THE FORMATION OF LOW-MASS TRANSIENT X-RAY BINARIES

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ABSTRACT

We consider constraints on the formation of low-mass X-ray binaries containing neutron stars (NLMXBs) arising from the presence of soft X-ray transients among these systems. For a neutron star of mass $M_1 \simeq 1.4 M_\odot$ at formation, we show that in short-period ($\lesssim 1$ –2 day) systems driven by angular momentum loss these constraints require the secondary at the beginning of mass transfer to have a mass of $1.3 M_\odot \lesssim M_2 \lesssim 1.5 M_\odot$ and to be significantly nuclear evolved, provided that supernova (SN) kick velocities are generally small compared with the pre-SN orbital velocity. As a consequence, a comparatively large fraction of such systems appear as soft X-ray transients even at short periods, as observed. Moreover, the large initial secondary masses account for the rarity of NLMXBs at periods $P \lesssim 3$ hr. In contrast, NLMXB populations forming with large kick velocities would not have these properties, suggesting that the kick velocity is generally small compared with the pre-SN orbital velocity in a large fraction of systems, consistent with a recent reevaluation of pulsar proper motions. The results also place tight constraints on the strength of magnetic braking: if magnetic braking is significantly stronger than the standard form, too many unevolved NLMXBs would form; if it is slower by only a factor of $\simeq 4$, no short-period NLMXBs would form at all in the absence of a kick velocity. The narrow range for M_2 found for negligible kick velocity implies restricted ranges near $4 M_\odot$ for the helium star antecedent of the neutron star and near $18 M_\odot$ for the original main-sequence progenitor. The pre-common-envelope period must lie near 4 yr, and we estimate the short-period NLMXB formation rate in the disk of the Galaxy as $\sim 2 \times 10^{-7} \text{ yr}^{-1}$. Our results show that the neutron star mass at short-period NLMXB formation cannot be significantly larger than $1.4 M_\odot$. Systems with formation masses of $M_1 \lesssim 1.2 M_\odot$ would have disrupted, so observations implying $M_1 \sim 1.4 M_\odot$ in some NLMXBs suggest that much of the transferred mass is lost from these systems.

Subject headings: accretion, accretion disks — binaries: close — stars: evolution — stars: neutron — X-rays: stars

1. INTRODUCTION

In a recent paper (King, Kolb, & Burderi 1996, hereafter Paper I) we considered the disk instability model for soft X-ray transients (SXTs). Following van Paradijs (1996) we noted that X-ray irradiation of the disk surface tends to suppress the instability, so that SXT behavior requires that the mass transfer rate is below a critical limit, which is itself a rising function of orbital period P . For $P \gtrsim 1$ –2 days, mass transfer is driven by the nuclear expansion of an evolved secondary and proceeds at low enough rates that almost all these systems appear as SXTs. For $P \lesssim 1$ –2 days, mass transfer is driven by angular momentum losses; in Paper I we showed that the resulting rates are higher than the critical rate, hence too high for SXT behavior unless the secondary star is already somewhat nuclear evolved before mass transfer begins. This effect is particularly marked in a neutron star low-mass X-ray binary (neutron star LMXB, or NLMXB): if we write (see Paper I) $\hat{m}_2$ for the ratio of the secondary's mass to that of a zero-age main-sequence (ZAMS) star filling the Roche lobe at the same orbital period, SXT behavior requires $\hat{m}_2 \lesssim 0.25$. By contrast, we found a much weaker limit $\hat{m}_2 \lesssim 0.75$ for SXT behavior in an LMXB with a $10 M_\odot$ black hole primary. These limits agree with the extreme mass ratios always found in SXTs. In Paper I we suggested that these limits also offered a potential explanation for the prevalence of black hole systems among SXTs (eight out of 14 with known P) as

compared with persistent LMXBs (one out of 29 with known P). As NLMXBs can be reliably identified by the presence of X-ray bursts, we consider this problem further by investigating constraints on NLMXB formation.

2. NLMXB FORMATION

The requirement that short-period SXT secondaries should be significantly evolved is a powerful clue in investigating LMXB (and particularly NLMXB) formation. For example, the very fact that at least four out of 22 NLMXBs in the relevant period range are transients (0748-676, Cen X-4, 1658-298, Aql X-1; see van Paradijs 1995) must imply that NLMXB secondaries usually have masses $M_2 \gtrsim 1 M_\odot$ at the onset of mass transfer. Lower initial mass secondaries would be essentially unevolved when mass transfer began, and a large population of them would require the four known SXTs among NLMXBs to be accompanied by a far greater number of persistent NLMXBs than observed. However, to get any NLMXBs at all at $P \lesssim 1$ –2 days simultaneously requires that some of the secondaries initially have masses $M_2 \lesssim 1.5 M_\odot$, so that angular momentum loss via magnetic braking can shrink the binary. In addition, for $M_2 \gtrsim 1.5 M_\odot$, these NLMXBs with a main-sequence donor will be unstable to thermal-timescale mass transfer once contact is reached; they transfer mass at super-Eddington rates and do not appear as NLMXBs (Kalogera & Webbink 1996a). Given these restrictions, all NLMXBs with $P \lesssim 1$ –2 days (and indeed most at longer periods too) must apparently form with M_2 in a very restricted range of $1 M_\odot \lesssim M_2 \lesssim 1.5 M_\odot$.

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The properties and the very existence of this group depend sensitively on the efficiency of magnetic braking. The paper by Iben, Tutukov, & Yungelson (1995) on general LMXB formation effectively rules out these systems because it assumes that magnetic braking does not operate for $M_2 > 1 M_\odot$. Similarly, Terman, Taam, & Savage (1996) do not consider them in detail as they somewhat artificially assume that $M_2 = 1.0 M_\odot$ separates systems captured by angular momentum losses from systems evolving toward longer orbital periods.

