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School mathematics and its everyday other?

Revisiting Lave's 'Cognition in Practice'

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Abstract. In the last three decades there have been a variety of studies of what is often referred to as 'everyday' or 'street' mathematics. These studies have documented a rich variety of arithmetic practices involved in activities such as tailoring, carpet laying, dieting, or grocery shopping. More importantly, these studies have helped to rectify outmoded models of rationality, cognition, and (school) instruction.

Despite these important achievements, doubts can be raised about the ways in which theoretical conclusions have been drawn from empirical materials. Furthermore, while these studies rightly criticised prevalent theories of rationality and cognition as too simplistic to account for everyday activities, it seems that some of the proposed alternatives suffer from similar flaws (i.e., are straightforward inversions of the to-be-opposed theories, rather than more nuanced views on complicated issues).

In this article we illustrate our sceptical view by discussing four case studies in Jean Lave's pioneering and influential 'Cognition in Practice' (1988). By looking at the case studies in detail, we investigate how Lave's conclusions relate to the empirical materials and offer alternative characterisations. In particular, we question whether the empirical studies demonstrate the existence of two different *kinds* of mathematics ('everyday' and 'school', or 'formal' and 'informal') and whether school instruction tries to replace the former with the latter.

Keywords. everyday mathematics; street mathematics; school mathematics; situated learning; Lave

Introduction

In the last three decades there have been a variety of studies of what is alternatively referred to as ‘everyday’ mathematics (Lave, 1988), ‘street’ mathematics (Nunes, Schliemann, and Carraher, 1993), ‘ethnomathematics’ (D’Ambrosio, 1985), or ‘out-of-school’ mathematics (Masingila, 1993). Influential studies have investigated the use of arithmetic by dieters and shoppers (Lave, 1988), Brazilian street vendors (Nunes *et al.*, 1993), candy sellers (Saxe, 1991), and South African carpenters (Millroy, 1992). Studies of street mathematics continue to be done (for example, Jurdak and Shahin, 1999; Noss, Pozzi, and Hoyles, 1999; Hoyles, Noss, and Pozzi, 2001; Gainsburg, 2006; for a detailed overview see Presmeg, 2007) – along with reflection on and debate over them (cf., Brenner and Moschkovich, 2002; Rowlands and Carson, 2002).

In our view, these studies can be read in (at least) two different ways: firstly, as predominantly *descriptive* studies documenting a variety of mathematical practices dealing with practical problems in concrete situations; secondly, but more importantly, as evidence in the prosecution of a much larger *critical* campaign, which wants to re-assess fundamental questions about the nature of mathematics, cognition, and rationality.

Our aim in this paper is raise doubts about whether successful connections are made between the empirical studies and these grander (critical) themes. We first summarise some main points of our argument, arguing that the key issue concerns the way in which general critical implications are drawn from specific case studies. We then review a leading and influential example, Jean Lave’s studies of everyday mathematics. We chose this example, because her studies were pioneering and

continue to be widely and approvingly quoted – not only in educational studies (see Henning, 2004), but also in other fields such as economics (Mousavi and Garrison, 2003), organisation studies (Callon and Muniesa, 2005), accounting (Vollmer, 2007), and social studies of science (Bowker and Star, 1999; Verran, 2001). That is to say, Lave's studies made a strategic contribution to the formation of the discourse of 'everyday mathematics' and set a precedent as to how to interpret case studies. Furthermore, her studies are among the most extensively documented in the field.

Since our argument concerns the detailed fit between case study materials and the implications drawn from them, it is impossible to develop our case without reviewing the details of fieldwork reports. Within a single paper only a few studies can be reviewed, and we focus on Lave's studies since they seem a pivotal contribution to a subsequently thriving strand of inquiry. That Lave's work is early in this approach does not mean it is outdated, for Lave's case studies seem no less robust, and often more detailed, than many of their successors, which nonetheless report much the same kind of conclusions. Nor have the problems that we find with Lave's studies been surpassed by her successors through improved forms of interpretation, and the themes of later studies are in general harmony with Lave's. Thus although Lave's studies were conducted some years ago, they still constitute a perspicuous site for posing the questions we think need to be asked.

