Apendix

PROOF OF LEMMA 1.

From Equation (2), we can obtain that $\frac{\partial \pi^{SC-0}(p)}{\partial p} = \overline{a} - 2kp + \overline{M}$, $\frac{\partial^2 \pi^{SC-0}(p)}{\partial p^2} = -2k < 0$. Thus, when $\frac{\partial \pi^{SC-0}(p)}{\partial p} = 0$, Equation (2) will arrive the maximal value. Then, we can obtain the optimal decisions shown in Lemma 1(i). Next, we find $\frac{\partial \pi^{SC-0*}}{\partial C} = \frac{C(b_0^2 e_0^2 k^2(1+r)^2 - 4b_0 k)}{2k} + \frac{4a_0 k + 2ab_0 e_0 k(1+r) - 2a_0 b_0 e_0^2 k^2 (1+r)^2 + 2b_0 e_0 k(2Q + kp_0 r(1+r))}{4k}$ and $\frac{\partial^2 \pi^{SC-0*}}{\partial C^2} = \frac{b_0^2 e_0^2 k^2 (1+r)^2 - 4b_0 k}{2k}$. When $C = 0$, $\frac{\partial \pi^{SC-0*}}{\partial C} > 0$. If $r > (2 - e_0\sqrt{b_0 k})/(e_0\sqrt{b_0 k})$, $\frac{\partial^2 \pi^{SC-0*}}{\partial C^2} > 0$. $\frac{\partial \pi^{SC-0*}}{\partial C}$ is increasing in C and always larger than zero, thus, π^{SC-0*} is increasing in C; otherwise, if $r \leq (2 - e_0\sqrt{b_0 k})/(e_0\sqrt{b_0 k})$, $\frac{\partial^2 \pi^{SC-0*}}{\partial C^2} < 0$. $\frac{\partial \pi^{SC-0*}}{\partial C}$ is linear decreasing in C, thus, there must be a C_0 that $\frac{\partial^2 \pi^{SC-0*}}{\partial C^2} = 0$ and when $C > C_0$, $\frac{\partial \pi^{SC-0*}}{\partial C} < 0$. Thus, π^{SC-0*} is firstly increasing in C and then decreasing in C. Therefore, we can obtain the results in Lemma 1(ii). ■

PROOF OF LEMMA 2.

From Equation (4), we can obtain $\frac{\partial \pi_p^{DR-0}(p)}{\partial p} = \overline{a} - 2kp + k\omega$, $\frac{\partial^2 \pi_p^{DR-0}(p)}{\partial p^2} = -2k < 0$. Thus, the platform's response function is $p^{DR-0*} = (\overline{a} + k\omega)/2k$. After submitting the response function to Equation (3), we can obtain $\frac{\partial \pi_m^{DR-0}(\omega)}{\partial \omega} = \frac{1}{2}(\overline{a} - krp_0 - 2k\omega + ke_0(a_0 - b_0 C))$, $\frac{\partial^2 \pi_m^{DR-0}(\omega)}{\partial \omega^2} = -k < 0$. Similarly, there is an optimal wholesale price $\omega^{DR-0*} = \frac{\overline{a} + \overline{M}}{2k}$ to maximize the manufacturer's profit. Thus, we obtain the optimal retail price as $p^{DR-0*} = \frac{3\overline{a} + \overline{M}}{4k}$, and the optimal profits of the manufacturer and the platform are $\pi_m^{DR-0*} = \frac{(\overline{a} - \overline{M})^2}{8k} + p_0 Q + (a_0 - b_0 C)(C - Qe_0)$ and $\pi_p^{DR-0*} = \frac{(\overline{a} - \overline{M})^2}{16k}$, respectively. ■

PROOF OF LEMMA 3.

From Equation (5), we can obtain $\frac{\partial \pi_p^{DM-0}(p)}{\partial p} = \overline{a}(1-\phi) + \overline{M} - 2pk(1-\phi)$, $\frac{\partial^2 \pi_p^{DM-0}(p)}{\partial p^2} = -2k(1-\phi) < 0$. Thus, there is an optimal retail price to maximize the manufacturer's profit. We can obtain the optimal retail price as $\frac{\partial \pi_p^{DM-0}(p)}{\partial p} = 0$. So, we have $p^{DM-0*} = (\overline{a}(1-\phi) + \overline{M})/(2k(1-\phi))$ in the decentralized solution with marketplace mode, and the maximal profits of the manufacturer and the platform are $\pi_m^{DM-0*} = \frac{(\overline{a}(1-\phi) - \overline{M})^2}{4k(1-\phi)} + p_0 Q + (a_0 - b_0 C)(C - Qe_0) - F$ and $\pi_p^{DM-0*} = \frac{\phi((\overline{a}(1-\phi))^2 - \overline{M}^2)}{4k(1-\phi)^2} + F$, respectively. ■

PROOF OF COROLLARY 1.