Below we investigate in detail constraints on NLMXBs forming after a common-envelope (CE) phase and a subsequent helium star supernova (SN). We consider first the fundamental case of a spherically symmetrical SN (§ 2.1), and we shall show that in this case secondary stars in NLMXBs are indeed significantly nuclear evolved at the onset of mass transfer. A generalization to the more realistic case when the neutron star receives a kick velocity during the SN (§ 2.2) reveals that the fraction of systems with unevolved secondaries increases with increasing magnitude of the kick. For a given magnetic braking strength, this places limits on the magnitude of the kick velocity. Possible alternative evolutionary channels leading to NLMXB formation are discussed in § 2.3.

2.1. NLMXBs from Spherically Symmetrical Helium Star Supernovae

To discover any limits on the initial mass M_2 of the secondary, we consider the constraints on the formation of LMXBs from helium star + main-sequence (MS) binary remnants of common-envelope evolution in the case of a spherically symmetrical SN.

2.1.1. Constraints on the Formation

Webbink & Kalogera (1994, hereafter WK) and Kalogera & Webbink (1996a) list a number of requirements that the various progenitor stages must satisfy in order to lead to LMXB formation. Of these, three turn out to be crucial in fixing M_2 :

1. The post-common-envelope binary must be wide enough to allow the helium star to evolve to core collapse (requirement 6 of WK).
2. The binary must survive the supernova event resulting from the core collapse of the helium star (requirement 7 of WK).
3. After the helium star progenitor of the LMXB primary explodes as a supernova, the binary must be able to reach interaction within the age of the Galaxy (requirement 8 of WK).

If M_2 is large enough and the post-SN separation is wide enough, requirement 3 will occur through the nuclear expansion of the secondary. Such systems require $M_2 \gtrsim 1 M_\odot$ to give main-sequence lifetimes t_{MS} less than the age t_{Gal} of the Galaxy and must have $P \gtrsim 1$ –2 days at the onset of mass transfer; P will increase further as the secondary expands. However, if M_2 is less than this and/or the post-SN binary is narrower, requirement 3 can come about through orbital angular momentum loss. After reaching contact with an initial orbital period of order 1–2 days, such systems evolve to either longer ($P \gtrsim 1$ day) or shorter ($P \lesssim 1$ day) periods depending on the competition between nuclear evolution and angular momentum loss (Pylyser & Savonije 1988a, 1988b).

For the remainder of this paper we concentrate on this latter group of systems, which are the only ones that can populate the short-period ($P \lesssim 1$ –2 day) region. It is convenient to apply constraints 1–3 in a different order. From constraint 3 we require $t_{\text{MB}}, t_{\text{GR}} < t_{\text{Gal}}$, where

$$t_{\text{MB}} = 2.2 \times 10^8 \frac{\gamma m_1}{(m_1 + m_2)^{1/3} m_2^4} P_d^{10/3} \left[1 - \left(\frac{P_c}{P} \right)^{10/3} \right] \text{yr}, \quad (1)$$

$$t_{\text{GR}} = 4.7 \times 10^{10} \frac{(m_1 + m_2)^{1/3}}{m_1 m_2} P_d^{8/3} \left[1 - \left(\frac{P_c}{P} \right)^{8/3} \right] \text{yr}, \quad (2)$$

are the timescales for detached orbital evolution under magnetic braking and gravitational radiation, with γ as a dimensionless efficiency parameter for the former case. These shrink the post-SN binary from its initial period P to the value P_c at which it first comes into contact with its Roche lobe ($P_d = P/d$, $m_1 = M_1/M_\odot$, $m_2 = M_2/M_\odot$, etc.). If the secondary is unevolved, an adequate approximation for P_c is $P_c \simeq 0.375 m_2$ days, while P_c is up to a factor of $\simeq 2.5$ larger if the secondary is fairly massive ($M_2 \gtrsim 1 M_\odot$) and close to the terminal main sequence.

We assume that the secondary has a structure such that a magnetic stellar wind brakes its rotation; since we are considering short-period LMXBs in which the secondaries are not too far from the main sequence, this requires $0.3 M_\odot \lesssim M_2 \lesssim 1.5 M_\odot$. Furthermore, we assume that the secondary is tidally locked to the orbit such that magnetic braking removes orbital (rather than only rotational) angular momentum. Applying simple scalings for the tidal synchronization timescale (e.g., Tassoul 1995) suggests that this is the case for the detached systems in the period range under consideration (see below). The efficiency parameter γ in equation (1) allows us to test the sensitivity of our results to the strength of magnetic braking. The standard case $\gamma = 1$ corresponds to the description by Verbunt & Zwaan (1981) when the radius of gyration is set to $(0.2)^{1/2}$ and the calibration parameter to unity.

Requirement 3 thus gives the longest post-SN period compatible with short-period LMXB formation. It is easy to show that the longest possible value of this period is given by equating t_{MB} to t_{Gal} with $P \gg P_c$, so that the term in square brackets in equation (1) is unity. Kepler's law now gives a condition on the separation a_{postSN} of the tidally circularized post-SN binary, i.e.,

$$a_{\text{postSN}} < 9.0 R_\odot \left[\frac{t_G (m_1 + m_2)^2 m_2^4}{\gamma m_1} \right]^{1/5}, \quad (3)$$

where $t_G = t_{\text{Gal}}/10^{10}$ yr.

