Online Supplement for “Speculation in a Two-stage Retail Supply Chain ”

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Appendix

2 Proof of Lemma 3

Proof. Note that, $E[P_r]$ is indeed a differentiable convex function of Q_s and Q_1 ; moreover, for $\gamma > 0$, it is trivial to show that Slater’s condition is satisfied. Thus, applying the KKT conditions produces optimal solutions for (6). With simple algebraic transformations, the optimal solution is achieved at

$$(1 - \frac{dQ_s}{dQ_1})(P - v)F(Q_1 - Q_s) = P - w + \mu(\gamma - \frac{dQ_s}{dQ_1}), \quad (9)$$

where $\mu (\geq 0)$ is the Lagrangian multiplier associated with the inequality constraint. If the inequality constraint is not binding, the KKT conditions reduce to (7). Obviously, for $\gamma = 1$, the inequality constraint cannot be binding at the optimal solution.

For general $\gamma \in (0, 1)$, if $\tilde{Q}_1(\gamma) < \hat{Q}_1(\gamma)$, the optimal solution of (6) cannot be the equilibrium decision of the retailer, since $Q_1 = \tilde{Q}_1(\gamma)$ and $Q_s = \gamma\tilde{Q}_1(\gamma)$ dominates all feasible solutions of (6).

Note that for $F^{-1}(\frac{P-w}{P-v}) < \hat{Q}_1(0)$, if the retailer orders $F^{-1}(\frac{P-w}{P-v})$, speculators can purchase $Q_s > 0$. In this case, the retailer should order more to earn a higher profit and thus $F^{-1}(\frac{P-w}{P-v})$ is not optimal.

It remains to consider the case of $\hat{Q}_1(0) \geq F^{-1}(\frac{P-w}{P-v}) = \tilde{Q}_1(0)$. We first show that

$$G(Q_1) = (1 - \frac{dQ_s}{dQ_1})F(Q_1 - Q_s)$$

increases in Q_1 for $E[P_r] = P$. If this holds and

$$G(Q_1)|_{Q_1=\hat{Q}_1(0)} > \frac{P-w}{P-v},$$

then (9) does not yield a solution with $E[P_r] = P$ and $Q_s \geq 0$.

Note that when $E[P_r] = P$,

$$\frac{\partial E[P_r]}{\partial Q_s} dQ_s + \frac{\partial E[P_r]}{\partial Q_1} dQ_1 = 0.$$

With (4) and mild algebraic transformation, we have

$$\frac{dQ_s}{dQ_1} = \frac{-\alpha \int_{y>Q_1-Q_s} \frac{1}{y} dF(y)}{\int \frac{1}{y} dF(y) - \alpha \int_{y \leq Q_1-Q_s} \frac{1}{y} dF(y)} < 0. \quad (10)$$

It can be shown that

$$\begin{aligned} \frac{dG(Q_1)}{dQ_1} &= (1 - \frac{dQ_s}{dQ_1})^2 f(Q_1 - Q_s) \left(\int \frac{1}{y} dF(y) - \alpha \int_{y \leq Q_1-Q_s} \frac{1}{y} dF(y) \right)^{-1} (Q_1 - Q_s)^{-1} \\ &\quad \left((1 - \alpha) \int_{y>Q_1-Q_s} \frac{Q_1 - Q_s}{y} dF(y) + \int_{y \leq Q_1-Q_s} \left(\frac{Q_1 - Q_s}{y} - 1 \right) dF(y) \right) > 0. \end{aligned}$$

Moreover, by the definition of $\tilde{Q}_1(0)$ and (10),

$$G(Q_1)|_{Q_1=\hat{Q}_1(0)} > F(Q_1)_{Q_1=\hat{Q}_1(0)} > F(Q_1)_{Q_1=\tilde{Q}_1(0)} = \frac{P-w}{P-v}.$$

Thus, the profit of the retailer increases in Q_1 with $E[P_r] = P$, $Q_s \geq 0$, and $\gamma = 1$.

□

3 Proof of Theorem 1

Proof. By Lemma 3, all three possible solution where the equilibrium order quantity, Q_1^* , may be achieved, are affected by the wholesale price. It is sufficient to consider the case when the equilibrium is given by (6).

When the equilibrium is the solution of (6),

$$G(Q_1) = (1 - \frac{dQ_s}{dQ_1}) F(Q_1 - Q_s) = \frac{P-w}{P-v}.$$

With increasing w , $G(Q_1)$ should decrease, leading to decreasing Q_1 by Lemma 3.

□

4 Proof of Theorem 3

Proof. Denote (w^0, Q_1^0) the equilibrium between the manufacturer and the retailer without speculation. With speculation, by setting the wholesale price at w^0 , the retailer orders no less than Q_1^0 , by Lemma 3. The profit of the manufacturer will not drop if the retailer orders Q_1^0 , and will increase if the retailer orders more.

Consider the retailer. For $\gamma = 1$, (Q_1^*, Q_s^*) is located on the curve defined by $E[P_r] = P$, according to Lemma 3; i.e., $\hat{Q}_1(1) \leq Q_1^* \leq \hat{Q}_1(0)$. With (4), it is easy to verify that

$$\begin{aligned}\hat{Q}_1(0) &= (a - bP)(\eta_0 + \eta), \\ \hat{Q}_1(1) &= \alpha \left(\int \frac{1}{y} dF(y) \right)^{-1}.\end{aligned}$$

Since $Q_1^0 < \hat{Q}_1(0)$, speculation drives up the order quantity of the retailer and the manufacturer benefits from speculation.

We next show that with $\gamma = 1$, speculation may hurt the retailer. Note that

$$\Pi_R(Q_1) = (P - w)Q_1 - (P - v)E[(Q_1 - Q_s) - D]^+.$$

Let $\epsilon = (P - w)$. Then, $\Pi_R(Q_1) < Q_1\epsilon$. As w approaches P , ϵ approaches 0, while Q_1^* approaches $\hat{Q}_1(1)$.

Now, consider a special case such that $Q_1^0 < \hat{Q}_1(1)$. Note that this is possible since Q_1^0 mainly depends on P , c , and c , while $\hat{Q}_1(1)$ does not. For example, if c is close to P , then w^0 is close to P and Q_1^0 is close to 0. But with speculation, $Q_1^* > \hat{Q}_1(1) > Q_1^0$ as w approaches P . In this case, the manufacturer makes a higher profit due to speculation, while the retailer suffers a loss. $\square$