We make the difference between profits of the manufacturer with marketplace mode and reselling mode.

$$\pi^{0*}_{difference} = \pi^{DM-0*}_{m} - \pi^{DR-0*}_{m} = \frac{(\overline{a}(1-\phi)-\overline{M})^2}{4k(1-\phi)} - F - \frac{(\overline{a}-\overline{M})^2}{8k} = \frac{1}{2}(2\overline{a}^2(1-\phi) + \frac{2\overline{M}^2}{1-\phi} - (\overline{a}+\overline{M})^2) - F$$

When $\phi = 1 - \frac{\overline{M}}{\overline{a}}$, we get the maximal level of $\pi^{0*}_{difference} = 4\overline{a}\overline{M} - (\overline{a}+\overline{M})^2 - F$. Obviously, $(\overline{a}+\overline{M})^2 > 4\overline{a}\overline{M}$, $\pi^{0*}_{difference}$ is decreasing in ϕ. Thus, we find when $\phi = \phi_0$, $\pi^{0*}_{difference} = 0$. Therefore, when $0 < \phi < \phi_0$, $\pi^{0*}_{difference} > 0$; otherwise, $\pi^{0*}_{difference} < 0$, where, $\phi_0 = 1 - \frac{(\overline{a}+\overline{M})^2 + 2F + \sqrt{((\overline{a}+\overline{M})^2 - 2F)^2 - 16\overline{a}^2\overline{M}^2}}{4\overline{a}^2}$. ■

PROOF OF PROPOSITION 1.

(i) When $\overline{q} + r\overline{q} - q^{SC-0*} - rq^{SC-0*} > 0$, we find $p < \widetilde{p}$, where $\widetilde{p} = (\overline{a} + 2\Delta a + \overline{M})/2k$. In this case, $\pi^{SC}(p) = p_0(Q + r\overline{q}) + p\overline{q} - (a_0 - b_0C)(e_0(Q + r\overline{q} + \overline{q}) - C) - \lambda_1(\overline{q} + r\overline{q} - q^{SC-0*} - rq^{SC-0*})$. From Equation (7), we can obtain $\frac{\partial \pi^{SC}_p(p)}{\partial p} = \overline{a} + \Delta a - 2kp + \overline{M} + \lambda_1 k(1+r)$, $\frac{\partial^2 \pi^{SC}_p(p)}{\partial p^2} = -2k < 0$. Thus, there is an optimal retail price that provides the maximal profits. We obtain the optimal retail price as $\frac{\partial \pi^{SC}_p(p)}{\partial p} = 0$. So, we have $p^{SC*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} + \frac{\lambda_1}{2}(1+r)$.

Comparing p^{SC*} and $\widetilde{p}$, we find when $\Delta a > \lambda_1 k(1+r)$, $p^{SC*} < \widetilde{p}$, thus the optimal retail price is $p^{SC*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} + \frac{\lambda_1}{2}(1+r)$, $\pi^{SC*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M}) + (\Delta a - \lambda_1 k(1+r))^2}{4k} + p_0 Q + (a_0 - b_0 C)(C - Qe_0)$; otherwise, $p^{SC*} \geq \widetilde{p}$. Thus, the optimal retail price is $p^{SC*} = \widetilde{p} = (\overline{a} + 2\Delta a + \overline{M})/2k$, $\pi^{SC*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})}{4k} + p_0 Q + (a_0 - b_0 C)(C - Qe_0)$.

(ii) When $\overline{q} + r\overline{q} - q^{SC-0*} - rq^{SC-0*} \leq 0$, we find $p \geq \widetilde{p}$. In this case, $\pi^{SC}(p) = p_0(Q + r\overline{q}) + p\overline{q} - (a_0 - b_0C)(e_0(Q + r\overline{q} + \overline{q}) - C) - \lambda_2(q^{SC-0*} + rq^{SC-0*} - \overline{q} - r\overline{q})$. From Equation (7), we can obtain $\frac{\partial \pi^{SC}_p(p)}{\partial p} = \overline{a} + \Delta a - 2kp + \overline{M} - \lambda_2 k(1+r)$, $\frac{\partial^2 \pi^{SC}_p(p)}{\partial p^2} = -2k < 0$. Thus, there is an optimal retail price that provides the maximal profits. We obtain the optimal retail price as $\frac{\partial \pi^{SC}_p(p)}{\partial p} = 0$. So, we have $p^{SC*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} - \frac{\lambda_1}{2}(1+r)$.

Comparing p^{SC*} and $\widetilde{p}$, we find when $\Delta a < -\lambda_2 k(1+r)$, $p^{SC*} > \widetilde{p}$, thus the optimal retail price is $p^{SC*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} - \frac{\lambda_2}{2}(1+r)$, $\pi^{SC*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M}) + (\Delta a + \lambda_2 k(1+r))^2}{4k} + p_0 Q + (a_0 - b_0 C)(C - Qe_0)$; otherwise, $p^{SC*} \leq \widetilde{p}$, thus the optimal retail price is $p^{SC*} = \widetilde{p} = (\overline{a} + 2\Delta a + \overline{M})/2k$, $\pi^{SC*} =$

$\frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})}{4k} + p_0 Q + (a_0 - b_0 C)(C - Q e_0)$.

Therefore, based on the solutions in case(i) and (ii), we divide Δa into three cases in PROPOSITION 1. ∎

PROOF OF PROPOSITION 2.