Before exploiting equation (3) further, we apply requirement 2. In a spherically symmetrical supernova explosion, it is well known that the remnant binary is disrupted if more than one-half of the binary mass is removed; i.e., unless

$$M_{\text{He}} - M_1 < \frac{1}{2}(M_{\text{He}} + M_2) \quad (4)$$

or

$$m_2 > m_{\text{He}} - 2m_1, \quad (5)$$

the binary will be disrupted. Here m_{He} is the helium star mass M_{He} in $M_\odot$. Moreover, the eccentricity e of the orbit

immediately after the SN is given by

$$e = \frac{m_{\text{He}} - m_1}{m_1 + m_2}. \quad (6)$$

Obviously equation (5) is equivalent to $e < 1$. The pre-SN separation and the post-SN separation of the tidally circularized orbit are related by (e.g., Verbunt 1993)

$$a_{\text{preSN}} = \frac{1}{1+e} a_{\text{postSN}}. \quad (7)$$

Using equations (3) and (7) we find that the pre-SN Roche lobe radius R_R of the He star must obey

$$R_R \simeq \frac{a_{\text{preSN}}}{2} < 4.5 R_\odot \frac{1}{1+e} \left[\frac{t_G (m_1 + m_2)^2 m_2^4}{\gamma m_1} \right]^{1/5}, \quad (8)$$

where we have assumed that R_R occupies one-half of the pre-SN binary separation.

Finally, requirement 1 demands that the largest radius $R_{\text{max}}(\text{He})$ of the helium star must fit inside the pre-SN binary. The evolution of the helium star is very rapid compared with the orbital evolution, so we assume that a_{preSN} is constant and the condition is $R_{\text{max}}(\text{He}) < R_R$. (If this condition fails, it is possible that some LMXBs nevertheless form after a second common-envelope phase; see Iben et al. 1995. We note, however, that these authors assume an unusually efficient envelope ejection compared with the standard common-envelope description, eq. [21] below. Application of the latter would almost always lead to a merger. Hence, we neglect this group of systems.)

Iben et al. (1995) fit $R_{\text{max}}(\text{He})$ as

$$R_{\text{max}} = 7700 R_\odot m_{\text{He}}^{-5.5} \quad (9)$$

for $m_{\text{He}} > 2.5 M_\odot$ (see also Habets 1986).

Hence, combining equation (9) with equation (8) we see that requirements 1 and 3 demand

$$(m_1 + m_2)^2 m_2^4 > \frac{\gamma}{t_G} (1+e)^5 \left(\frac{3.87}{m_{\text{He}}} \right)^{55/2} m_1, \quad (10)$$

thereby defining a lower limit for m_2 for given m_1, m_{He} (or a lower limit for m_{He} for given m_1, m_2).

2.1.2. Mass Limits for NLMXBs

Observational evidence strongly suggests that neutron stars emerge from the supernova that creates them with a mass $m_1 = 1.4$. Figure 1 shows the constraints (eqs. [5], [10]) for this mass, with $t_G = 1$ and $\gamma = 1$. (For $m_2 < 0.3$, where magnetic braking is thought not to operate, eq. [10] was replaced by the corresponding constraint using t_{GR} instead of t_{MB} .) As can be seen, the two constraints cross at a value $m_2 = 1.22$, $m_{\text{He}} = 4.02$. Hence, $m_2 \geq 1.22$ for NLMXB formation through this channel. We now go one step further and ask under what conditions the binary can begin mass transfer with a secondary that has spent less than a fraction f ($0 < f \leq 1$) of its lifetime t_{MS} on the main sequence, i.e., with

$$t_{\text{MB}} < f t_{\text{MS}}. \quad (11)$$

Clearly this is a more stringent requirement than requirement 3, where the right-hand side is $t_{\text{Gal}} > t_{\text{MS}}$, and it therefore requires a larger mass m_2 . Taking

$$t_{\text{MS}} = 10^{10} m_2^{-3} \text{ yr} \quad (12)$$

FIG. 1.—Constraints on NLMXB formation in the donor mass (M_2)-He star mass (M_{He})-plane in the absence of kick velocities. The straight line with positive slope represents the SN survival condition (eq. [5]) and is a lower limit for M_2 . The curve with negative slope represents another lower limit for M_2 (or, equivalently, M_{He}), the Galactic age constraint (eq. [10]) (for $M_2 > 0.3 M_\odot$; as magnetic braking does not operate in systems with fully convective secondaries, we plot for $M_2 < 0.3 M_\odot$, the corresponding limit obtained by replacing t_{MB} with t_{GR}). The horizontal line represents an upper (stability) limit for M_2 (where $1.4 M_\odot$ was assumed for the neutron star mass). These limits restrict the allowed range of NLMXB formation to the small triangular (shaded) area enclosed by these curves. Also shown is the lower limit for M_2 (or M_{He}) if the system's age is less than t_{MS} (dotted curve; $f = 1$ in eq. [13]), or less than $0.25 t_{\text{MS}}$ (dash-dotted curve; $f = 0.25$) at mass transfer turn-on. The allowed range for short-period NLMXBs with main-sequence donors is only the narrow-hatched area.

and following the same line of reasoning that led to the constraint (eq. [10]), i.e., replacing t_G in equation (10) by (f/m_2^3) , we find

$$(m_1 + m_2)^2 m_2 > \frac{\gamma}{f} (1+e)^5 \left(\frac{3.87}{m_{\text{He}}} \right)^{55/2} m_1, \quad (13)$$

which must hold simultaneously with equation (10). Condition (13) is best understood as a lower limit for the He star mass for any given m_1, m_2 . In Figure 1 we show the constraint for $f = 1$ (dotted curve) and $f = 0.25$ (dash-dotted curve), with standard magnetic braking ($\gamma = 1$). Instead of the rough approximation (eq. [12]), we used the fitting formulae given by Kalogera & Webbink (1996a) to determine t_{MS} . We see that condition (13) with $f = 1$ intersects the allowed mass range $1.2 \lesssim m_2 \lesssim 1.5$ set by the Galactic age constraint and the requirement of thermally stable mass transfer, respectively. Systems with $1.2 \lesssim m_2 \lesssim 1.5$ to the left of the dotted line in Figure 1 (wide-hatched area) turn on mass transfer with a giant donor at a long orbital period ($P \gtrsim 1$ –2 days). As in most of them, mass transfer is unstable (e.g., Kalogera & Webbink 1996a) and they would probably not appear as long-period NLMXBs; we do not consider this group further. Systems with $1.3 \lesssim m_2 \lesssim 1.5$ to the right of this line (narrow-hatched area) will evolve to short orbital periods. The location of the dash-dotted line in Figure 1 clearly demonstrates that all secondaries of systems in this group are somewhat evolved.