The motivation of studies of 'everyday' mathematics

Why did studies of everyday or street mathematics have such an impact on mathematics education and other fields such as cultural anthropology or cognitive science? In our view the main contribution of these studies was that they illustrated the variety of arithmetic procedures that have been developed as part of, for example,

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supermarket shopping, tailoring, carpet laying, or candy selling – thereby deflating the idea that mathematical competences can only be acquired by attending (Western) schools. In short, these studies ‘discovered’ mathematics in both non-Western ‘traditional’ societies as well as the ‘academic hinterland’ (Lave, 1988, p. 3) of the West.

However, this descriptive aspect alone cannot account for the wide-reaching impact that these studies had. To understand this influence, one has to see these studies as part of a critical project that was a reaction against then prevailing theoretical regimes of rationalism, cognitivism, and formalism in the social and human sciences. These studies thus can be seen as (1) a reaction against the way that rationality was conceived (especially in cultural anthropology); (2) a critique of the cognitivist conceptions of learning and knowledge (with its focus on the individual and its neglect of the situation); and (3) an expression of the view that (Western) schools play a key role in transmitting these preconceptions about rationality and thereby alienate large parts of the society.

Firstly, studies of everyday and street mathematics, especially those conducted in ‘non-developed’ or ‘traditional’ societies, can be seen as a reaction against the supposedly dominant view that (Western) science and mathematics are the hallmark of rationality, i.e., are superior to other forms of human knowledge. According to this view, mathematics is the supreme embodiment of rationality:

Problem solving through the use of mathematics has become a cultural symbol for human rationality and ‘right’ thinking and is often considered the underlying mechanism of thought itself. (de la Rocha, 1986, p. xii)

Consequently, cultures without professional mathematics were seen as inferior to Western cultures:

Nonliterate peoples are often explicitly characterized as simpleminded or childlike, as only capable of concrete thought and not of abstraction or generalization, as of lesser intelligence, as incapable of analytic thought, and as without formal reasoning or logic. In any context these descriptions are heavily judgemental; in the context of mathematics, they are condemning. (Ascher and Ascher, 1986, p. 128)

Studies of everyday/street mathematics attempted to counter the equation ‘being rational = being good at formal (Western) mathematics’. By demonstrating that even in cultures with no professional mathematics a wide variety of mathematical procedures have been developed as part of practices such as carpentry, tailoring, or candy selling, these studies demonstrated that ‘traditional’ cultures are by no means intellectually inferior to Western culture.

A second aim of studies of everyday and street mathematics was to attack the way that cognitive science conceived of understanding and learning by comparing the mind to a computer and reasoning to the running of a computerised algorithm. This conception was markedly individualist (determined to understand the general and formal properties of the individual mind) and acontextual (attempting to isolate individuals from all external influences by placing them in experimental laboratories). The cognitive science view of cognition and learning thus nestled very comfortably with assumptions about the supposed supremacy of formal (mathematical) reasoning. The idea of ‘everyday cognition’ (Rogoff and Lave, 1984), which feeds into that of ‘everyday mathematics’, challenged the idea that actual reasoning (as opposed to artificially restricted laboratory exercises) can be represented by formal computational schemes. ‘Everyday cognition’ also opposed individualism (by shifting the emphasis

to the socio-cultural milieu) and acontextualism (by emphasising the situated character of learning and cognition).

A third aim of these studies had to do with school mathematics and the view that the aforementioned ideas about rationality permeate the teaching of mathematics to schoolchildren. In other words, cognitivist or formalistic conceptions of rationality were seen as having been ‘institutionalised’ in Western schools (see, e.g., Lave, 1990, p. 310; Lave, 1993, p. 24). As Taylor (1996, p. 163) puts it: “modern school mathematics continues to be influenced strongly by the rationalist myth of cold reason”. Schools were thus seen as perpetuating an ideology of mathematics as definitive of rationality with the result that those doing badly in school mathematics see themselves not just failing at a particular subject in school, but as being generally less ‘intelligent’. Studies of everyday mathematics demonstrated that people who do not perform well in school mathematics (and therefore might *think* that they can’t do mathematics) are frequently efficient and successful in solving these everyday and street problems (i.e., ‘really’ can do mathematics). These studies aimed to highlight the alienating effects of classroom instruction and to establish that the acquisition of rationality does not require schooling.

