Supplementary Materials for Smoothed Lexis Diagrams with Applications to Lung and Breast Cancer Trends in Taiwan

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APPENDIX A. ALGORITHM (MHRJ FOR THE POSTERIOR)

Let $B_{(n)} = \{(n, a_{0,0}, \dots, a_{n,n}) \mid (a_{0,0}, \dots, a_{n,n}) \in \tilde{B}_n\}$. Let $\theta^{(t)} = (n, a_{0,0}, \dots, a_{n,n}) \in B_{(n)}$ be the current state. We describe the transition from $\theta^{(t)} \in B_{(n)}$ to a new point $\theta^{(t+1)}$ by the following algorithms.

Randomly select one of three types of moves H , H^+ , or H^- . Here H is a transition of element in $B_{(n)}$, H^+ a transition of element from $B_{(n)}$ to $B_{(n+1)}$, and H^- a transition of element from $B_{(n)}$ to $B_{(n-1)}$, respectively. Denote by E_H^n , $E_{H^+}^n$ and $E_{H^-}^n$ the probabilities of selecting the three different types of moves — H , H^+ , and H^- — when the current state of the Markov chain is in $B_{(n)}$. We set $E_{H^-}^2 = E_{H^+}^{n_0} = 0$. Consider $E_H^n = 1 - E_{H^+}^n - E_{H^-}^n$, $E_{H^+}^n = c \min\{1, p(n+1)/p(n)\}$ and $E_{H^-}^n = c \min\{1, p(n-1)/p(n)\}$, where p is as given in Section 2.2 and c is a sample parameter as described in Section 3. We note that in the following, ν denotes the posterior density introduced in Section 2.1.

If the move of type H^+ is selected, then

1. randomly choose g and h from $\{1, \dots, n\}$;
2. generate $W_0 \sim \text{Uniform}(M_1, l_1)$ and $W_{n+1} \sim \text{Uniform}(M_1, l_2)$;
3. generate $W_i \sim \text{Uniform}(M_1, M_2)$ for $i = 1, \dots, n$;
4. generate $V_0 \sim \text{Uniform}(M_1, l_3)$ and $V_n \sim \text{Uniform}(M_1, l_4)$;
5. generate $V_j \sim \text{Uniform}(M_1, M_2)$ for $j = 1, \dots, n-1$;
6. let $s^{(t)} = (n+1, \mathbf{a}_0, \dots, \mathbf{a}_{g-1}, \check{\mathbf{a}}, \mathbf{a}_g, \dots, \mathbf{a}_n) \in B_{(n+1)}$ with $\check{\mathbf{a}} = (W_0, \dots, W_{n+1})$ and $\mathbf{a}_j = (a_{0,j}, \dots, a_{n-1,j}, V_j, a_{n,j}, \dots, a_{n,j})$ for $j = 0, \dots, n$;

7. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)}) \times p(n) \times \prod_{d=1}^4 (l_d - M_1) \times (M_2 - M_1)^{2n-1}}{\nu(\theta^{(t)}) \times p(n+1)} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

If the move of type H^- is selected, then

1. randomly choose g and h from $\{1, \dots, n-1\}$;
2. let $s^{(t)} = (n-1, \mathbf{a}_0, \dots, \mathbf{a}_{g-1}, \mathbf{a}_{g+1}, \dots, \mathbf{a}_n) \in B_{(n-1)}$ with $\mathbf{a}_j = (a_{0,j}, \dots, a_{h-1,j}, a_{h+1,j}, \dots, a_{n,j})$ for $j = 0, \dots, g-1, g+1, \dots, n$;
3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)}) \times p(n)}{\nu(\theta^{(t)}) \times p(n-1) \times \prod_{d=1}^4 (l_d - M_1) \times (M_2 - M_1)^{2n-3}} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

If the move of type H is selected, then randomly choose a updating procedure from the following.

i) Update $a_{0,0}$.

1. generate $\tilde{a}_{0,0} \sim q_1$;
2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{0,0}$ replaced by $\tilde{a}_{0,0}$;
3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

ii) *Update* $a_{0,n}$.

1. generate $\tilde{a}_{0,n} \sim q_2$;
2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{0,n}$ replaced by $\tilde{a}_{0,n}$;
3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

iii) *Update* $a_{n,0}$.

1. generate $\tilde{a}_{n,0} \sim q_3$;
2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{n,0}$ replaced by $\tilde{a}_{n,0}$;
3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

iv) *Update* $a_{n,n}$.

