds \geq \varepsilon_3, \text{ whence } k \geq N+1.$$

Noting that

$$\tau_{2k}(\theta) - \tau_{2k-1}(\theta) \geq T, \text{ if } \theta \in \Omega_2 \cap \Omega_3 \cap \Omega_k^2,$$

we have

$$\begin{aligned} E \left[\sum_{k=1}^{\infty} I_{\Omega_2 \cap \Omega_3} \int_{\tau_{2k-1}}^{\tau_{2k}} \rho(s)ds \right] &\geq E \left[\sum_{k=N+1}^{\infty} I_{\Omega_2 \cap \Omega_3 \cap \Omega_k^2} \int_{\tau_{2k-1}}^{\tau_{2k-1}+T} \rho(s)ds \right] \\ &\geq \varepsilon_3 E \left[\sum_{k=N+1}^{\infty} I_{\Omega_2 \cap \Omega_3 \cap \Omega_k^2} \right] \\ &\geq \varepsilon_3 \sum_{k=N+1}^{\infty} \varepsilon_2 \\ &= \infty, \end{aligned} \tag{S15}$$

which is a contradiction. So (S11) must hold. Then (S5) must hold.

Finally, from Step 1–Step 3, we obtain the conclusion of Proposition P1. The proof is thus complete. $\blacksquare$

Please note that since the above proving processes are recalled from [R2], they have not been included in the revised version for conciseness of the paper.

Comment 3: *Since Example 2 is focused on a practical model, the physical meaning of Figure 5 and Figure 6 should be given.*

Response 3: Thanks for this significant comment. To clearly explain the physical meaning of Figure 5 and Figure 6 in the paper, a detailed modeling process of Example 2 is presented in the following step by step.

- By defining $\bar{x}_1 = V_c$, $\bar{x}_2 = i_L$, the state equation of the circuit system can be expressed as

$$\begin{cases} \dot{\bar{x}}_1 = \frac{-\frac{1}{R_1} + d}{C} \bar{x}_1 - \frac{1}{C} \bar{x}_2, \\ \dot{\bar{x}}_2 = \frac{1}{L} \bar{x}_1 - \frac{R_2}{L} \bar{x}_2 - \frac{1}{L} \bar{V}, \end{cases} \tag{S16}$$

where the voltage $\bar{V}$ denotes the control input.

- Our control goal is to regulate the circuit system to the desired equilibrium $(\bar{x}_{10}, \bar{x}_{20}, \bar{V}_0) = \left(\frac{\bar{x}_{20}}{-\frac{1}{R_1} + d}, \bar{x}_{20}, -R_2 \bar{x}_{20} + \bar{x}_{10} \right)$.
- To design a state feedback controller, we then introduce the change of coordinates

$$x_1 = \bar{x}_1 - \bar{x}_{10}, \quad x_2 = \bar{x}_2 - \bar{x}_{20}, \quad V = \bar{V} - \bar{V}_0,$$

from which and (S16), we obtain

$$\begin{cases} \dot{x}_1 = \frac{-\frac{1}{R_1} + d}{C}x_1 - \frac{1}{C}x_2, \\ \dot{x}_2 = \frac{1}{L}\bar{x}_1 - \frac{R_2}{L}x_2 - \frac{1}{L}V. \end{cases} \quad (\text{S17})$$

- It is known that the input voltage V may change due to the external noises. That is, one may suppose that the input voltage V is given by $V = V_1 + aV_1\dot{B}(t)$, where $\dot{B}(t)$, the formal derivative of a standard Wiener process $B(t)$, is the white noise and a is an unknown constant. Thus, Eq.(S17) can further be expressed as

$$\begin{cases} \dot{x}_1 = \frac{-\frac{1}{R_1} + d}{C}x_1 - \frac{1}{C}x_2, \\ \dot{x}_2 = \frac{1}{L}x_1 - \frac{R_2}{L}x_2 - \frac{1}{L}(V_1 + aV_1\dot{B}(t)). \end{cases} \quad (\text{S18})$$