From Equation (8), we can obtain $\frac{\partial \pi_p^{DR}(p)}{\partial p} = \overline{a} + \Delta a - 2kp + k\omega$, $\frac{\partial^2 \pi_p^{DR}(p)}{\partial p^2} = -2k < 0$. Thus, there is an optimal retail price that provides the maximal profits. We obtain the optimal price as $\frac{\partial \pi_p^{DR}(p)}{\partial p} = 0$. So, we have $p^{DR*} = (\overline{a} + \Delta a + k\omega)/2k$. We then determine the wholesale prices ω.

(i) When $\overline{q} + r\overline{q} - q^{DR-0*} - rq^{DR-0*} > 0$, we find $\omega < \widetilde{\omega}$, where $\widetilde{\omega} = (\overline{a} + 2\Delta a + \overline{M})/2k$. In this case, $\pi_m^{DR}(\omega) = p_0(Q + r\overline{q}) + \omega\overline{q} - (a_0 - b_0 C)(e_0(Q + r\overline{q} + \overline{q}) - C) - \lambda_1(\overline{q} + r\overline{q} - q^{DR-0*} - rq^{DR-0*})^+$ and $\pi_p^{DR}(p) = (p - \omega)\overline{q}$. Then, we get $\frac{\partial \pi_p^{DR}(\omega)}{\partial \omega} = (\overline{a} + \Delta a - 2k\omega + \overline{M} + \lambda_1 k(1+r))/2$, $\frac{\partial^2 \pi_p^{DR}(\omega)}{\partial \omega^2} = -2k < 0$. Thus, there is an optimal wholesale price that provides the maximal profits. We obtain the optimal wholesale price as $\frac{\partial \pi_p^{DR}(\omega)}{\partial \omega} = 0$. So, we have $\omega^{DR*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} + \frac{\lambda_1}{2}(1 + r)$. Thus, we obtain the optimal retail price as $p^{DR*} = \frac{3\overline{a}+3\Delta a+\overline{M}}{4k} + \frac{\lambda_1}{4}(1 + r)$.

Comparing ω^{DR*} and $\widetilde{\omega}$, we find when $\Delta a \geq \lambda_1 k(1+r)$, $\omega^{DR*} < \widetilde{\omega}$, thus the optimal wholesale price and retail price are $\omega^{DR*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} + \frac{\lambda_1}{2}(1+r)$ and $p^{DR*} = \frac{3\overline{a}+3\Delta a+\overline{M}}{4k} + \frac{\lambda_1}{4}(1+r)$, the maximal profits of the manufacturer and the platform are $\pi_m^{DR*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})+(\Delta a-\lambda_1 k(1+r))^2}{8k} + p_0 Q + (a_0 - b_0 C)(C - Q e_0)$ and $\pi_p^{DR*} = \frac{(\overline{a}+\Delta a-\overline{M}-\lambda_1 k(1+r))^2}{16k}$, respectively; otherwise, the optimal wholesale price and retail price are $\omega^{DR*} = \widetilde{\omega} = (\overline{a} + 2\Delta a + \overline{M})/2k$ and $p^{DR*} = (3\overline{a} + 4\Delta a + \overline{M})/4k$, the maximal profits of the manufacturer and the platform are $\pi_m^{DR*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})}{8k} + p_0 Q + (a_0 - b_0 C)(C - Q e_0)$ and $\pi_p^{DR*} = \frac{(\overline{a}-\overline{M})^2}{16k}$, respectively.

(ii) When $\overline{q} + r\overline{q} - q^{DR-0*} - rq^{DR-0*} \leq 0$, we find $\omega \geq \widetilde{\omega}$. In this case, $\pi_m^{DR}(\omega) = p_0(Q + r\overline{q}) + \omega\overline{q} - (a_0 - b_0 C)(e_0(Q + r\overline{q} + \overline{q}) - C) - \lambda_2(q^{DR-0*} + rq^{DR-0*} - \overline{q} - r\overline{q})^+$ and $\pi_p^{DR}(p) = (p - \omega)\overline{q}$. Then, we get $\frac{\partial \pi_p^{DR}(\omega)}{\partial \omega} = (\overline{a} + \Delta a - 2k\omega + \overline{M} - \lambda_2 k(1+r))/2$, $\frac{\partial^2 \pi_p^{DR}(\omega)}{\partial \omega^2} = -2k < 0$. Thus, there is an optimal wholesale price that provides the maximal profits. We can obtain the optimal wholesale price as $\frac{\partial \pi_p^{DR}(\omega)}{\partial \omega} = 0$. So, we have $\omega^{DR*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} - \frac{\lambda_2}{2}(1 + r)$. Thus, we obtain the optimal retail price as $p^{DR*} = \frac{3\overline{a}+3\Delta a+\overline{M}}{4k} - \frac{\lambda_2}{4}(1 + r)$.