Since we are critical of some of the claims that are made on the basis of studies of everyday mathematics, we emphasise that we too oppose the identified intellectual targets (cognitivism and formalism) of these studies. However, we are less than sanguine about the way the opposition to these theoretical positions *is realised through the case studies*. We wonder whether the interpretations of studies of everyday/street mathematics exhibit an overreaction against the theories that they want to attack, leading them to assert the direct opposite to them. However, if the to-

be-opposed theory is simplistic and naïve, then its direct opposite will be correspondingly simplistic and naïve. We therefore argue that the importance of studies of everyday mathematics is limited because their interpretive function is largely restricted to flat denials of very simplistic theories – where it is often not clear whether the opponents really hold such simple views. As Pea (1991, p. 484), reviewing Lave (1988), observed:

Nevertheless, other arguments may, to many readers, border on caricatures of the disciplines under critique, as when cognitive psychology is designated as ‘functionalist’ in its core orientation to mind as a ‘self-perpetuating, closed, input-output system’.

If one does not start with a view that only doing mathematics counts as thinking and that the only way that mathematics can be learned is through formal schooling, then many of the findings of these studies become not wrong, but uncontroversial and commonplace. These studies did record that:

- the calculations in everyday situations are not undertaken for their own sake but in service of practical activities;
- the calculations that people encounter in many everyday situations typically involve small and ‘round’ numbers;
- the most usual arithmetic practices are addition, subtraction, and multiplication of ‘simple’ numbers (e.g., doubling or trebling);
- where calculation is called for in everyday situations, it does not take the form of the (school-taught) pencil-and-paper application of a place-holding algorithm;

- instead, most calculation involves the use of ‘mental arithmetic’ (which features such procedures such as memorised results, rounding up, estimating, or approximating);
- when we compare schooled and unschooled children, we find that they perform equally on the simple problems that they encounter as part of their everyday lives – but that schooled children perform better at complicated problems (‘school problems’).

Without targeting a simplistic view of human rationality, none of these statements seem remotely controversial, stating facts familiar from commonplace experience in, for example, grocery shopping. As Carraher and Schliemann (2002, p. 133) recently observed:

In retrospect, the observation that mathematical learning occurs out of school may seem obvious. Indeed, one might wonder how anyone could ever have thought otherwise!

A fundamental question seems to be: Do studies of everyday and street mathematics point to the existence of *two different kinds of mathematics* (a formal one taught in school, its everyday, informal other having evolved autonomously and independently and being fundamentally different)? This seems to be the view expressed in titles such as “*Street Mathematics and School Mathematics*” (Nunes *et al.*, 1993; our emphasis) or “*Everyday and Academic Mathematics in the Classroom*” (Brenner and Moschkovich, 2002; our emphasis) and in quotations such as these:

[...] recent research, mainly carried on by anthropologists, shows evidence of practices which are typically mathematical, such as counting, ordering, sorting, measuring and weighing, done in *radically different ways* than

those which are commonly taught in the school system. (D'Ambrosio, 1985, p. 44; our emphasis)

[...] the *chasm* between school and everyday life. (Lave, 1992, p. 76; our emphasis)

[...] the *wide gap* that exists between mathematics practice in school and in out-of-school situations. (Masingila, 1993, p. 18; our emphasis)

Lave (1988) has shown how arithmetic activity in the real world does not reflect the formal procedures taught in the classroom. [...] *The gap* between this formally taught system and the strategies needed in the world outside *is wide*. (Aubrey, 1997, p. 57; our emphasis)

In our view the distinction between 'school' and 'everyday/street' *mathematics* is at best misleading, at worst, mistaken – and to speak of a radical or wide gap or chasm is hyperbole, rather than careful description. We will argue that rather than saying that there are two different kinds of mathematics, one could as well say that one and the same kind of mathematics is used differently in two different *situations* (it is subject to more 'relaxed' standards in many everyday situations, to 'stricter' ones in many school situations). Evidence for such a view can be found in a recent summary:

The goal of making everyday mathematical activities a part of classroom practices can be accomplished in many different ways. It can mean bringing into the classroom (a) the *objects* of study of everyday mathematics, such as checkbooks to be balanced or architectural problems to be solved; (b) aspects of the *social organisation* of nonschool settings, such as collaboration; (c) the *discursive practices* of a work setting, such as peer reviews and evaluations; or (d) the *participation structures* from everyday activities, such as conversation, discussion, and debate. (Brenner and Moschkovich, 2002, p. vi; emphasis in original)

The new aspects of ‘everyday mathematics’ that are emphasised here are not in the mathematical forms of computation or their results, but in the *context or purpose* of the overall activity. The only mathematical difference that we could find evidenced in studies of everyday mathematics is that between doing calculations using (school-taught) ‘place-holding algorithms written out with pen-and-paper’ and what we have called ‘mental arithmetic’.

