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STRESS ANALYSIS OF HEMISPHERICAL SHELLS WITH SINGLE NONRADIALLY PENETRATED NOZZLES

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ABSTRACT

A systematic study is presented of the stress distribution of a cylindrical shell nonradially penetrated into a shallow hemispherical shell under internal pressure, for different angles of inclination of the nozzle. The results from the combined analytical, computational and experimental investigations for a 20-deg nozzle are compared, to demonstrate the accuracy of the theoretical method presented.

INTRODUCTION

The problem of investigating the stresses in compound shells has become an area of increasing interest especially because of safety requirements in nuclear reactor structures and pressure vessels. Since nonradial nozzle-to-spherical shell attachments are commonly used, means for calculating stresses and strains for such attachments are important in shell structure design.

This paper presents three combined studies which have been conducted to obtain stress distributions for internal pressure in a hemispherical shell having a 20-deg nonradial penetrating nozzle: an analytical study, a finite element analysis, and an experimental investigation. Consideration of different angles of inclination is included. The analytical method is based on closed-form type analyses for cylindrical and shallow spherical shells, using Flugge' (1962) and Vlasov's (1964) equations. Numerical results for the stresses in the compound-shell are obtained using a dedicated computer program developed by the authors. A finite element study is developed, employing the PAFEC (1992) computer program. The calculated results are compared with measured results obtained from an experimental stress analysis of a steel model by means of electrical resistance strain gages. Similar studies have been carried out by Maxwell (1971) and Gwaltney (1973).

NOMENCLATURE

R_c	mean radius of cylindrical shell	ϕ	meridional angular coordinate of spherical shell
R_s	mean radius of spherical shell	θ	circumferential angular coordinate of spherical shell
t_c	thickness of cylindrical shell		
t_s	thickness of spherical shell	$q = r/l$	

E	Young's modulus	$\ell^4 = (R_s t_s)^2 / 12(1 - \nu^2)$
ν	Poisson's ratio	$S = pR_s / 2t_s$ membrane stress in spherical shell
p	internal pressure	$K_s = Et_s^3 / 12(1 - \nu^2)$ flexural rigidity of spherical shell
r	radius of latitude circle	$k^2 = t_c^2 / 12R_c^2$
Ω	angle of obliquity	$\nabla^2(\) = (\)_{,\alpha\alpha} + (\)_{,\beta\beta}$ for cylindrical shell
α	axial coordinate of cylindrical shell (= distance along shell / R_c)	$\nabla^2(\) = (\)_{,rr} + (1/r)(\)_{,r} + (1/r^2)(\)_{,\theta\theta}$ for spherical shell
β	angular coordinate of cylindrical shell	

ANALYTICAL METHOD

This method of analysis is more completely described by Yu et. al. (1971), (1973). Besides the assumptions used in classical thin elastic shells applied to both the nozzle and the sphere, for practical reasons the following additional assumptions are made:

- for small angles of inclination of the nozzle, the intersection of the middle surface of the three shells can be approximated as a plane curve
- $R_c/R_s \ll 1$
- t_c and t_s are constant, although not necessarily equal
- both shells have same E and ν
- welds keep both shells rigidly connected but do not add to shell thickness

A sketch of the configuration of the compound shell is shown in Fig. 1.

Fig. 1 Configuration of the compound shell.

For the cylindrical shell, the displacements in the axial, circumferential and normal directions can be expressed as follows:

$$\begin{aligned}
 u &= u_0 + u_1 + [\Phi_{,\alpha\beta\beta} - \nu\Phi_{,\alpha\alpha\alpha}] \\
 v &= -[\Phi_{,\beta\beta\beta} + (2 + \nu)\Phi_{,\alpha\alpha\beta}] \\
 w &= w_1 + \nabla^4 \Phi
 \end{aligned} \tag{1}$$

where u_0 is an undetermined rigid body displacement, and $u_1 = (1/2 - \nu)\alpha p R_c^2 / Et_c$ and $w_1 = (1 - 1/2\nu)p R_c^2 / Et_c$ are displacements due to the internal pressure. The remaining terms are associated with self-equilibrated edge loadings. $\Phi(\alpha, \beta)$ is the displacement function given by the differential equation of Vlasov (1964) :

$$\nabla^4 \nabla^4 \Phi + \left[(1 - \nu^2) k^2 \right] \Phi_{,\alpha\alpha\alpha\alpha} = 0 \quad (2)$$