Comparing ω^{DR*} and $\widetilde{\omega}$, we find when $\Delta a < -\lambda_2 k(1+r)$, $\omega^{DR*} \geq \widetilde{\omega}$, thus the optimal wholesale price and retail price are $\omega^{DR*} = \frac{\overline{a}+\Delta a+\overline{M}}{2k} - \frac{\lambda_2}{2}(1+r)$ and $p^{DR*} = \frac{3\overline{a}+3\Delta a+\overline{M}}{4k} - \frac{\lambda_2}{4}(1+r)$, the maximal profits of the manufacturer and the platform are $\pi_m^{DR*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})+(\Delta a+\lambda_2 k(1+r))^2}{8k}+p_0Q+(a_0-b_0C)(C-Qe_0)$ and $\pi_p^{DR*} = \frac{(\overline{a}+\Delta a-\overline{M}+\lambda_2 k(1+r))^2}{16k}$, respectively; otherwise, the optimal wholesale price and retail price are $\omega^{DR*} = \widetilde{\omega} = (\overline{a}+2\Delta a+\overline{M})/2k$ and $p^{DR*} = (3\overline{a}+4\Delta a+\overline{M})/4k$, the maximal profits of the manufacturer and the platform are $\pi_m^{DR*} = \frac{(\overline{a}-\overline{M})(\overline{a}+2\Delta a-\overline{M})}{8k}+p_0Q+(a_0-b_0C)(C-Qe_0)$ and $\pi_p^{DR*} = \frac{(\overline{a}-\overline{M})^2}{16k}$, respectively.

Therefore, based on the solutions in case(i) and (ii), we divide Δa into three cases in PROPOSITION 2. ■

PROOF OF COROLLARY 2.

The difference of the profits in the centralized and decentralised situations with reselling mode is as follows:

$\pi_{SC-DR} = \pi^{SC*} - (\pi_m^{DR*} + \pi_p^{DR*}) =$

$$\begin{cases} \dfrac{(\overline{a}+\Delta a)^2 - 2\overline{M}(\overline{a}+\Delta a) - 2\lambda_2 k(1+r)(\overline{a}-\Delta a) + (\overline{M}+\lambda_2 k(1+r))^2}{16k} & -\overline{a} \le \Delta a \le -\lambda_2 k(1+r) \\ \dfrac{(\overline{a}-\overline{M})(\overline{a}-\overline{M}+4\Delta a)}{16k} & -\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r) \\ \dfrac{(\overline{a}+\Delta a)^2 - 2\overline{M}(\overline{a}+\Delta a) + 2\lambda_1 k(1+r)(\overline{a}-\Delta a) + (\overline{M}-\lambda_1 k(1+r))^2}{16k} & \Delta a > \lambda_1 k(1+r) \end{cases} \quad (1)$$

Under the two conditions that (i) $\overline{a}+\Delta a > 0$, that ensured the maximal market size of online channel with demand disruptions is larger than zero; (ii) $\overline{a}-\overline{M} > 0$, that ensured the optimal demand $d^* = q^* = \overline{a} - kp^{DR-0*} > 0$. We find that in case 1 ($-\overline{a} \le \Delta a \le -\lambda_2 k(1+r)$), when $\Delta a < \min\{\frac{\overline{M}-\overline{a}}{4}, -k\lambda_2(1+r)\}$ and $\frac{\overline{a}-\Delta a-2\sqrt{\Delta a(\overline{M}-\overline{a})}}{k(1+r)} < \lambda_2 < \frac{-\Delta a}{k(1+r)}$, $\pi_{SC-DR} < 0$; in case 2 ($-\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r)$), when $\min\{\frac{\overline{M}-\overline{a}}{4}, -k\lambda_2(1+r)\} \le \Delta a < \frac{\overline{M}-\overline{a}}{4}$, $\pi_{SC-DR} < 0$; in case 3, $\pi_{SC-DR} < 0$ is never existed. To conclude, when $\Delta a < \frac{\overline{M}-\overline{a}}{4}$ and $\lambda_2 > \frac{\overline{a}-\Delta a-2\sqrt{\Delta a(\overline{M}-\overline{a})}}{k(1+r)}$, the profit with reselling mode in the decentralized situation is larger than that in the centralized situation. ■

PROOF OF PROPOSITION 3.

(i) When $\overline{q}+r\overline{q}-q^{DM-0*}-rq^{DM-0*} > 0$, we find $p < \widetilde{p}_{dm}$, where $\widetilde{p}_{dm} = \frac{(\overline{a}+2\Delta a)(1-\phi)+\overline{M}}{2k(1-\phi)}$. In this case, $\pi_m^{DM}(p) = p_0(Q+r\overline{q}) + (1-\phi)p\overline{q} - (a_0-b_0C)(e_0(Q+r\overline{q}+\overline{q})-C) - F - \lambda_1(\overline{q}+r\overline{q}-q^{DM-0*}-rq^{DM-0*})^+$, $\pi_p^{DM}(\phi) = \phi p\overline{q} + F$. From Equation (8), we can obtain $\frac{\partial \pi_m^{DM}(p)}{\partial p} =$

$(\overline{a}+\Delta a)(1-\phi)-2kp(1-\phi)+\overline{M}+\lambda_1 k(1+r)$, $\frac{\partial^2\pi_m^{DM}(p)}{\partial p^2}=-2k(1-\phi)<0$. Thus, there is an optimal retail price that provides the maximal profits. We can obtain the optimal retail price as $\frac{\partial\pi_m^{DM}(p)}{\partial p}=0$. So, we have $p^{DM*}=\frac{(\overline{a}+\Delta a)(1-\phi)+\overline{M}+\lambda_1 k(1+r)}{2k(1-\phi)}$.