In other words, in the limit of negligible kick velocities, secondaries in a large fraction of NLMXBs with $P \lesssim 1$ –2 days are close to the end of their MS lifetimes and are partially nuclear evolved ($\dot{m}_2 < 1$) when mass transfer starts. The resulting low mass transfer rates (Pylyser & Savonije 1988b) then probably explain why a comparatively

large fraction of NLMXBs of both types appear as transients. The simple scalings of Paper I suggest that SXT behavior requires $\hat{m}_2 \lesssim 0.25$; full population synthesis calculations are needed in order to quantify the resulting SXT fraction.

We note that Kalogera & Webbink (1996b) find that short-period LMXBs do not form at all without kick velocities. The reason for this difference from our conclusions is a modification of the standard form (eq. [1]) of magnetic braking such that t_{MB} varies as m_2^{-2} (rather than m_2^{-4}) and the introduction of a reduction factor for secondaries more massive than $1 M_\odot$. Even if the reduction factor is ignored (which is $\simeq 7$ for $m_2 = 1.5$), their t_{MB} is still longer by a factor of $\simeq 6$ for $m_2 = 1.5$. This corresponds to $\gamma = 6$ in equation (13) or, equivalently, to $f = \frac{1}{6}$. Inspection of Figure 1 shows that the critical line corresponding to $f = \frac{1}{6}$ would be to the right of the dash-dotted $f = \frac{1}{4}$ line; hence, no parameter space would be left for short-period NLMXB formation. Physically this means that because of the correspondingly slower evolution the systems would not reach contact within the age of the Galaxy.

This underlines the sensitivity of our result to the strength of magnetic braking for main-sequence stars in the critical mass range where the convective envelope disappears. To illustrate this further, we consider the lower limit a_{crit} we obtain for the post-SN orbital distance, $a_{\text{postSN}} \simeq 2(1+e)R_R$, when the requirement $R_R > R_{\text{max}}(\text{He})$ is combined with equations (9) and (5):

$$a_{\text{postSN}} > a_{\text{crit}} = 5.1 R_\odot (1+e) \left(\frac{4.3}{m_2 + 2m_1} \right)^{5.5}. \quad (14)$$

Note that this limit does not depend on magnetic braking and that a_{crit} is essentially determined by m_2 ($m_1 = 1.4$, $0 \leq e \leq 1$). For NLMXBs with main-sequence donors of mass m_2 to exist at all, the detached evolution time t_{MB} needed to shrink the orbit from a_{crit} into contact must be shorter than t_{MS} . In contrast, to ensure that a large fraction of the secondaries are significantly evolved when mass transfer turns on, t_{MB} must be longer than a certain minimum fraction $f \simeq \frac{1}{4}$ of t_{MS} . This places a tight limit on the allowed strength of magnetic braking for systems with secondary mass m_2 , initial orbital distance a_{crit} , and neutron star mass $m_1 = 1.4$:

$$0.25 \lesssim \frac{t_{\text{MB}}(a_{\text{crit}}, M_2)}{t_{\text{MS}}} \leq 1, \quad (15)$$

independent of the functional dependence of t_{MB} on mass and period.

2.1.3. Mass Limits for Progenitor Systems

The narrow range

$$1.3 \lesssim m_2 \lesssim 1.5 \quad (16)$$

also implies a very restricted range for m_{He} : from Figure 1 we see that this must obey

$$4.0 \lesssim m_{\text{He}} \lesssim 4.3. \quad (17)$$

Using the analytic fit

$$\log m_{\text{He}} = 1.273 \log m_{\text{prog}} - 0.979 \quad (18)$$

from Kalogera & Webbink (1996b) to stellar models by Schaller et al. (1992) we see that the mass of the MS progenitor of the helium (and ultimately neutron) star must be

in the range

$$17.5 \lesssim m_{\text{prog}} \lesssim 18.5. \quad (19)$$

Immediately before the CE phase, the primary has evolved beyond core He exhaustion and has lost mass in the form of stellar winds. Interpolation of Schaller et al.'s models suggests that primaries from the mass range (eq. [19]) will then have masses m_{preCE} in the range

$$15.2 \lesssim m_{\text{preCE}} \lesssim 15.9. \quad (20)$$

Wind mass loss carrying the specific angular momentum of the mass-losing component increases the orbital separation as the inverse of the total mass. Hence, the MS parent binary orbit is closer by a factor of $\simeq 0.9$ than the immediate pre-CE binary orbit.

Simple treatments of common-envelope evolution (e.g., WK) lead to the relation

$$a_{\text{preCE}} \simeq \frac{2m_{\text{preCE}}(m_{\text{preCE}} - m_{\text{He}} + \alpha \lambda r_R m_2/2)}{\alpha \lambda r_R m_{\text{He}} m_2} a_{\text{postCE}}, \quad (21)$$

where a_{preCE} , a_{postCE} are the binary separation before and after the common-envelope phase, α is the fraction of the orbital binding energy used to drive off the envelope, λ is a weighting factor for the gravitational binding energy of the envelope to the core, and $r_R = r_R(m_{\text{preCE}}, m_2)$ is the fractional size of the primary's Roche lobe at the start of the CE phase. We find with the mass limits (eqs. [16], [17], [20]) and $r_R \simeq \frac{1}{2}$, $\lambda \simeq \frac{1}{2}$

$$a_{\text{preCE}} \simeq \frac{8}{\alpha} \frac{m_{\text{preCE}}}{m_2} \frac{m_{\text{preCE}} - m_{\text{He}}}{m_{\text{He}}} \simeq \frac{250}{\alpha} a_{\text{postCE}}. \quad (22)$$

