

# Supplementary Materials for “Selection of terms in random coefficient regression models”

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## Simulation of the incomplete data

The data in Tables S1, S2 and S3 correspond to a simulation study to evaluate the performance of the procedure proposed in the manuscript entitled “Selection of terms in random coefficient regression models” in situations with missing observations.

We set  $\sigma^2 = 1$  and  $\rho = 0.4, 0.6$  and  $0.8$  and considered 80 different settings based on combinations of different values for  $n$  (5, 10, 25, 50 and 100),  $\sigma_a^2$  (0.5, 1, 2 and 4),  $\sigma_b^2$  (0.5, 1 and 2) and  $\rho_{ab}$  (0.25, 0.75). We assumed that each unit had a 50% probability of missing one observation; the position of the missing observation was chosen at random among the five available. For each setting we simulated 1000 samples and in each case we fitted second degree polynomials to the individual profiles. In this study we did not consider the procedure based on the sample covariance matrix.

S 1: Percentage of correct decisions for analysis based on individual profiles for models A as well as B and C with  $\rho = 0.4$

n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C	n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C
5	4	2	0.75	53	66	71	25	1	1	0.75	85	78	80
5	4	2	0.25	52	62	74	25	1	1	0.25	85	79	77
5	2	2	0.75	49	63	70	25	1	0.5	0.75	63	74	77
5	2	2	0.25	51	60	73	25	1	0.5	0.25	65	74	77
5	2	1	0.75	27	41	56	25	0.5	1	0.75	84	79	79
5	2	1	0.25	29	42	57	25	0.5	1	0.25	86	79	76
5	1	2	0.75	51	65	69	25	0.5	0.5	0.75	64	74	79
5	1	2	0.25	49	62	69	25	0.5	0.5	0.25	66	75	77
5	1	1	0.75	28	42	54	50	4	2	0.75	83	72	72
5	1	1	0.25	30	41	53	50	4	2	0.25	85	72	70
5	1	0.5	0.75	10	21	38	50	2	2	0.75	81	72	70
5	1	0.5	0.25	11	22	38	50	2	2	0.25	83	69	71
5	0.5	1	0.75	27	41	53	50	2	1	0.75	83	68	72
5	0.5	1	0.25	30	42	58	50	2	1	0.25	84	71	69
5	0.5	0.5	0.75	12	22	37	50	1	2	0.75	82	70	74
5	0.5	0.5	0.25	10	21	38	50	1	2	0.25	84	70	69
10	4	2	0.75	84	83	84	50	1	1	0.75	82	68	69
10	4	2	0.25	84	84	86	50	1	1	0.25	82	68	72
10	2	2	0.75	83	84	85	50	1	0.5	0.75	76	70	69
10	2	2	0.25	83	85	85	50	1	0.5	0.25	78	67	70
10	2	1	0.75	63	72	79	50	0.5	1	0.75	84	69	70
10	2	1	0.25	58	73	80	50	0.5	1	0.25	86	70	68
10	1	2	0.75	84	83	84	50	0.5	0.5	0.75	80	71	70
10	1	2	0.25	83	82	85	50	0.5	0.5	0.25	78	68	72
10	1	1	0.75	62	75	81	100	4	2	0.75	92	83	82
10	1	1	0.25	63	73	82	100	4	2	0.25	92	83	82
10	1	0.5	0.75	31	55	68	100	2	2	0.75	91	82	83
10	1	0.5	0.25	32	50	70	100	2	2	0.25	92	82	83
10	0.5	1	0.75	64	74	80	100	2	1	0.75	92	81	79
10	0.5	1	0.25	63	72	81	100	2	1	0.25	90	81	82
10	0.5	0.5	0.75	32	56	66	100	1	2	0.75	91	83	84
10	0.5	0.5	0.25	28	52	68	100	1	2	0.25	90	82	84
25	4	2	0.75	88	78	77	100	1	1	0.75	92	80	83
25	4	2	0.25	89	77	78	100	1	1	0.25	91	82	82
25	2	2	0.75	88	79	78	100	1	0.5	0.75	89	82	79
25	2	2	0.25	90	79	77	100	1	0.5	0.25	89	83	80
25	2	1	0.75	86	78	79	100	0.5	1	0.75	93	81	83
25	2	1	0.25	87	78	79	100	0.5	1	0.25	92	81	83
25	1	2	0.75	88	79	79	100	0.5	0.5	0.75	87	82	81
25	1	2	0.25	87	77	79	100	0.5	0.5	0.25	88	82	83