Recently there have been steps towards a more subtle view of the relationship between street and school mathematics. For example, Carraher and Schliemann (2002, p. 134) do not speak of two different *kinds* of arithmetic, but point to “the *subtle* differences between arithmetic algorithms in and out of school” (our emphasis). With this paper we hope to contribute further to such a more nuanced view, which we think will result in a better understanding of what can be learned from studies of everyday and street mathematics.

Case studies of ‘everyday’ mathematics

We now look closely at four of Lave’s studies, offering an alternative characterisation of the materials, especially with respect to the implications the studies carry. We argue that Lave assembles excessively stark contrasts from relatively small and heterogeneous differences and that alternative understandings of the examples are readily available.

Tailoring

Lave’s early study of the use of arithmetic by Liberian tailors (Lave 1977a,b,c, 1980, 1982a,b, 1990; Reed and Lave, 1979) is in many ways a typical example of studies of arithmetic in other cultures (e.g., Gay and Cole, 1967). Lave’s study records two

things that cannot be challenged: firstly, that *some* arithmetic competence can be learned without schooling; secondly, those who attend school learn some arithmetic competences that are not so easily picked up in out-of-school situations.

Lave studied sixty-three tailors who were given arithmetic problems, half involving numbers and problems actually encountered in their daily work ('tailoring problems'), half involving problems of equal difficulty that were rarely encountered in their work ('nontailoring problems'). According to Lave, tailors with no schooling solved on average 91% of the former and 70% of the latter, while tailors with 5-10 years of schooling averaged 95% and 91% respectively (Lave, 1977a, p. 179). The message of this was clear: don't presume that all arithmetic procedures used in everyday life originate in school; some arithmetic procedures can be picked up outside schools.

However, another aspect, less emphasized by Lave, is that there *are* arithmetic problems ('nontailoring problems'), where it does make a difference whether someone has attended school or not (91% versus 70% success rate). What this suggests is that only certain *kinds* of arithmetic procedures are picked up in tailoring. This is confirmed through Lave's ethnographic observations, where she specifies the *limited range* of arithmetic procedures employed in tailoring:

Through observation and participation in the activities of the tailor shop, we found that tailors must be able to read off inches on the tape measure; measure various lengths; and quarter, halve, double, and quadruple measurements as they cut out the four main pieces of a pair of trousers before assembling them into a finished product. Multiplying and dividing by 2 and 4 are therefore by far the most common operations. (Reed and Lave, p. 581, endnote 3)

In other words, tailors do not encounter a wide range of arithmetic operations, but only a small range of them.

Dieting

After studying the use of arithmetic in another culture, the Adult Math Project (AMP) investigated arithmetic practices in American culture, for example, as part of dieting (de la Rocha, 1985, 1986; Lave, 1986a,b, 1988, 1990; Lave, de la Rocha, Faust, Murtaugh, and Migalski, 1982; Lave, Murtaugh, and de la Rocha, 1984). This study demonstrated that people can be inventive in developing purpose-built arithmetic applications for activities that interest them. It also showed, like the tailoring study, that the kinds of problems encountered are of a narrow range. However, in the discussion of these findings, we also find the more problematic tendency to exaggerate differences in order to create stark contrasts (e.g., between ‘formal’ and ‘informal’, or between ‘quantitative’ and ‘non-quantitative’).

Lave’s collaborator de la Rocha investigated Weight Watcher dieters’ use of arithmetic (i.e., their calculations and measurements). The calculations necessary for dieting “were very simple, arithmetically speaking” (de la Rocha, 1986, p. 17) and dieters employed a variety of ‘calculating devices’:

They measure a precise amount of food and transfer the measured amount to a glass or bowl. Once the measured food is in its container, they note its position relative to some feature of the container such as a decorative pattern. By using the same glass or bowl over and over, and always filling it to the same position, they eliminate the need for continual measuring. (de la Rocha, 1985, p. 194)

Like the tailoring study, this provided a basis for a critique of simplistic accounts of mathematics: if you think that there is only one way of measuring in the world, think

again! People are incredibly inventive in devising purpose-specific solutions to their arithmetic problems. Dieters do not employ ‘universal’ (manufactured) measuring devices for every measurement they have to do, and they try to avoid repeated measurements by ‘storing’ the appropriate amounts in purpose-specific devices. These rather uncontroversial observations are, however, seen to point to two different kinds of procedures, ‘universal’ or ‘formal’ ones (that are presumably taught in school) and ‘everyday’ or ‘informal’ ones:

[...] many problems that might have been solved by quantitative means were solved in non-quantitative ways that accomplished the same end. For example, an old cracked coffee cup became ‘my rice cup’ and replaced the standard measuring cup in the preparation of rice, a circumstance leading to the disappearance of numbers from the preparation process. (de la Rocha, 1986, p. 18)

In our view the contrast between ‘quantitative’ and ‘non-quantitative’ means is exaggerated and possibly misleading, since it seems to mischaracterise the measuring done through ‘my rice cup’. We would argue that using ‘my rice cup’ to measure rice is still ‘quantitative’ measuring, since it measures the same *quantity* as the standard measuring cup (it replaces the measuring device, not the system of measurement). Obviously, there is a big difference between ‘measuring’ and ‘estimating’ (rather than using any measuring device, a dieter may just estimate how much 100g of rice might be), but this is not the same as the difference between a ‘universal’ and ‘specific’ measuring device (i.e., between a manufactured measuring cup and ‘my rice cup’). Estimating could be said to lead “to the disappearance of numbers from the preparation process”, but the use of ‘my rice cup’ does not, since it retains the knowledge that ‘my rice cup’ *numbers* the correct quantity of rice. If ‘my rice cup’ holds 100g and a recipe requires 200g of rice, the dieter need not resort to the

standard measuring device, but can simply figure that the recipe calls for two ‘rice cups’. The initial measuring operation is presupposed in the use of the rice cup and the absence of further mention of numbers of grams (replaced by, possibly, multiples of rice cups) does not ‘eradicate numbers’ from the process. Similarly, when dieters in the supermarket shop for items in a size that equals the Weight Watchers serving size (de la Rocha, 1985, p. 197), this is not a ‘non-quantitative’ but a ‘quantitative’ method.

In sum, de la Rocha’s study demonstrates that dieters have ways of customising their measurements to make them more efficient or convenient (e.g., avoiding the need to make repeated measurements by ‘storing’ the first measurement in a specific device). However, these considerations do not point to a contrast in terms of ‘quantitative’ versus ‘non-quantitative’ methods (which, in our view, stands for formal/school versus informal/everyday). In other words, the ‘informal’, ‘everyday’ methods that dieters came up with do not stand in any way in conflict or contradiction with ‘generic’, ‘universal’ ways of measuring. The study exhibits that these different ways of measuring measure standard amounts and that dieters are aware of their equivalence (using first a ‘universal’ measuring device, subsequently replaced by ‘my rice cup’).

Money-management

The tendency to create stark contrasts from small differences is also illustrated by the study of money-management in households conducted by Katherine Faust (reported in, e.g., Lave, 1988; Lave *et al.*, 1982), which described some ways in which money is handled in families, in particular, the variety of ‘stashers’ people create to handle their money:

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Money [...] is brought into the family as income, held temporarily by various compartmentalized means ('stashers'), and finally used to meet expenditures. The flow of money occurs in varied media of exchange, including cash, checks, and credit cards [...]. (Lave, 1988, p. 131)

Just as dieters' use of measuring devices is tied to recurrent practical problems, so household income is organised in terms of specific purposes (day-to-day-expenses, 'treats', savings for children's future education, etc.). These commonplace observations are used to create yet another contrast, between 'universal' and 'non-universal' uses of money. Thus, on the one hand, Lave (1988, p. 131) argues that "Western cultures have a universal monetary system and medium of exchange that in principle provides a universal standard". On the other hand, she finds that:

Incoming funds shaped into stashes and by media for transactions, are used to create, in practice, special purpose monies. It will be argued here that they are used to create categories that may not be treated as equivalent, and that these prohibitions have the same moral character as those surrounding special purposes monies in other societies. Participants in the AMP gave the impression that a universal standard of value and medium of exchange was *not* an advantage [...]. (pp. 131-132; emphasis in original)