1. generate $\tilde{a}_{n,n} \sim q_4$;
2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{n,n}$ replaced by $\tilde{a}_{n,n}$;
3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

v) *Update* $a_{0,j}$ for $j=1, \dots, n-1$.

1. generate $\tilde{a}_{0,j} \sim \text{Uniform}(M_1, l_1)$ for $j=1, \dots, n-1$;

2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{0,1}, \dots, a_{0,n-1}$ replaced by $\tilde{a}_{0,1}, \dots, \tilde{a}_{0,n-1}$, respectively;

3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

vi) Update $a_{n,j}$ for $j=1, \dots, n-1$.

1. generate $\tilde{a}_{n,j} \sim \text{Uniform}(M_1, l_2)$ for $j=1, \dots, n-1$;

2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{n,1}, \dots, a_{n,n-1}$ replaced by $\tilde{a}_{n,1}, \dots, \tilde{a}_{n,n-1}$, respectively;

3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

vii) Update $a_{i,0}$ for $i=1, \dots, n-1$.

1. generate $\tilde{a}_{i,0} \sim \text{Uniform}(M_1, l_3)$ for $i=1, \dots, n-1$;

2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{1,0}, \dots, a_{n-1,0}$ replaced by $\tilde{a}_{1,0}, \dots, \tilde{a}_{n-1,0}$, respectively;

3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)}, & \text{with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)}, & \text{o.w.} \end{cases}$$

viii) Update $a_{i,n}$ for $i=1, \dots, n-1$.

1. generate $\tilde{a}_{i,n} \sim \text{Uniform}(M_1, l_4)$ for $i=1, \dots, n-1$;

2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{1,n}, \dots, a_{n-1,n}$ replaced by $\tilde{a}_{1,n}, \dots, \tilde{a}_{n-1,n}$,

respectively;

3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)} & , \text{ with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)} & , \text{ o.w.} \end{cases}$$

ix) Update $a_{1,j}, \dots, a_{n-1,j}$ for any $j=1, \dots, n-1$.

1. randomly choose g from $\{1, \dots, n-1\}$;

2. generate $\tilde{a}_{i,g} \sim \text{Uniform}(M_1, M_2)$ for $i=1, \dots, n-1$;

2. let $s^{(t)}$ be the vector $\theta^{(t)}$ with $a_{1,g}, \dots, a_{n-1,g}$ replaced by $\tilde{a}_{1,g}, \dots, \tilde{a}_{n-1,g}$,

respectively;

3. set the next state

$$\theta^{(t+1)} = \begin{cases} s^{(t)} & , \text{ with prob. } \rho = \min \left\{ 1, \frac{\nu(s^{(t)})}{\nu(\theta^{(t)})} \right\}, \\ \theta^{(t)} & , \text{ o.w.} \end{cases}$$

APPENDIX B.

For the FIBC study, we consider cases with ICD-9-CM codes 1740–1746, 1748, and 1749, and morphological code 85003 (infiltrating duct carcinoma). For lung cancer, we consider cases with ICD-9-CM codes 1620, 1622–1625, 1628, and 1629. For lung ADC, the morphological codes include 81403, 81413, 81433, 82113, 82503, 82513, 82553, 82603, 83103, 83233, 84403, 84803, 84813, 84903, and 85003. For the lung SCC, the morphological codes include 80523, 80703, 80713, 80723, 80733, 80743, 80763, 80823, 80833, and 80843.

APPENDIX C.

Tables C1, C2, C3, and C4: These tables show characteristics of the MCMC for each analysis. Each row describes the characters of one Bayesian analysis. The first column lists the four sets of priors used for the analysis, given in Section 3. Columns 2 and 3 give, respectively, the mean and maximum of $\hat{F} \times 10^5$. Columns 4, 5, and 6 give the Gelman-Rubin statistic monitoring chain convergence, the number of iterations, and the CUP time for this chain, respectively. Column 7 gives the posterior predictive p -value.

Tables C1-1, C2-1, C3-1, and C4-1: These tables report sensitivity analysis regarding Tables C1, C2, C3, and C4, respectively. For example, Table C1-1 reports the difference between any two Bayesian analyses in Table C1, using the criterion $T = \max \left\{ \left| \hat{F}_{\text{prior } i}(x_k, y_k) - \hat{F}_{\text{prior } j}(x_k, y_k) \right| \mid k = 1, \dots, 1020 \right\} \times 10^5$, the largest difference happens between set of priors 2 and 3 and has the value of 7.312.