Comparing p^{DM*} and $\widetilde{p}_{dm}$, we find when $\Delta a>\frac{\lambda_1 k(1+r)}{1-\phi}$, $p^{DM*}<\widetilde{p}_{dm}$, the optimal retail price is $p^{DM*}=\frac{(\overline{a}+\Delta a)(1-\phi)+\overline{M}+\lambda_1 k(1+r)}{2k(1-\phi)}$, and the maximal profits of the manufacturer and the platform are $\pi_m^{DM*}=\frac{((\overline{a}+\Delta a)(1-\phi)-\overline{M})^2-\lambda_1 k(1+r)(2\Delta a(1-\phi)-\lambda_1 k(1+r))}{4k(1-\phi)}+p_0Q+(a_0-b_0C)(C-Qe_0)-F$ and $\pi_p^{DM*}=\frac{\phi((\overline{a}+\Delta a)^2(1-\phi)^2-(\overline{M}+\lambda_1 k(1+r))^2)}{4k(1-\phi)^2}+F$; otherwise, the optimal retail price is $p^{DM*}=\widetilde{p}_{dm}=\frac{(\overline{a}+2\Delta a)(1-\phi)+\overline{M}}{2k(1-\phi)}$, the maximal profits of the manufacturer and the platform are $\pi_m^{DM*}=\frac{1}{4k(1-\phi)^2}((1-\phi)\overline{a}-\overline{M})((1-\phi)(\overline{a}+2\Delta a)-\overline{M})+p_0Q+(a_0-b_0C)(C-Qe_0)-F$ and $\pi_p^{DM*}=\frac{((1-\phi)\overline{a}-\overline{M})((1-\phi)\overline{a}+\overline{M}+2\Delta a(1-\phi))}{4k(1-\phi)^2}+F$, respectively.

(ii) When $\overline{q}+r\overline{q}-q^{DM-0*}-rq^{DM-0*}\leq 0$, we find $p\geq\widetilde{p}_{dm}$. In this case, $\pi_m^{DM}(p)=p_0(Q+r\overline{q})+(1-\phi)p\overline{q}-(a_0-b_0C)(e_0(Q+r\overline{q}+\overline{q})-C)-F-\lambda_2(q^{DM-0*}+rq^{DM-0*}-\overline{q}-r\overline{q})^+$, $\pi_p^{DM}(\phi)=\phi p\overline{q}+F$. From Equation (8), we can obtain $\frac{\partial\pi_m^{DM}(p)}{\partial p}=(\overline{a}+\Delta a)(1-\phi)-2kp(1-\phi)+\overline{M}-\lambda_2 k(1+r)$, $\frac{\partial^2\pi_m^{DM}(p)}{\partial p^2}=-2k(1-\phi)<0$. Thus, there is an optimal retail price that provides the maximal profits. We can obtain the optimal retial price as $\frac{\partial\pi_m^{DM}(p)}{\partial p}=0$. So, we have $p^{DM*}=\frac{(\overline{a}+\Delta a)(1-\phi)+\overline{M}-\lambda_2 k(1+r)}{2k(1-\phi)}$.

Comparing p^{DM*} and $\widetilde{p}_{dm}$, we find when $\Delta a<-\frac{\lambda_2 k(1+r)}{1-\phi}$, $p^{DM*}>\widetilde{p}$, thus the optimal retail price is $p^{DM*}=\frac{(\overline{a}+\Delta a)(1-\phi)+\overline{M}-\lambda_2 k(1+r)}{2k(1-\phi)}$, and the maximal profits of the manufacturer and the platform are $\pi_m^{DM*}=\frac{((\overline{a}+\Delta a)(1-\phi)-\overline{M})^2+\lambda_2 k(1+r)(2\Delta a(1-\phi)+\lambda_2 k(1+r))}{4k(1-\phi)}+p_0Q+(a_0-b_0C)(C-Qe_0)-F$ and $\pi_p^{DM*}=\frac{\phi((\overline{a}+\Delta a)^2(1-\phi)^2-(\overline{M}-\lambda_2 k(1+r))^2)}{4k(1-\phi)^2}+F$; otherwise, the optimal retail price is $p^{DM*}=\widetilde{p}_{dm}=\frac{(\overline{a}+2\Delta a)(1-\phi)+\overline{M}}{2k(1-\phi)}$, the maximal profits of the manufacturer and the platform are $\pi_m^{DM*}=\frac{1}{4k(1-\phi)^2}((1-\phi)\overline{a}-\overline{M})((1-\phi)(\overline{a}+2\Delta a)-\overline{M})+p_0Q+(a_0-b_0C)(C-Qe_0)-F$ and $\pi_p^{DM*}=\frac{((1-\phi)\overline{a}-\overline{M})((1-\phi)\overline{a}+\overline{M}+2\Delta a(1-\phi))}{4k(1-\phi)^2}+F$, respectively.

Therefore, based on the solutions in case(i) and (ii), we divide Δa into three cases which is shown in PROPOSITION 3. ∎

PROOF OF PROPOSITION 4.