It seems that Lave treats the 'special purpose monies' as a counter-example to the supposed universality of money. In other words, Lave seems to think that budgeting conflicts with the 'universal' role of money. However, Lave gives a rather misleading picture of what the universality of money in Western culture involves. If we understand by this the requirement that money is, within the currency domain, a universal medium of exchange, then we do not see that special purpose monies in any way detracts from the universality of money, but in fact presupposes it. In other words, there is no evidence that people suppose that money is non-interchangeable

between one stash and another (i.e., that one cannot borrow a ten dollar bill from the 'mad money' stash to add to the day-to-day expenses stash). Of course, monetarily equivalent amounts can have very different significances: ten dollars for a six year old is different from ten dollars to a thirty-year old investment banker; four hundred dollars to pay the mortgage is different from four hundred dollars spent on a lunch. The differences that Lave describes are not in numerical value of the currency or its purchasing power, but in respect of the relativity of terms like 'large' or 'expensive' to the financial circumstances (specified in terms of the usual numerical and value currency equivalents) of the individual. Ten dollars is a large amount to a small child, but not for an investment banker. A four hundred dollar lunch may be expensive, while a four hundred dollar monthly mortgage might be small. How we judge the meaning or practical utility of ten dollars depends on the situation.

Lave seems to be contrasting two different views of money, the 'universalist' (formal) and the 'non-universalist' (informal), attributing the former view to theorists and the latter to 'just plain folks'. In contrast, we would argue that the 'universality' of money is presupposed by all participants (since having accounts dedicated to specific purposes does not deprive the money thus stored of its standard exchange function).

What we have been trying to show is that studies of everyday mathematics (not just Lave's) tend to create strong contrasts, but often do not take the time to spell out the exact nature of the implied contrast. In this case, Lave argues that her study critiques the idea of money as a universal standard, but this is only so in a very limited (and rather misleading) sense: people's organisation of money is not 'universal' (i.e., equal across different people and situations), but nothing in Lave's study needs to be seen as counterevidence to what economists consider the universal exchangeability of money.

Supermarket shopping

The most famous study of the Adult Math Project, an analysis of the arithmetic procedures used as part of grocery shopping, was conducted by Michael Murtaugh (Lave 1982b, 1986a,b, 1988; Lave *et al.*, 1982, 1984; Murtaugh 1984, 1985a,b). This study mirrors the results from the other studies: the range of arithmetic problems encountered as part of grocery shopping is limited, but shoppers are successful at performing the calculations that *do* occur. Our aim, again, is to show how the ethnographic project is used to create a misleading contrast between ‘everyday’ and ‘school’ arithmetic.

Twenty-five adult grocery shoppers were accompanied on shopping trips. Since one tape was lost (Murtaugh, 1984, p.245), the data reported in the study concerns 24 shoppers¹. This is the summary:

Of 803 grocery items purchased by 24 shoppers, 312 involved explicit problem solving through consideration of alternative brands and sizes. Of these 312 cases, 125 involved some arithmetic calculation; this represented about 16% of the total items purchased. Most of these cases – 77 in all – involved price comparisons among different brands within the same product-class. [...] In only four of the 77 cases did the shopper proceed to select the more expensive alternative. (pp. 81-82)

Perhaps the most important thing to note is that almost two thirds of the items purchased in the supermarket were bought ‘without thinking’ / ‘automatically’ and

¹ Unfortunately, the reported numbers are not consistent throughout the reviewed articles. That is to say, there are differences between, for example, Lave *et al.* (1984), Lave (1988), and Murtaugh (1985a), which makes a clear understanding of what the study actually shows slightly problematic.

only a very small number of items purchased (16%) involved any arithmetic ‘calculation’. In a sense, then, these findings demonstrate that there is *very little arithmetic* in grocery shopping and that most of it is routine. This impression is enforced if we consult the table of “frequencies of arithmetic usage by grocery shoppers” (Murtaugh, 1985b, p. 187):

<u>Price Comparisons</u>	<u>180</u>
simple comparison	102
with coupon	10
quantity comparison	3
‘best buy’ calculation	65
 <u>Nonprice Comparisons</u>	 <u>33</u>
number of items	11
other	22

We would argue that comparing the price of two items of equal size in order to buy the cheaper one (‘simple comparison’) is at best a borderline case of ‘calculation’. The same applies to comparing the unit price displayed below items of different sizes, which was done in 16 of the 65 ‘best buy’ cases (Murtaugh, 1985b, p. 189). Although these comparisons involve numbers, they do not involve ‘calculation’ (unless seeing that one number is bigger than another is counted as a calculation). Thus as far as we can tell from these numbers, it seems that only 49 out of the 803 cases (6%) involved what might typically be called a ‘calculation’.