Since there is little orbital evolution before the helium star undergoes its supernova explosion, and the post-SN separation is within a factor of 2 of the pre-SN separation, we can translate this into limits on a_{preCE} or, equivalently, the orbital period of the progenitor binary. We get an upper limit from requirement 3 (see eq. [8]), and a lower limit from requirement 1, using equation (9) and the fact that $m_{\text{He}} < 4.3$. (This limit prevents NLMXB progenitors emerging from the common-envelope phase with the MS secondary already close to filling its Roche lobe, which is WK's condition 6; the requirement here gives a tighter limit. This in turn justifies our assumption above that the post-SN-period P was much larger than the contact period P_c .) The resulting constraints are close, i.e., $\sim 2300 R_\odot/\alpha$ and $\sim 1300 R_\odot/\alpha$, respectively. Together they show that the orbital distance a_{prog} and period P_{prog} of the MS progenitor $\sim 18 M_\odot + 1.4 M_\odot$ binary must lie in the range

$$1200 R_\odot \lesssim a_{\text{prog}}/\alpha \lesssim 2100 R_\odot \quad (23)$$

and

$$3 \text{ yr} \lesssim \alpha^{3/2} P_{\text{prog}} \lesssim 7 \text{ yr}. \quad (24)$$

The formation channel we have considered requires that the progenitor fills its Roche lobe after core-helium exhaustion but before it would explode as an SN. For an $18 M_\odot$ MS star this implies $1200 \lesssim a_{\text{prog}}/R_\odot \lesssim 1600$ or $3 \lesssim P_{\text{prog}} \text{ yr}^{-1} \lesssim 4.6$ (Kalogera & Webbink 1996b), thus reducing the limits (eqs. [23], [24]) even further. If most short-period NLMXBs in the Galaxy form via helium-star supernovae, this restricted range of allowed initial orbits and the very narrow limits (eqs. [16], [19]) then show that only an

extremely small progenitor population can produce these systems, explaining their comparative rarity. Following Iben et al. (1995), a very crude estimate of the Galactic short-period NLMXB formation rate ν can be obtained from

$$\nu = 0.2\Delta \log a_{\text{prog}} \frac{\Delta m_{\text{prog}}}{m_{\text{prog}}^{2.5}} \frac{\Delta m_2}{m_{\text{prog}}} \text{yr}^{-1}, \quad (25)$$

where Δx denotes the allowed range of the quantity x . Equation (25) relies on standard assumptions for the distribution of initial orbital parameters in newly forming ZAMS binaries, in particular, on a flat distribution in $\log M_2/M_{\text{prog}}$. Using the limits obtained above we find $\nu \simeq 2 \times 10^{-7} \text{yr}^{-1}$. Replacing equation (25) by the initial parameter distribution of ZAMS binaries used in Kalogera & Webbink (1996b) gives a similar value for ν .

2.2. The Role of Kick Velocities

In reality, supernova explosions are probably not spherically symmetrical and give the compact primary remnant a kick velocity. In this case, the main differences from the considerations in § 2.1 are twofold (see, e.g., Kalogera 1996a). First, a suitably directed kick can unbind binaries that would have been stable according to equation (5) or keep together binaries that would have disrupted according to this inequality. The latter case requires a kick velocity comparable to the pre-SN orbital velocity of the companion, directed almost parallel to its instantaneous motion, and is therefore rather rare unless the kick velocity is close to an optimum value near the pre-SN orbital velocity. Second, relation (7) no longer holds, and a_{postSN} can be either smaller or larger than a_{preSN} (but never larger than $2a_{\text{preSN}}$). These two effects not only widen the allowed region of NLMXB formation in the $M_2 - M_{\text{He}}$ plane of Figure 1 but, in particular, allow the formation of NLMXBs with almost unevolved secondaries.

In the absence of a theoretical understanding of the origin, magnitude, and direction of kick velocities, the standard approach in population synthesis considerations (e.g., Brandt & Podsiadlowski 1995; Terman et al. 1996; Kalogera & Webbink 1996b) is to assume that the kicks are isotropic and that their magnitude derives from a given distribution function, characterized by a certain rms value σ_k . As demonstrated extensively by, e.g., Kalogera (1996a), the resulting probability distributions can be expressed in terms of the two governing dimensionless parameters, the ratio ξ of kick velocity to relative orbital velocity in the pre-SN orbit, $\xi = \sigma_k/v_{\text{orbit}}$, and the ratio β of post-SN and pre-SN binary mass, $\beta = (m_1 + m_2)/(m_{\text{He}} + m_2)$.

Given the stochastic nature of the problem, only a full population synthesis can provide a quantitative estimate of the fraction of NLMXBs with significantly nuclear evolved secondaries at turn-on of mass transfer. To illustrate the main effect of kick velocities and to gain a very rough estimate of the maximum kicks we can tolerate and still maintain the large fraction of systems with evolved secondaries found for spherically symmetrical SNs, we make use of the analytic expression for the distribution of binaries over $\alpha_c = a_{\text{postSN}}/a_{\text{preSN}}$ derived by Kalogera (1996a) under the assumption of a Maxwellian kick velocity distribution. In Figure 2 we show the same limits as plotted in Figure 1, but with the factor $(1 + e)$ in equations (10) and (13) replaced by the approximate median value of α_c , 1.75, 1.25, and 0.75, for

FIG. 2.—Same as Fig. 1, but for weak ($\xi = 0.1$, top panel), moderate ($\xi = 0.3$, middle panel), and strong ($\xi = 1.0$, bottom panel) kick velocities. For the $\xi = 1$ case, the SN survival line was replaced by a line with $\beta = \text{const.} = 0.4$ (see § 2.2).

small ($\xi = 0.1$), moderate ($\xi = 0.3$), and large kick velocities ($\xi = 1.0$), respectively. These median values depend only weakly on β . For strong kicks, the critical line (eq. [5])—which is equivalent to $\beta = 0.5$ —was replaced by the line $\beta = 0.4$, as the survival probability of these systems is only a factor of 2 smaller than the one for the most stably bound systems (corresponding to $\beta \simeq 0.75$). The resulting enclosed area in the $M_2 - M_{\text{He}}$ plane can be thought of as representative for the effective parameter space of NLMXB formation.