S 2: Percentage of correct decisions for analysis based on individual profiles for models A as well as B and C with  $\rho = 0.6$

n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C	n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C
5	4	2	0.75	53	66	74	25	1	1	0.75	85	75	72
5	4	2	0.25	52	64	76	25	1	1	0.25	85	73	75
5	2	2	0.75	49	68	74	25	1	0.5	0.75	63	72	74
5	2	2	0.25	51	64	71	25	1	0.5	0.25	65	72	73
5	2	1	0.75	27	49	63	25	0.5	1	0.75	84	73	74
5	2	1	0.25	29	47	62	25	0.5	1	0.25	86	75	75
5	1	2	0.75	51	66	75	25	0.5	0.5	0.75	64	72	71
5	1	2	0.25	49	65	75	25	0.5	0.5	0.25	66	73	74
5	1	1	0.75	28	48	65	50	4	2	0.75	83	63	62
5	1	1	0.25	30	51	60	50	4	2	0.25	85	67	66
5	1	0.5	0.75	10	31	47	50	2	2	0.75	81	64	64
5	1	0.5	0.25	11	31	43	50	2	2	0.25	83	66	65
5	0.5	1	0.75	27	47	62	50	2	1	0.75	83	64	64
5	0.5	1	0.25	30	46	62	50	2	1	0.25	84	66	63
5	0.5	0.5	0.75	12	30	44	50	1	2	0.75	82	66	65
5	0.5	0.5	0.25	10	29	47	50	1	2	0.25	84	63	64
10	4	2	0.75	84	80	82	50	1	1	0.75	82	65	62
10	4	2	0.25	84	82	82	50	1	1	0.25	82	64	66
10	2	2	0.75	83	83	83	50	1	0.5	0.75	76	67	62
10	2	2	0.25	83	81	81	50	1	0.5	0.25	78	62	63
10	2	1	0.75	63	75	81	50	0.5	1	0.75	84	63	63
10	2	1	0.25	58	75	79	50	0.5	1	0.25	86	64	61
10	1	2	0.75	84	80	80	50	0.5	0.5	0.75	80	62	65
10	1	2	0.25	83	81	84	50	0.5	0.5	0.25	78	63	64
10	1	1	0.75	62	75	81	100	4	2	0.75	92	76	75
10	1	1	0.25	63	74	80	100	4	2	0.25	92	75	75
10	1	0.5	0.75	31	62	76	100	2	2	0.75	91	75	73
10	1	0.5	0.25	32	61	75	100	2	2	0.25	92	73	77
10	0.5	1	0.75	64	76	82	100	2	1	0.75	92	76	75
10	0.5	1	0.25	63	76	78	100	2	1	0.25	90	76	74
10	0.5	0.5	0.75	32	62	74	100	1	2	0.75	91	76	74
10	0.5	0.5	0.25	28	62	74	100	1	2	0.25	90	76	74
25	4	2	0.75	88	74	72	100	1	1	0.75	92	79	76
25	4	2	0.25	89	74	73	100	1	1	0.25	91	78	76
25	2	2	0.75	88	75	74	100	1	0.5	0.75	89	76	75
25	2	2	0.25	90	70	72	100	1	0.5	0.25	89	75	75
25	2	1	0.75	86	73	74	100	0.5	1	0.75	93	76	79
25	2	1	0.25	87	71	72	100	0.5	1	0.25	92	76	73
25	1	2	0.75	88	75	74	100	0.5	0.5	0.75	87	75	74
25	1	2	0.25	87	73	74	100	0.5	0.5	0.25	88	78	76