This important fact is seldom emphasized. That is to say, Lave summarises these observations as pointing to “a startling 98% – virtually error free – arithmetic in the

supermarket” (Lave *et al.*, 1984, p. 82²), a statement frequently repeated in the literature (e.g., Aubrey, 1997, p. 57; Harris, 1997, p. 199; Pozzi, Noss, and Hoyles, 1998, p. 106; Hoyles *et al.*, 2001, p. 7; Civil, 2002, p. 43; Mousavi and Garrison, 2003, p. 148) – without emphasising that this applies to *only 49 cases of mostly very basic calculation*³. We would argue that a more accurate summary would be to say that it demonstrated the complete absence of anything but elementary arithmetic from the supermarket.

We do not deny that the supermarket study could be used to correct claims that argue that schooling is an indispensable prerequisite for successful shopping. However, we wonder whether the shoppers’ achievements are exaggerated (since the low frequency and limited range of the encountered problems is rarely stressed) and used to force a contrast between shoppers’ ‘informal’ and schools’ ‘formal’ methods. This can be seen in the way that the success rate of shoppers in the supermarket is compared with that on a pen-and-paper test:

We compared shoppers’ arithmetic in the supermarket with their performance on an extensive paper-and-pencil arithmetic test, covering integer, decimal, and fraction arithmetic, and using addition, subtraction, multiplication, and division operations, based on a test from the Torque

² We are unsure how Lave *et al.* arrive at this number (98%). 73 out of 77 ‘best buy’ calculations would be 95%, while 121 out of 125 arithmetic calculations would be 97%.

³ Thus we would argue that the following summary is seriously misleading, since it suggests that the shoppers performed several hundred calculations (rather than only 49 of them, which they solved correctly): “In several hundred grocery items purchased under observation by the Adult Math Project, 98% of the calculations were correct.” (Lave, 1986b, 147)

Project at MIT. The twenty-five shoppers varied in the amount of their schooling and in the time since their schooling was completed. Their scores averaged 59% on the arithmetic test, compared with a startling 98% – virtually error free – arithmetic in the supermarket. (Lave *et al.*, 1984, p. 82)

This is perhaps the most famous ‘finding’ of Lave, again frequently cited in the literature (e.g., Roth, 1992, p. 629; Devlin, 1999; Belzen, 2000, p. 9; de Abreu, 2002, p. 331; Sato, 2004, p. 32; Orton, 2004, p. 122). However, we think that the success of everyday or street mathematics is overstated, since it is by no means clear that the 59% and 98% success rates are *comparable*.

We have already pointed to the low frequency of actual cases of calculation in the supermarket. Furthermore, the kinds of calculations performed are not usually very complex. That is to say, the 49 cases of proper ‘best buy’ calculation, involved predominantly either (a) ‘doubling’ or ‘trebling’ (Murtaugh, 1985b, p. 190) or (b) ‘differing’, which consisted of

calculat[ing] the difference between items on one or both dimensions and compare the results. For example, a shopper considered two rolls of paper towels, one costing 82 cents, the other 79 cents. The shopper noted the number of sheets in each roll, 119 versus 104, and proceeded to reformulate the problem, saying ‘that would be three cents more and you get 11 more, 15 more sheets.’ She concluded that the larger roll was ‘probably better, a better buy’. (Murtaugh, 1985b, pp. 190-191)

The 98% success rate thus reflects the fact that the shoppers can successfully employ the two and three times table and are able to add and subtract small numbers. While this shows that shoppers’ arithmetic competence is sufficient for shopping purposes, it is also clear that schools aim to teach procedures that can be used to solve a much wider range of problems, including more complicated ones. Thus the paper-and-

pencil test included more complex arithmetic problems, e.g., $975 + 987 + 956$ or 437×305 (Lave, 1988, p. 74), which are not problems encountered as part of grocery shopping sums and which are more difficult to solve with mental arithmetic. Given that one success rate applies to a rather limited range of small number calculations, and the other to more complicated calculations, is it surprising that they are different?