Assuming that the area is a measure of the corresponding relative formation rate, Figure 2 suggests that in the case of standard magnetic braking (eq. [1]) systems with unevolved secondaries still constitute only a small fraction of NLMXBs for $\xi = 0.1$, represent the majority for $\xi = 0.3$, and entirely dominate for $\xi = 1.0$.

More efficient magnetic braking would increase the dominance of unevolved systems even further, whereas a favorable combination of weaker magnetic braking and a kick velocity distribution with small (or moderate) mean velocity could ensure both the formation of the (otherwise forbidden) class of short-period NLMXBs and the predominance of nuclear-evolved main-sequence donors. However, a population subject to a large mean kick velocity would necessarily contain a large fraction of systems where the secondary is close to contact in the post-SN orbit and hence essentially unevolved at mass transfer turn-on—whatever the strength of magnetic braking. The reason for this is twofold. First, the limit (eq. [14]) formally allows post-SN orbital periods shorter than the contact period P_c for $m_{\text{He}} \gtrsim 4.7$, and with strong kicks a large fraction of such

binaries would survive the SN. Second, the SN-induced orbital reduction factor α_c is very small for the majority of systems.

In view of this, the large fraction of soft X-ray transients observed among NLMXBs provides a strong argument not only against a magnetic braking stronger than our standard case (eq. [1]) but also against a mean kick velocity of order $350\text{--}400\text{ km s}^{-1}$ ($\xi \simeq 1$) invoked by Lyne & Lorimer (1994) from observed pulsar proper motions. Figure 2 shows that kick velocities must on average be small compared with the pre-SN orbital velocity, probably $\xi \lesssim 0.1$. This is consistent with a more recent reevaluation of the initial velocity distribution of radio pulsars that confirms the existence of the high-velocity tail found by Lyne & Lorimer but suggests that the distribution has its maximum at zero velocity, hence a smaller average value (Hansen & Phinney 1996; Hartmann 1996).

2.3. Alternative Evolutionary Channels

Short-period NLMXBs might also form from systems with initially fairly massive main-sequence secondaries above the limit for thermally stable mass transfer, $1.5\text{--}2 \lesssim m_2 \lesssim 3$, provided these survive the initial phase of thermal timescale (hence super-Eddington) mass transfer. (Systems with $m_2 \gtrsim 3$ would probably develop a delayed dynamical instability; see Hjellming 1989.) Kalogera & Webbink (1996a, 1996b) pointed out that such systems could reappear as stable NLMXBs with a sub-Eddington mass transfer rate and donor mass $1 \lesssim m_2 \lesssim 1.5$. An analysis similar to the one in § 2.1.2 (using t_{GR} instead of t_{MB}) reveals that a significant fraction of them would emerge from the CE phase almost semidetached. They would turn on mass transfer with an unevolved secondary, again in conflict with the observed comparatively large fraction of transients among NLMXBs. Hence we conclude that this channel cannot contribute significantly to the formation of short-period NLMXBs.

Recently, Kalogera (1996b) has described yet another formation mechanism for LMXBs where no CE phase is involved. Instead, the orbital shrinkage is achieved by a suitably directed kick when the primary star in the wide progenitor binary explodes as an SN before it reaches its Roche lobe. Population synthesis models show (Kalogera 1996b) that the production of short-period LMXBs via this channel is altogether negligible compared with the standard He-star SN case if the kick velocities are large (σ_k of order $300\text{--}400\text{ km s}^{-1}$) but might account for a formation rate comparable to the one derived in § 2.1.2 if σ_k is close to an optimum value $\simeq 50\text{ km s}^{-1}$. However, in the latter case a relatively large fraction of LMXBs would start the X-ray phase with a very small (below $0.3 M_\odot$) donor mass, hence with essentially unevolved secondaries. Again, this suggests that the direct SN mechanism represents only a minor channel for the formation of short-period NLMXBs (whereas it might be important to produce long-period systems with $P_d \gtrsim 100$ days) even if kick velocities are generally small.

3. NLMXBs BELOW THE PERIOD GAP

The difference between the period histograms of LMXBs and cataclysmic variables (CVs), in which the primary is a white dwarf, has long been remarked (e.g., White & Mason 1985; van den Heuvel & van Paradijs 1988; Verbunt & van den Heuvel 1995; Kolb 1996). The most prominent differ-

ence is a total (or near total) lack of LMXBs in the $80 < P < 120$ minute region below the famous CV period gap. A K-S test reveals that there are also statistically significant differences above the gap. The hypothesis that the LMXB and CV samples are drawn from the same underlying distribution can be rejected at a confidence level greater than 99.99% in the period range 80 minutes to 2 days (Fig. 3, *top panel*) and at $\simeq 99.96\%$ in the range 3 hr to 2 days (Fig. 3, *bottom panel*). This cannot be explained by selection effects discriminating against short orbital periods, since many of the X-ray periods were turned up by satellites such as *EXOSAT*, which had a 4 day orbit. The differences conflict with the simple picture of CVs and LMXBs as essentially the same in terms of their secular evolution, apart from the substitution of a white dwarf by a neutron star or black hole primary. However, our arguments above show that this simple picture is inaccurate, particularly for NLMXBs. CVs emerge from common-envelope evolution with a full range of secondary masses down to $m_2 \sim 0.1$. The vast majority of these stars are essentially unevolved, and the post-CE separations are so small in many cases that the secondaries are close to their Roche lobes. The majority of CVs probably start mass transfer at periods below the gap (more than 67% according to King et al. 1994). None of these features hold for NLMXBs, as we have seen: the secondaries are confined to the narrow range $1.3 \lesssim m_2 \lesssim 1.5$ initially, a large fraction of them must be significantly nuclear evolved, the post CE (and SN) Roche lobes are considerably larger than the secondaries, and they start mass transfer at periods in the range $10\text{ hr} \lesssim P \lesssim 30\text{ hr}$.