S 3: Percentage of correct decisions for analysis based on individual profiles for models A as well as B and C with  $\rho = 0.8$

n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C	n	$\sigma_a^2$	$\sigma_b^2$	$\rho_{ab}$	A	B	C
5	4	2	0.75	53	70	74	25	1	1	0.75	85	69	69
5	4	2	0.25	52	70	76	25	1	1	0.25	85	69	71
5	2	2	0.75	49	70	75	25	1	0.5	0.75	63	68	71
5	2	2	0.25	51	70	74	25	1	0.5	0.25	65	66	70
5	2	1	0.75	27	56	66	25	0.5	1	0.75	84	69	71
5	2	1	0.25	29	54	67	25	0.5	1	0.25	86	71	71
5	1	2	0.75	51	67	74	25	0.5	0.5	0.75	64	69	69
5	1	2	0.25	49	68	76	25	0.5	0.5	0.25	66	69	69
5	1	1	0.75	28	53	66	50	4	2	0.75	83	58	55
5	1	1	0.25	30	55	62	50	4	2	0.25	85	60	57
5	1	0.5	0.75	10	38	52	50	2	2	0.75	81	60	59
5	1	0.5	0.25	11	37	53	50	2	2	0.25	83	56	58
5	0.5	1	0.75	27	54	64	50	2	1	0.75	83	57	57
5	0.5	1	0.25	30	52	68	50	2	1	0.25	84	59	59
5	0.5	0.5	0.75	12	37	52	50	1	2	0.75	82	60	60
5	0.5	0.5	0.25	10	37	54	50	1	2	0.25	84	59	59
10	4	2	0.75	84	80	81	50	1	1	0.75	82	56	61
10	4	2	0.25	84	79	79	50	1	1	0.25	82	56	61
10	2	2	0.75	83	80	80	50	1	0.5	0.75	76	59	63
10	2	2	0.25	83	79	79	50	1	0.5	0.25	78	58	59
10	2	1	0.75	63	75	79	50	0.5	1	0.75	84	61	60
10	2	1	0.25	58	77	80	50	0.5	1	0.25	86	59	61
10	1	2	0.75	84	79	80	50	0.5	0.5	0.75	80	60	57
10	1	2	0.25	83	77	80	50	0.5	0.5	0.25	78	58	58
10	1	1	0.75	62	79	78	100	4	2	0.75	92	71	72
10	1	1	0.25	63	77	78	100	4	2	0.25	92	71	70
10	1	0.5	0.75	31	66	76	100	2	2	0.75	91	68	71
10	1	0.5	0.25	32	67	76	100	2	2	0.25	92	69	71
10	0.5	1	0.75	64	77	80	100	2	1	0.75	92	72	68
10	0.5	1	0.25	63	73	81	100	2	1	0.25	90	70	70
10	0.5	0.5	0.75	32	67	75	100	1	2	0.75	91	69	70
10	0.5	0.5	0.25	28	65	73	100	1	2	0.25	90	72	71
25	4	2	0.75	88	69	72	100	1	1	0.75	92	73	69
25	4	2	0.25	89	71	68	100	1	1	0.25	91	73	72
25	2	2	0.75	88	71	72	100	1	0.5	0.75	89	68	71
25	2	2	0.25	90	69	70	100	1	0.5	0.25	89	69	70
25	2	1	0.75	86	69	71	100	0.5	1	0.75	93	72	73
25	2	1	0.25	87	68	69	100	0.5	1	0.25	92	73	72
25	1	2	0.75	88	70	68	100	0.5	0.5	0.75	87	71	72
25	1	2	0.25	87	69	71	100	0.5	0.5	0.25	88	70	71

## Aorta diameter example

The data for the SGA pre term neonates are presented in Table S4.

An Excel spreadsheet with the estimates for the parameters of all models considered in Table S6 as well as data for the examples and R codes for the simulation and analyses may be downloaded from