Table C1. FIBC

Set of priors	Mean	Max	$\hat{R}$	Iterations	Time (min.)	P -value
1	58.33	135.7	1.039	200,000	7	0.600
2	58.32	133.1	1.042	200,000	7	0.508
3	58.48	136.2	1.088	400,000	14	0.500
4	58.32	136.1	1.008	800,000	27	0.620

Table C1-1. Sensitivity analysis regarding Table C1

Set of priors	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
T	2.580	6.738	1.352	7.312	3.045	7.123

Table C2. FADC

Set of priors	Mean	Max	$\hat{R}$	Iterations	Time (min.)	P -value
1	36.79	167.0	1.015	200,000	7	0.385
2	36.79	167.0	1.015	200,000	6	0.418
3	36.79	167.8	1.013	200,000	7	0.385
4	36.79	167.1	1.003	200,000	7	0.401

Table C2-1. Sensitivity analysis regarding Table C2

Set of priors	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
T	3.385×10^{-4}	1.081	0.727	1.081	0.727	0.354

Table C3. MADC

Set of priors	Mean	Max	$\hat{R}$	Iterations	Time (min.)	P -value
1	47.50	207.4	1.002	200,000	7	0.306
2	47.50	207.4	1.002	200,000	6	0.345
3	47.47	207.4	1.016	200,000	7	0.334
4	47.47	207.4	1.016	200,000	7	0.319

Table C3-1. Sensitivity analysis regarding Table C3

Set of priors	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
T	0.001	0.394	0.394	0.393	0.393	7.575×10^{-4}

Table C4. MSCC

Set of priors	Mean	Max	$\hat{R}$	Iterations	Time (min.)	P -value
1	50.23	220.1	1.096	1,600,000	53	0.029
2	50.17	219.2	1.030	6,400,000	218	0.027
3	46.07	153.5	1.013	200,000	6	0.195
4	46.07	153.5	1.013	200,000	6	0.183

Table C4-1. Sensitivity analysis regarding Table C4

Set of priors	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
T	4.897	68.50	68.50	67.51	67.51	5.248×10^{-4}

APPENDIX D.

Figure D1. Derivative (rate of change) of age-specific rates by year of birth, plotted in Figure 3. Light green horizontal lines have slopes of zero.

Figure D2. Derivative (rate of change) of age-specific rates by year of diagnosis, plotted in Figure 4. Light green horizontal lines have slopes of zero.

Figure D3. Derivative (rate of change) of year-specific rates by age of diagnosis, plotted in Figure 5. Light green horizontal lines have slopes of zero.

Figure D4. Derivative (rate of change) of cohort-specific rates by age of diagnosis, plotted in Figure 6. Light green horizontal lines have slopes of zero.

APPENDIX E.

Figure E1. Smoking rate among people age 18 or more in Taiwan (downloaded from <http://tobacco.hpa.gov.tw/Show.aspx?MenuId=581>, the Health Promotion Administration, Ministry of Health and Welfare, Taiwan, on August 15, 2013).

APPENDIX F.

Figure F1-1. Age-specific rates by year of birth based on 1-year tabulated standard Lexis diagrams (rates versus age of diagnosis, observations within each birth-cohort are connected).

Figure F1-3. Age-specific rates by year of birth based on 3-year tabulated standard Lexis diagrams (rates versus age of diagnosis, observations within each birth-cohort are connected).

Figure F2-1. Age-specific rates by year of diagnosis based on 1-year tabulated standard Lexis diagrams (rates versus age of diagnosis, observations within each year are connected).

Figure F2-3. Age-specific rates by year of diagnosis based on 3-year tabulated standard Lexis diagrams (rates versus age of diagnosis, observations within each year are connected).

Figure F3-1. Year-specific rates by age of diagnosis based on 1-year tabulated standard Lexis diagrams (rates versus year of diagnosis, observations within each age group are connected).

Figure F3-3. Year-specific rates by age of diagnosis based on 3-year tabulated standard Lexis diagrams (rates versus year of diagnosis, observations within each age group are connected).

Figure F4-1. Cohort-specific rates by age of diagnosis based on 1-year tabulated standard Lexis diagrams (rates versus birth-cohort, observations within each age group are connected).

Figure F4-3. Cohort-specific rates by age of diagnosis based on 3-year tabulated standard Lexis diagrams (rates versus birth-cohort, observations within each age group are connected).