In using Eqs.(1), the rigid body displacements associated with $\Phi(\alpha, \beta)$ were replaced by:

$$u^* = -d_2 \cos \beta; v^* = -(d_1 + d_2 \alpha) \sin \beta; w^* = (d_1 + d_2 \alpha) \cos \beta \quad (3)$$

where the d's are constants. Therefore, the displacements of the cylindrical shell are obtained by linear combinations of Eqs.(3) and (1) with the appropriate terms associated with $\Phi(\alpha, \beta)$ deleted. The forces, moments and rotations are calculated from Vlasov's (1964) displacement equations. The solutions of the spherical shell chosen for the problem being considered include oscillatory solutions, membrane and inextensional solutions.

For a shallow spherical shell subjected to edge loadings, Reissner's (1946) equations are:

$$\begin{aligned} \nabla^4 F - (Et_s/R_s) \nabla^2 w &= 0 \\ \nabla^4 w + (1/K_s R_s) \nabla^2 F &= 0 \end{aligned} \quad (4)$$

where $F(r, \theta)$ is the stress function and $w(r, \theta)$ is the normal displacement. The following Johnson's (1967) oscillatory functions satisfy Eqs.(4):

$$\begin{aligned} w &= \sum_{n=0}^N [A_n \ker_n(q) + B_n \operatorname{kei}_n(q)] \cos n\theta \\ F &= (Et_s \ell^2 / R_s) \sum_{n=0}^N [A_n \operatorname{kei}_n(q) - B_n \ker_n(q)] \cos n\theta \end{aligned} \quad (5)$$

The total displacements of the shallow spherical shell under internal pressure and self-equilibrated loadings along the coordinate curve, $\phi = \text{constant}$, (which are used to derive expressions for forces, moments and rotations), were taken in the case of method used, as the respective sums of those given by Eqs.(5), inextensional displacements corresponding to nonoscillatory solutions for Eqs.(4) given by Flugge (1962), with terms corresponding to rigid body motions discarded, and the displacements due to internal pressure. The latter are expressed as:

$$u_3 = 0; v_3 = 0; w_3 = (1 - \nu) R_s^2 p / 2Et_s \quad (6)$$

The boundary conditions imposed on each shell are those required for equilibrium and continuity of the composite shell.

According to Yu et.al. (1971), (1973) and Maxwell (1971) no deviation in the nature of the stress distributions in the compound-shells with and without intrusions has been observed. Therefore, for the analytical study only an outwardly protruding nozzle attached to a spherical shell is considered. Three displacements, one rotation, three force components and one moment for each of the two members yield eight junction conditions. Because the junction curve does not coincide with coordinate curves for the shells, the boundary conditions were satisfied by the method of least square boundary point matching.

The computations were performed using a computer program developed by the authors, with the following data: $R_c = 21.5\text{mm}$, $R_s = 257\text{mm}$, $t_c = 4.8\text{mm}$, $t_s = 6.3\text{mm}$, $\Omega = 20^\circ$, $E = 2.05 \times 10^5 \text{ MPa}$, $\nu = 0.3$, $M = 19$ (number of matching points), $N = 10$ (number of terms retained in the series). The stress components at $\theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ$ were normalised by dividing by the membrane stress in the spherical shell. The dimensionless stresses at $\theta = 0^\circ, 90^\circ$ are plotted in Figs.2 and 3.

a) inner surface

b) outer surface

Fig.2 Stress distribution 0-deg plane

a) inner surface

b) outer surface

Fig.3 Stress distribution 90-deg plane

FINITE ELEMENT ANALYSIS

For the same spherical shell having a nonradial penetrating nozzle, the analysis was carried out using the PAFEC (1992) finite element computer program. Since the primary objective of this study was to determine the stress distributions at the junction, only one-half of the nozzle and one-quarter of the spherical vessel were modeled. In view of the symmetry of the geometry and applied load about the X-Y plane (Fig.4), a finite element mesh consisting of 768 elements and 2433 nodes was generated for the model, using the curved shell element of Ahmad et.al.(1990). This is defined by eight nodal points having six degrees of freedom at each node. Consistent boundary conditions were chosen to reflect half symmetry of the finite element mesh and the applied internal pressure, with an arbitrary chosen value of 1MPa.