<http://www.ime.usp.br/~jmsinger/Rocha&Singer2016.zip>

S 4: Aorta diameter per unit weight (mm/kg) of SGA pre-term neonates

Subject ID	Week post-conception														
	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
30						7.16	6.81			5.51			4.72		
31							7.08	7.98		7.74			6.50	5.56	
32						7.44	8.31	9.38	10.00	9.20			8.04		
33							7.68		6.56		5.51		4.61		
34								6.50						4.38	
35					7.63	8.63	9.26	7.98		6.64		5.00		4.72	
36					6.64	8.37		8.18	7.57		6.64				
37							7.05	6.27			6.09	5.92	5.73		4.81
38						8.54	8.39					4.92			
39				8.31	7.42		6.17			4.56		3.84			
40				9.76			9.06		7.30		5.35				
41				6.60			6.36	5.80	5.03			4.41		3.90	
42									7.80	7.90		7.05		6.04	
43				6.39											
44								5.85	5.87		4.69				
45								5.98		5.27					
46								6.20	5.83		4.77				
47								6.27		5.26		5.03		4.38	
48						6.89	8.92	7.82	8.50	8.26		5.09			
49										7.31		6.08		5.25	
50								7.05	6.79	6.41		4.94			
51									6.89						
52								8.57	9.42	9.03		8.21		7.52	7.30
53									4.93	5.64	5.70	5.47	5.28		4.66
54									6.13			4.92		4.49	
55									5.95		5.49		4.63		
56									6.84	6.15		5.09		4.56	
57									7.24	6.45				3.94	3.62
58						8.87	10.48	9.85	11.00	10.46	10.00				
59								9.71	9.08					5.86	
60											6.19				
61														5.76	4.91

S 5: Estimates of the individual intercepts, slopes and quadratic coefficients and corresponding t-values for subjects in the SGA group (aorta diameter example)

Subject	Intercept		Slope		Quadratic	
	Estim	t-value	Estim	t-value	Estim	t-value
30	6.63	1.09	-2.53	1.34	0.75	1.68
31	7.22	-4.49	2.73	4.78	-2.83	-1.11
32	9.20	16.34	2.38	7.04	-2.95	-3.02
33	7.10	0.59	-2.44	1.25	0.25	1.28
35	7.89	8.96	-1.40	-0.07	-0.89	-0.91
36	8.20	3.24	-0.39	2.48	-2.94	-1.72
37	6.65	1.89	-0.59	2.63	-0.29	0.55
39	5.54	-6.65	-2.38	-4.20	0.59	1.95
40	8.28	1.27	-3.14	-3.43	-1.63	-0.93
41	5.70	0.03	-1.20	-0.36	-0.10	0.77
42	7.86	-12.27	0.34	3.48	-1.19	0.37
47	6.64	3.88	-1.44	-4.34	-0.30	-0.20
48	7.51	1.05	-1.73	-2.36	-1.13	-0.74
50	6.39	-4.73	-2.25	0.23	-1.84	0.33
52	8.92	12.50	-0.52	2.77	-0.36	0.46
53	4.61	-14.58	2.45	4.65	-1.50	-0.47
54	6.12	-0.96	-1.16	-1.98	-0.02	1.00
55	5.98	-2.11	-1.39	1.36	0.00	1.06
56	6.70	0.79	-1.94	-1.30	0.13	1.27
57	6.55	2.55	-1.42	-1.53	-0.38	-0.20
58	5.61	-13.24	-10.49	-2.20	-5.22	-4.22
59	7.49	2.83	-2.40	-2.83	-0.07	0.96
61	7.93	2.03	-3.13	-7.40	0.39	1.84
Estimate	6.99		-1.48		-0.94	
Std. error	0.09		0.23		0.21	
p-value	<b>0.000</b>		<b>0.000</b>		<b>0.000</b>	