The difference of the manufacturer's profits with reselling mode and marketplace mode is as follows:

$\pi_{DR-DM} = \pi_m^{DR*} - \pi_m^{DM*}$

$$\begin{cases} \dfrac{(1-2\phi)(\overline{a}+\Delta a)^2 - 2\overline{M}(\overline{a}+\Delta a) + 2\Delta a\lambda_2 k(1+r)}{8k} + \dfrac{(\overline{M}^2 + (\lambda_2 k(1+r))^2)(1+\phi)}{8k(1-\phi)} + F & -\overline{a} \le \Delta a \le -\dfrac{\lambda_2 k(1+r)}{1-\phi} \\ \dfrac{\overline{a}(1-2\phi)(\overline{a}+2\Delta a) - 2\overline{M}(\overline{a}+\Delta a) - (\Delta a + \lambda_2 k(1+r))^2}{8k} + \dfrac{\overline{M}^2(1+\phi)}{8k(1-\phi)} + F & -\dfrac{\lambda_2 k(1+r)}{1-\phi} < \Delta a \le -\lambda_2 k(1+r) \\ \dfrac{\overline{a}(1-2\phi)(\overline{a}+2\Delta a) - 2\overline{M}(\overline{a}+\Delta a)}{8k} + \dfrac{\overline{M}^2(1+\phi)}{8k(1-\phi)} + F & -\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r) \\ \dfrac{\overline{a}(1-2\phi)(\overline{a}+2\Delta a) - 2\overline{M}(\overline{a}+\Delta a) - (\Delta a - \lambda_1 k(1+r))^2}{8k} + \dfrac{\overline{M}^2(1+\phi)}{8k(1-\phi)} + F & \lambda_1 k(1+r) < \Delta a \le \dfrac{\lambda_1 k(1+r)}{1-\phi} \\ \dfrac{(1-2\phi)(\overline{a}+\Delta a)^2 - 2\overline{M}(\overline{a}+\Delta a) - 2\Delta a\lambda_1 k(1+r)}{8k} + \dfrac{(\overline{M}^2 + (\lambda_1 k(1+r))^2)(1+\phi)}{8k(1-\phi)} + F & \Delta a > \dfrac{\lambda_1 k(1+r)}{1-\phi} \end{cases} \quad (2)$$

We discuss the difference in five cases,

Case i: $-\overline{a} \le \Delta a \le -\frac{\lambda_2 k(1+r)}{1-\phi}$. When $\Delta a = -\overline{a}$, $\pi_{DR-DM}(-\overline{a}) = \frac{(\overline{M}^2+(\lambda_2 k(1+r))^2)(1+\phi)}{8k(1-\phi)} - \frac{\overline{a}\lambda_2 k(1+r)}{4k} + F$. Thus, we find that if $\frac{-\lambda_2 k(1+r)}{1-\phi} \le \overline{a} \le \max\{\frac{-\lambda_2 k(1+r)}{1-\phi}, \widetilde{a}\}$, $\pi_{DR-DM}(-\overline{a}) > 0$; otherwise, $\pi_{DR-DM}(-\overline{a}) \le 0$, where $\widetilde{a} = \frac{((\lambda_2 k(1+r))^2+\overline{M}^2)(1+\phi)}{2\lambda_2 k(1+r)(1-\phi)}$.

From Equation (13), we can get $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = \frac{1}{4k}((1-2\phi)(\overline{a}+\Delta a) - \overline{M} + \lambda_2 k(1+r))$, $\frac{\partial^2 \pi_{DR-DM}}{\partial \Delta a^2} = \frac{1}{4k}(1-2\phi) > 0$. Thus, this is a convex programming problem. When $\Delta a < \Delta a_{min1}$, π_{DR-DM} is decreasing in Δa, where $\Delta a_{min1} = -\overline{a} + \frac{\overline{M}-\lambda_2 k(1+r)}{1-2\phi}$. When $\Delta a = \Delta a_{min1}$, $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = 0$, which means π_{DR-DM} has the minimal solution, and $\pi_{DR-DM} = \frac{\overline{M}\lambda_2 k(1+r)(1-\phi) - \phi^2(\overline{M}^2+(\lambda_2 k(1+r))^2)}{4k(1-\phi)(1-2\phi)} - \frac{\overline{a}\lambda_2 k(1+r)}{4k} + F$. When $F < \frac{\overline{a}\lambda_2 k(1+r)}{4k} - \frac{\overline{M}\lambda_2 k(1+r)(1-\phi) - \phi^2(\overline{M}^2+(\lambda_2 k(1+r))^2)}{4k(1-\phi)(1-2\phi)}$, we find $\pi_{DR-DM} < 0$; otherwise, when $F \ge \frac{\overline{a}\lambda_2 k(1+r)}{4k} - \frac{\overline{M}\lambda_2 k(1+r)(1-\phi) - \phi^2(\overline{M}^2+(\lambda_2 k(1+r))^2)}{4k(1-\phi)(1-2\phi)}$, we find $\pi_{DR-DM} \ge 0$. Therefore, when $\Delta a_{min1} < \Delta a \le -\frac{\lambda_2 k(1+r)}{1-\phi}$, π_{DR-DM} is increasing in Δa.