We investigate differences between the CV and NLMXB period histogram that arise alone from these effects in Figure 4. In particular, we test if the population of NLMXBs below $P \sim 2$ hr is much smaller than for CVs, as the lack of the enormous influx of newly formed systems boosting the CV distribution there would suggest. Despite this lack, the expected intrinsic period distribution (Fig. 4, *middle panel*) derived from a typical evolutionary sequence (Fig. 4, *top panel*) still predominantly populates the short-period range, simply because the period evolves more

FIG. 3.—Normalized cumulative period distribution for CVs and LMXBs in selected period ranges, with data taken from Ritter & Kolb (1997). *Top panel*: Systems with orbital periods between 80 minutes and 2 days (249 CVs, 30 LMXBs). *Bottom panel*: Systems with orbital periods between $\simeq 3$ hr and 2 days (142 CVs, 27 LMXBs). The arrows indicate the period where the difference between the distributions is largest.

FIG. 4.—Possible shape of the NLMXB period distribution if all systems form with a secondary that is somewhat nuclear evolved and has a rather high mass. As a representative evolutionary sequence, we use a calculation by Singer (Singer et al. 1993; Ritter 1994) with an initial donor mass $M_2 = 1.2 M_\odot$, initial central hydrogen mass fraction 0.36 (i.e., with age $= 0.5 t_{\text{MS}}$ at turn-on), and a (constant) mass $M_1 = 1.0 M_\odot$ for the compact object. The mass transfer rate $\dot{M}$ as a function of orbital period P for this sequence is shown in the top panel. The sequence was originally used to represent CV evolution; a slightly different initial primary mass ($1.4 M_\odot$ instead of $1.0 M_\odot$) does not affect the main conclusions drawn in the text. The middle panel depicts the intrinsic discovery probability $\propto P/\dot{P}$ vs. $\log P$, a quantity representing the intrinsic period distribution of a population of such binaries with similar initial configurations at mass transfer turn-on. In the bottom panel we plot the corresponding discovery probability obtained by multiplying by a visibility factor $\propto \dot{M}$ to account for observational selection.

slowly there (number density $\propto \dot{P}^{-1}$). However, these short-period systems are suppressed for any visibility function $\propto (-\dot{M}_2)^\alpha$ with $\alpha \gtrsim 1$ (Fig. 4, *bottom panel*). Such a visibility function corresponds, for example, to a flux-limited sample taken from a disklike population. In addition, the detection probability function obtained in this way shows a pronounced peak at long orbital periods (here $P \simeq 8$ hr), a feature consistent with the observed LMXB overpopulation at long orbital periods compared with CVs (see Fig. 3, *bottom panel*). This peak is a consequence of the high mass transfer rate immediately after contact is reached in NLMXBs, since the systems are close to instability ($M_2 \sim M_1$). The complexity of quantifying the relevant selection effects makes it difficult to decide if $\alpha \gtrsim 1$ properly describes the NLMXB population. However, only if indeed $\alpha < 1$ is an additional mechanism needed to account for the lack of short-period systems. One such mechanism is the evaporation of the secondary star by pulsar irradiation from a rapidly rotating neutron star spun-up by accretion (van den Heuvel & van Paradijs 1988). Remarkably, this implicitly assumes that all LMXBs cross the gap, i.e., assumes the result we have demonstrated above.

4. CONCLUSIONS

We have shown that short-period neutron star low-mass X-ray binaries forming from helium-star supernovae without kick velocities must have secondaries with masses in the narrow range $1.3 M_\odot \lesssim M_2 \lesssim 1.5 M_\odot$.

For standard strength magnetic braking, the secondary star is already significantly nuclear evolved when mass transfer begins, explaining why the resulting mass transfer rates are in many cases low enough for a substantial fraction of these systems to appear as soft X-ray transients even at short ($P \lesssim 1$ –2 days) orbital periods (see Paper I). In contrast, if the neutron star receives a strong kick velocity at birth, many NLMXBs would form with unevolved low-mass donors.

The observed large fraction of SXTs among NLMXBs then forces us to conclude that kick velocities must be small compared to the pre-SN orbital velocity ($\lesssim 10\%$, i.e., $\lesssim 50$ km s $^{-1}$) for a large fraction of progenitor systems. This is consistent with a recent reevaluation of observed pulsar proper motions that suggest that the distribution of neutron star velocities at birth has a maximum at zero.

Similarly, short-period NLMXBs forming from both initially thermally unstable systems (Kalogera & Webbink 1996a, 1996b) and via the direct SN channel (Kalogera 1996b) would have a large fraction of (low-mass) unevolved secondaries, suggesting that neither of these channels contributes significantly to the short-period NLMXB population.