Based on the above analysis, it is clear that when $x_1 \rightarrow 0$, $x_2 \rightarrow 0$, and $V_1 \rightarrow 0$ (see Figure 5 and Figure 6 in the paper), we have $\bar{x}_1 \rightarrow \bar{x}_{10}$, $\bar{x}_2 \rightarrow \bar{x}_{20}$, $\bar{V} \rightarrow \bar{V}_0$. This implies that the states from Example 2 asymptotically reach the desired equilibrium and the expected heat generated by the circuit system is achieved and maintained according to the calculation method of electric power.

Response to Reviewer #2

Comment: *This paper considers the stabilization problem for a class of stochastic feedforward nonlinear systems under unknown growth rate. The topic is interesting, this reviewer has the following comments*

Response: Thanks very much for the reviewer's encouraging comments.

Comment 1: *Is it possible that the unknown constant θ in the growth rate is a time-varying state, i.e., dependent on the system state ?*

Response 1: Thank the reviewer for the insightful comment. When the unknown constant θ in the growth rate is dependent on the system state, a dynamic gain is always introduced to cope with such nonlinearities of those concerned systems (see, e.g., [R3–R5]). However, we would like to point out that this dynamic gain method cannot be extended to stochastic systems. The main reason lies in the fact that to the best of the authors' knowledge, there has been no effective proof method guaranteeing the monotonic property of the dynamic gain, and the nonnegativity of the derivative of the dynamic gain in the context of stochastic cases. Thus, in the paper we are mainly devoted to the case of the unknown constant θ . On the other hand, from the viewpoint of modern control theory, it is certainly interesting and important to consider the problem of state feedback stabilization of stochastic feedforward nonlinear systems with *state-dependent growth rates*, which has been explicitly mentioned in the Conclusion Section of the revised paper and will be pursued in our future research.

Comment 2: *Please give some comparisons with the existing results.*

Response 2: Thank the reviewer for the valuable comment. Compared with the existing works, the main advantages of our paper are illustrated as follows.

- For stochastic time-varying systems, although the condition of weakly positive definition plays a significant role to develop the stochastic stability theories in [R6], such a condition is still restrictive (please see the motivating example in Section 2 of the revised version). Thus, in our paper a new definition of the generalized weakly positive definite function has been given, based on which we have provided a new LaSalle-type theorem for stochastic time-varying systems (please see Definition 3 and Lemma 1 in the revised version).
- The system considered in this paper is more common than some existing ones, for instance, the diffusion terms in [R7–R10] are identically equal to zero. In addition, even though the nonlinearities of systems in [R11–R14] are stronger than that of the system considered in this paper, the growth rates in Assumption 3.1 of [R14] and Assumption 1 of [R11–R13] are all limited to known constants. It is worth pointing out that the growth rate is *unknown* in our work, which indicates the presence of completely unknown parameters and implies the essential difference from those in [R11–R14].

Please note that the above-mentioned advantages have been stressed in the Introduction Section and Remark 3 of the revised version by blue color, respectively.

Comment 3: *The references are not complete, some latest results about feedforward systems should be included such as: Global stabilization of a class of uncertain upper-triangular systems under sampled-data control, International Journal of Robust and Nonlinear Control, 23(6), 620-637, 2013; Global output feedback stabilization of a class of nonlinear systems*

via linear sampled-data control, IEEE Transactions on Automatic Control, 57(11), 2934-2939, 2012

Response 3: Thank the reviewer for having brought these latest results to our attention. We have carefully read the mentioned latest results and other related ones (e.g., [R8]) on feedforward systems, which are valuable works for us to improve the current paper and have been added into the References (please see References Du, Qian, & Li (2013); Qian & Du (2012); and Du, Qian, He, & Cheng (2013) in the revised paper).

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