Therefore, when $\frac{-\lambda_2 k(1+r)}{1-\phi} \le \overline{a} \le \max\{\frac{-\lambda_2 k(1+r)}{1-\phi}, \widetilde{a}\}$, there is a unique $\Delta a_1^* < \Delta a_{min1}$ that, if $\Delta a < \Delta a_1^*$, $\pi_{DR-DM} > 0$, that reselling mode is better than marketplace mode for the manufacturer; otherwise, when $\overline{a} > \max\{\frac{-\lambda_2 k(1+r)}{1-\phi}, \widetilde{a}\}$, if $-\overline{a} \le \Delta a < \Delta a_{min1}$, $\pi_{DR-DM} < 0$, that marketplace mode is better than reselling mode for the manufacturer.

Case ii: $-\frac{\lambda_2 k(1+r)}{1-\phi} < \Delta a \le -\lambda_2 k(1+r)$. Similarly, $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = \frac{1}{4k}((1-2\phi)\overline{a} - \Delta a - \overline{M} - \lambda_2 k(1+r))$, $\frac{\partial^2 \pi_{DR-DM}}{\partial \Delta a^2} = -\frac{1}{4k} < 0$. Thus, this is a concave programming problem. When $\Delta a < \Delta a_{max1}$, π_{DR-DM} is increasing in Δa, where $\Delta a_{max1} = \overline{a}(1-2\phi) - \lambda_2 k(1+r) - \overline{M} > -\lambda_2 k(1+r)$. Thus, in this case, π_{DR-DM} is always increasing in Δa.

Case iii: $-\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r)$. Similarly, $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = \frac{1}{4k}((1-2\phi)\overline{a} - \overline{M})$,

$\frac{\partial^2 \pi_{DR-DM}}{\partial \Delta a^2} = 0$. Thus, the objective is a linear function and the constraint is larger than zero based on our assumptions. Therefore, in this case, π_{DR-DM} is always increasing in Δa.

Case iv: $\lambda_1 k(1+r) < \Delta a \leq \frac{\lambda_1 k(1+r)}{1-\phi}$. Similarly, $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = \frac{1}{4k}((1-2\phi)\overline{a} - \Delta a - \overline{M} + \lambda_1 k(1+r))$, $\frac{\partial^2 \pi_{DR-DM}}{\partial \Delta a^2} = -\frac{1}{4k} < 0$. Thus, this is a concave programming problem. When $\Delta a < \Delta a_{max2}$, π_{DR-DM} is increasing in Δa, where $\Delta a_{max2} = \overline{a}(1-2\phi) + \lambda_1 k(1+r) - \overline{M} > \lambda_1 k(1+r)$. Therefore, in this case, π_{DR-DM} is always increasing in Δa.

Case v: $\Delta a > \frac{\lambda_1 k(1+r)}{1-\phi}$. Similarly, $\frac{\partial \pi_{DR-DM}}{\partial \Delta a} = \frac{1}{4k}((1-2\phi)(\overline{a} + \Delta a) - \overline{M} - \lambda_1 k(1+r))$, $\frac{\partial^2 \pi_{DR-DM}}{\partial \Delta a^2} = \frac{1}{4k}(1-2\phi) > 0$. Thus, this is a convex programming problem. When $\Delta a > \Delta a_{min2}$, π_{DR-DM} is increasing in Δa, where $\Delta a_{min2} = -\overline{a} - \frac{\lambda_2 k(1+r)+\overline{M}}{1-2\phi} < \frac{\lambda_1 k(1+r)}{1-\phi}$. Thus, in this case, π_{DR-DM} is always increasing in Δa, and $\lim_{\Delta a \to +\infty} \pi_{DR-DM} = +\infty$.

To conclude when $\Delta a < \Delta a_{min1}$, π_{DR-DM} is always decreasing in Δa. Otherwise, when $\Delta a > \Delta a_{min1}$, π_{DR-DM} is always increasing in Δa. Thus, for the conditions that (i) $\pi_{DR-DM}(\Delta a_{min1}) < 0$; (ii) $\lim_{\Delta a \to +\infty} \pi_{DR-DM} > 0$; (iii) when $\frac{-\lambda_2 k(1+r)}{1-\phi} \leq \overline{a} \leq \max\{\frac{-\lambda_2 k(1+r)}{1-\phi}, \widetilde{a}\}$, $\pi_{DR-DM}(-\overline{a}) > 0$; otherwise, $\pi_{DR-DM}(-\overline{a}) \leq 0$. We can draw the conclusion in PROPOSITION 4. ■

PROOF OF PROPOSITION 5.

Without demand disruptions, $p^{SC-0*} = \frac{\overline{a}+\overline{M}}{2k}$.

(i) With reselling mode, $\frac{\partial \pi_p^{DR-0}(p)}{\partial p} = \overline{a} - 2kp + k\omega$. thus, $p^{DR-0} = \frac{\overline{a}+k\omega}{2k}$. After letting $p^{SC-0*} = p^{DR-0}$, we find $\omega = \frac{\overline{M}}{k}$. Thus, we find that when $r < r_0$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated, where $r_0 = \frac{e_0(a_0-b_0C)}{p_0-e_0(a_0-b_0C)}$.