Ignoring the uncertain survivors of thermally unstable mass transfer, the very special formation conditions for the case with negligible kick velocity also show that only a very restricted progenitor population (essentially $18 M_\odot + 1.4 M_\odot$ binaries with periods $P_{\text{prog}} \sim 4$ yr) can form NLMXBs, explaining their rarity in the Galaxy. We estimate a total formation rate $\sim 2 \times 10^{-7}$ yr $^{-1}$. The formation conditions are much more restricted than for cataclysmic variables. In particular, short-period NLMXBs must all begin mass transfer at periods $\gtrsim 12$ hr, in contrast to CVs, of which a majority start mass transfer at periods $\lesssim 1$ –2 hr. The resulting NLMXB period histogram has far fewer systems at short periods than the CV version, in agreement with observation.

The smallness of the area of the $M_{\text{He}} - M_2$ plane (Fig. 1) allowing short-period NLMXB formation is very striking. The fact that the resulting population has several properties in good agreement with observation is implicit confirmation that the assumed formation conditions are realistic. In particular, it is clear that the neutron star mass at formation cannot be significantly larger than $1.4 M_\odot$: if it were, the allowed area would become much larger, sharply decreasing the predicted relative population of transient systems. However, we can say nothing about the lower limit on the formation mass, as systems with $m_1 \lesssim 1.2$ would not appear as NLMXBs (the allowed area in Fig. 1 would disappear). This result suggests another conclusion: with initial conditions $1.2 \lesssim m_1 \lesssim 1.4$, $1.2 \lesssim m_2 \lesssim 1.5$, evolution without mass loss would lead us to expect neutron star masses $\gtrsim 2.4 M_\odot$ in short-period systems or those where direct estimates of m_2 give a low mass, such as Cen X-4 (Shabaz, Naylor, & Charles 1993). As there is no observational support for such masses, this suggests that a large fraction of the mass transferred in the NLMXB phase is lost from the binary. The most likely way for this to occur is through mass loss from the accretion disks in these systems (cf. Begelman, McKee, & Shields 1983; Czerny & King 1989). If the mass is lost at disk radii much greater than the size of the neutron star, the central accretion rate can in principle be smaller than the mass transfer rate, so it may be simplistic to infer the latter from the observed X-ray flux. This in turn would mean that

the presence of transient behavior would pose a somewhat less stringent upper limit on the mass transfer rate than we inferred in Paper I.

This paper has discussed the formation of LMXBs containing neutron stars. The constraints on the formation of black hole LMXBs appear to be weaker (cf., e.g., Romani 1994, 1996), as are the conditions for them to appear as transients (Paper I). We shall investigate this problem in a future paper.

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REFERENCES

- Begelman, M. C., McKee, C. F., & Shields, G. A. 1983, *ApJ*, 271, 70
 Brandt, N., & Podsiadlowski, P. 1995, *MNRAS*, 274, 461
 Czerny, M., & King, A. R. 1989, *MNRAS*, 241, 839
 Habets, G. M. H. J. 1986, *A&A*, 167, 61
 Hansen, C. M. S., & Phinney, S. L. 1996, preprint
 Hartmann, J. W. 1996, *A&A*, in press
 Hjellming, M. S. 1989, Ph.D. thesis, Univ. Illinois
 Iben, I., Tutukov, A. V., & Yungelson, L. R. 1995, *ApJS*, 100, 233
 Kalogera, V. 1996a, *ApJ*, 471, 352
 ———. 1996b, *ApJ*, in press
 Kalogera, V., & Webbink, R. F. 1996a, *ApJ*, 458, 301
 ———. 1996b, *ApJ*, in press
 King, A. R., Kolb, U., & Burderi, L. 1996, *ApJ*, 464, L127 (Paper I)
 King, A. R., Kolb, U., de Kool, M., & Ritter, H. 1994, *MNRAS*, 269, 907
 Kolb, U. 1996, in *IAU Coll. 158, Cataclysmic Variables and Related Objects*, ed. A. Evans & J. H. Wood (Dordrecht: Kluwer), 433
 Lyne, A. G., & Lorimer, D. R. 1994, *Nature*, 369, 127
 Pylyser, E. H. P., & Savonije, G. J. 1988a, *A&A*, 191, 57
 ———. 1988b, *A&A*, 208, 52
 Ritter, H. 1994, *Mem. Soc. Astron. Italiana* 65, 173
 Ritter, H., & Kolb, U. 1997, *Catalog of CVs, LMXBs and Rel. Objects*, 6th ed., in preparation
 Romani, R. W. 1994, in *ASP Conf. Ser. 56, Interacting Binary Stars*, ed. A. Shafter (San Francisco: ASP), 196
 Romani, R. W. 1996, in *IAU Symp. 165, Compact Stars in Binaries*, ed. J. van Paradijs et al. (Dordrecht: Kluwer), 93
 Schaller, G., Schaerer, D., Meynet, G., & Maeder, A. 1992, *A&AS*, 96, 269
 Shabaz, T., Naylor, T., & Charles, P. A. 1993, *MNRAS*, 265, 655
 Singer, R., Kolb, U., & Ritter, H. 1993, *Astronomische Gesellschaft Abstract Series*, 9, 39
 Tassoul, J.-L. 1995, *ApJ*, 444, 338
 Terman, J. L., Taam, R. E., & Savage, C. O. 1996, *MNRAS*, 281, 552
 van den Heuvel, E. P. J., & van Paradijs, J. 1988, *Nature*, 334, 227
 van Paradijs, J. 1995, in *X-Ray Binaries*, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 536
 ———. 1996, *ApJ*, 464, L139
 Verbunt, F. 1993, *ARA&A*, 31, 93
 Verbunt, F., & van den Heuvel, E. P. J. 1995, in *X-Ray Binaries*, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 457
 Verbunt, F., & Zwaan, C. 1981, *A&A*, 100, L7
 Webbink, R. F., & Kalogera, V. 1994, in *The Evolution of X-Ray Binaries*, ed. S. S. Holt & C. S. Day (New York: AIP), 321 (WK)
 White, N. E., & Mason, K. O. 1985, *Space Sci. Rev.*, 40, 167