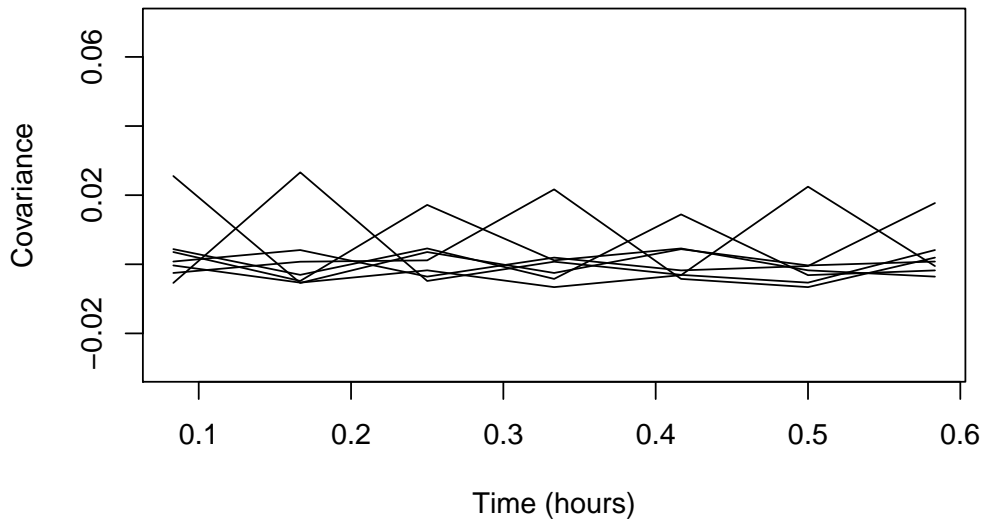
S 6: Terms selected for alternative models (aorta diameter example)

Models	AGA						SGA						Common Random Coefficients					
	Fixed coefficients			Random Coefficients			Fixed coefficients			Random Coefficients			Random Coefficients			Common Random Coefficients		
	Intercept	Slope	Quad	Intercept	Slope	Quad	Intercept	Slope	Quad	Intercept	Slope	Quad	Intercept	Slope	Quad	Intercept	Slope	Quad
Model 1	x	x		x			x			x								
Model 2	x	x					x									x		
Model 3	x	x		x						x								
Model 4	x	x					x								x			
Model 5	x	x					x											
Model 6	x	x													x			
Model 7	x	x		x											x			
Model 8	x	x		x											x			
Model 9	x	x					x											x
Model 10	x	x					x											
Model 11	x	x					x											
Model 12	x	x					x								x			
Model 13	x	x					x								x			
Model 14	x	x																
Model 15	x	x																
Model 16	x	x																
Model 17	x	x																
Model 18	x	x																
Model 19	x	x																
Model 20	x	x																
Model 21	x	x																
Model 22	x	x																

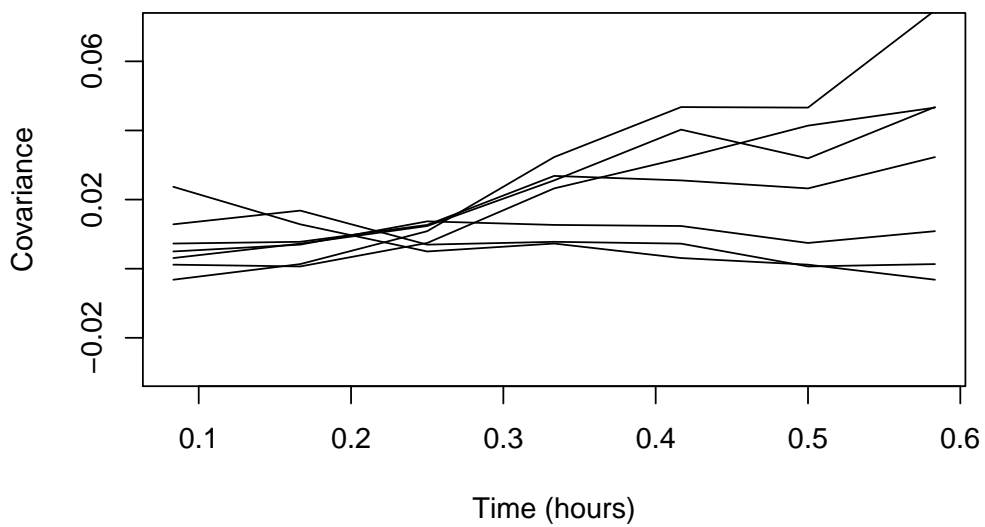


# Mucociliary speed example

S 1: Profile plots of the rows of the sample covariance matrix for data with random intercept (mucociliary speed data)



S 2: Profile plots of the rows of the sample covariance matrix for data with random intercept and slope (mucociliary speed data)



S 3: Profile plots of the rows of the sample covariance matrix for data with random intercept, slope and quadratic coefficient (mucociliary speed data)