(ii) With marketplace mode, $\frac{\partial \pi_p^{DM-0}(p)}{\partial p} = \overline{a}(1-\phi) + \overline{M} - 2pk(1-\phi)$. Because of $p^{SC-0*} \neq p^{DM-0}$, the manufacturer and the platform can not be coordinated. ■.

PROOF OF PROPOSITION 6.

With demand disruptions,

$$p^{SC*} = \begin{cases} \dfrac{\overline{a}+\Delta a+\overline{M}}{2k} - \dfrac{\lambda_2(1+r)}{2} & -\overline{a} \le \Delta a \le -\lambda_2 k(1+r) \\ \dfrac{\overline{a}+2\Delta a+\overline{M}}{2k} & -\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r) \\ \dfrac{\overline{a}+\Delta a+\overline{M}}{2k} + \dfrac{\lambda_1(1+r)}{2} & \Delta a > \lambda_1 k(1+r) \end{cases} \tag{3}$$

With reselling mode, $\frac{\partial \pi_p^{DR}(p)}{\partial p} = \overline{a} + \Delta a + k\omega - 2kp$. Thus, $p^{DR} = \frac{\overline{a}+\Delta a+k\omega}{2k}$. We discuss the coordination in three cases,

Case i: $-\overline{a} \le \Delta a \le -\lambda_2 k(1+r)$. After letting $p^{SC*} = p^{DR}$, we find that when $r < r_1$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated, where $r_1 = \frac{e_0(a_0-b_0C)-\lambda_2}{p_0-e_0(a_0-b_0C)+\lambda_2}$.

Case ii: $-\lambda_2 k(1+r) < \Delta a \le \lambda_1 k(1+r)$. After letting $p^{SC*} = p^{DR}$, we find that when $r < r_2$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated, where $r_2 = \frac{e_0 k(a_0-b_0C)+\Delta a}{k(p_0-e_0(a_0-b_0C))}$.

Case iii: $\Delta a > \lambda_1 k(1+r)$. After letting $p^{SC*} = p^{DR}$, we find that when $r < r_3$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated, where $r_3 = \frac{e_0(a_0-b_0C)+\lambda_1}{p_0-e_0(a_0-b_0C)-\lambda_1}$.

To conclude, when $r < r_1$, the manufacturer and the platform can be coordinated; when $r_1 \le r < r_2$, if $\Delta a > -\lambda_2 k(1+r)$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated; when $r_2 \le r < r_3$, if $\Delta a > \lambda_1 k(1+r)$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated. Therefore, based on the solutions of these three cases, we can get PROPOSITION 6. ■

PROOF OF PROPOSITION 7.

With marketplace mode with demand disruptions, we discuss the coordination in five cases,,

Case i: $-\overline{a} \le \Delta a \le -\frac{\lambda_2 k(1+r)}{1-\phi}$. There is not ϕ that satisfying $p^{DM*} = p^{SC*}$. Thus, the manufacturer and the platform can not be coordinated.

Case ii: $-\frac{\lambda_2 k(1+r)}{1-\phi} < \Delta a \le -\lambda_2 k(1+r)$. There is not exist ϕ that satisfying $p^{DM*} = p^{SC*}$.

Thus, the manufacturer and the platform can not be coordinated.

Case iii: $-\lambda_2 k(1+r) < \Delta a \leq 0$. After letting $p^{DM*} = p^{SC*}$, we find when $\phi = \frac{\Delta a}{\Delta a+\overline{\overline{M}}}$, the manufacturer and the platform can be coordinated. Thus, when $r < r_2$, the manufacturer and the platform can not be coordinated; otherwise, the manufacturer and the platform can be coordinated, where $r_2 = \frac{e_0 k(a_0-b_0C)+\Delta a}{k(p_0-e_0(a_0-b_0C))}$.

Case iv: $0 < \Delta a \leq \lambda_1 k(1+r)$. After letting $p^{DM*} = p^{SC*}$, we find when $\phi = \frac{\Delta a}{\Delta a+\overline{\overline{M}}}$, the manufacturer and the platform can be coordinated. Thus, when $r < r_2$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the platform can not be coordinated.

Case v: $\lambda_1 k(1+r) < \Delta a \leq \frac{\lambda_1 k(1+r)}{1-\phi}$. After letting $p^{DM*} = p^{SC*}$, we find when $\phi = \frac{\lambda_1 k(1+r)}{\lambda_1 k(1+r)+\overline{\overline{M}}}$, the manufacturer and the platform can be coordinated. Thus, when $r < r_3$, the manufacturer and the platform can be coordinated; otherwise, the manufacturer and the the manufacturer and the platform can not be coordinated, where $r_3 = \frac{e_0(a_0-b_0C)+\lambda_1}{p_0-e_0(a_0-b_0C)-\lambda_1}$.

Case vi: $\Delta a > \frac{\lambda_1 k(1+r)}{1-\phi}$. There is not ϕ that satisfying $p^{DM*} = p^{SC*}$. Thus, the manufacturer and the platform can not be coordinated.

Therefore, based on the solutions of these six cases, we can get PROPOSITION 7. ■