The plot of the finite element mesh and its distortion is presented in Fig.4. Distributions of hoop or circumferential stresses (S_{XX}) and axial or meridional stresses (S_{YY}), depending whether the region considered is the cylinder or the sphere have been plotted for 0, 45, 90, 135, 180-deg planes.

Fig.4 Distorsion of the finite element mesh

Figs.5 and 6 present the stresses for 0 and 90-deg planes.

a) inner surface

b) outer surface

Fig.5 Stress distribution 0-deg plane

Fig.6 Stress distribution 90-deg plane

The calculated stress components were normalized and compared with the analytical results. The maximum error was less than 15%, with the calculated stresses being smaller. Although the discrepancy is acceptable for practical design purposes, it would be necessary to take particular care in modeling the junction. It should be noted that the stresses near the base of the hemisphere are higher because of the fixed boundary conditions.

The contour plots of the von Mises stresses are shown in Fig.7. The major conclusion is that the critical stress occurs in the acute inside corner of the intersection.

Fig.7 Stress contour plots (von Mises)

EXPERIMENTAL INVESTIGATION

An experimental investigation was conducted to provide data for use in analytical and finite element method development and assessment. Therefore a machined steel model was extensively strain-gaged in the high stress regions. The dimensions of the model coincided with those for the models used in the numerical application of the analytical method and finite element analysis (Fig.8). The nozzle was welded to the hemisphere with a full penetration weld. It should be noted that there is essentially no fillet at the junction; thus, an idealized configuration was obtained so that maximum benefit could be derived from the results. The model was instrumented with foil resistance strain-gage rosettes bonded to both inner and outer surfaces of the shells. The gages were arranged on five axial planes around the nozzle and extended in great circles on the shell (Fig.9).

Fig.8 Cross-sectional view of hemisphere and nozzle assembly

Fig.9 Gages lines for nonradial nozzle-to-shell attachment

On all surfaces, except the inside surface of the nozzles, the nearest rosettes were located 1mm from the intersection. After gaging, the hemisphere was bolted to a thick steel plate, using a rubber gasket for a seal. The pressure load was applied hydraulically. Strain gage readings were at zero load, one-third load, two-thirds load, full load and then again at zero load, so that linearity and drift of strain gages could be checked.

The strain data obtained from the test were reduced by means of a computer program, to give normalized stresses in tabulated form at each rosette position. An examination of stress distributions along 0 and 90-deg planes (Figs.2 and 3) reveals that the theoretical and experimental data are qualitatively compatible in the general trends. The slight deviations are believed to be a result of the 76.2mm intrusion.

CONCLUSIONS

The results obtained for stress distributions in the attachment region of a hemispherical shell with a single nonradial penetrating nozzle using analytical, computational and experimental methods, although similar, are not the same. Differences between theoretical and measured values were noticeable for the stresses in the immediate vicinity of the intersection curve.

Therefore, the comparison of these results led to the following conclusions:

- the analytical model can be improved by finding more appropriate equations which approximate the junction curve
- the finite element model can also be improved by using 3-D isoparametric elements in the attachment region (for distances having the same order as the thickness of the shells), due to the fact that the stress state is tridimensional and has a complex configuration. However it will be useful to carry out mesh convergence studies
- the analyses have been restricted to linear elastic theory. Due to the large stress gradients along the junction curve, the linear theory may fail, so non-linear effects might play an important role
- for different angles of obliquity of the nozzle ($\Omega = 10^\circ, 15^\circ, 30^\circ$), the calculated results obtained have not been compared with the experimental values; therefore, further study is required to find out the maximum angle that can be utilized to obtain accurate results.

However, the method should find important use in shell-structure design within the established range of applicability, over which usefull solutions can be found,

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