The different nature of an arithmetic ‘problem’ in the supermarket and an arithmetic problem in a school exam can further be demonstrated by the fact that shoppers abandon complicated calculations in the supermarket and use estimates (rather than precise results) – both of which are not acceptable in a school test. If a calculation gets complicated, shoppers just give it up:

In supermarket arithmetic an alternative to arithmetic problem-solving is abandonment of the arithmetic and resolution of snags through exercise of other options. Abandonment of a calculation may occur when it becomes too complicated for solution within grocery-shopping activity in the supermarket setting. (Lave *et al.*, 1984, p. 90)

For example, one shopper compared one type of barbecue source of 18 ounces for \$0.89 with another of 23 ounces for \$1.17 (which is quite a complicated calculation, not at all easy to do in one’s head). However, she did not perform the calculation, but simply abandoned it (Lave *et al.*, 1984, p. 91). We do not wish to criticise shoppers for abandoning calculations (why would we?), but want to point out that on a paper-and-pencil test abandoning a problem would count as a ‘failure’.

Similarly, Lave observes that shoppers are not interested in ‘precise’ calculations but deal with estimates. In particular, they may only be interested whether item *A* offers a better deal than item *B*, but not necessarily try to calculate the exact difference. Thus for many purposes 59 cents becomes 60 cents and one dollar 12 cents could count as

one dollar (Lave *et al.*, 1984, p. 87). Again, there is nothing to be disapproved of in terms of practical activities, nor need school mathematics entail that these ‘convenience’ measures are in any way improper in practical decision making, but offering these as results in a paper-and-pencil test would constitute a mistake (or at least a misunderstanding of the kind of operations the test was calling for). As Murtaugh (1985a, p. 193) himself observes that “[n]ot all of them would be recognized as acceptable solutions on a school arithmetic test”⁴.

In sum, the supermarket study documented the kinds of arithmetic problems encountered as part of shopping and demonstrated that shoppers are able to solve them with sufficient success and precision for their practical purposes (not necessarily to a school-test standard of numerical precision). It also reminded us that most arithmetic calculations in everyday situations are done using mental arithmetic, rather than through pen-and-paper place-holding algorithms. However, these observations are subsequently exaggerated, since it is suggested that everyday mathematics (with a success rate of 98%) is therefore superior to school mathematics (with a success rate of 59%), whereas we have tried to show that these success rates are not like-for-like. Lave’s polemical aim is, presumably, to establish that practical activities (such as

⁴ In fact, one could argue that a different ‘success rate’ of supermarket arithmetic is buried in the descriptions. For example, Murtaugh (1985a, p. 193) provides us with a table of “ways of handling ‘best buy’ problems”. Of the 61 cases, 16 would not appear in a school test (since they were solved by ‘read unit price labels’); 15 of them would be marked as a mistake (‘ignore size difference’ and ‘indicate inputs only’); out of the remaining 30 cases, 4 were miscalculated, leaving us with a success rate of 57% – even worse than the 59% success rate on the test from the Torque Project (we are simply trying to point to the difficulty of arriving at, and interpreting, the reported numbers).

shopping) are a better environment for the expression of individual competences than the (artificial?) setting of the classroom, but this could only be inferred from the differential success rates – were these accepted – at the price of overlooking that the main difference between the two environments is the criteria of success (not the relations between numbers).

The contrast with ‘school’ mathematics

The pivotal role of the case studies of everyday mathematics is to provide a contrast with school mathematics teaching. However, any genuine comparison between everyday applications of arithmetic and school arithmetic will be complicated, since the aims and purposes of teaching mathematics in schools are multiple and complex, and the instructional import of any moment of mathematical activity may have only an indirect relationship to ‘everyday problem solving’ of the sort found in everyday activities such as dieting or shopping. We have thus been struck by how little discussion about the variety of *aims* of school mathematics we could find in Lave’s papers. We found one place in which Lave acknowledges that studies of everyday arithmetic pertain only to *one* of the many aims of schools:

[...] it should be clear that these data speak to only one goal of school arithmetic instruction – the teaching of skills used in everyday life in this society today. Goals of preparing future engineers and physicists are not herein addressed; nor are the goals of teaching arithmetic structures for their own sake, both of which are strongly held functions of public education. (Lave, 1982b, p. 6)

We think that this restriction of the possible implications of studies of everyday mathematics is subsequently disregarded. Instead, it seems that there is an implicit assumption that the most important aim and prevailing business of school

mathematics is to teach skills used in basic practices of everyday life. That is, the ‘success’ of school mathematics is only evaluated by looking at whether school-taught algorithms are used as part of everyday activities such as dieting or shopping. The absence of the use of these school-taught procedures is seen as evidence for the ‘failure’ of school mathematics. However, as Dienes (1960, p. 21) observed many years ago: “If the requirements of everyday life determined the contents of our mathematics syllabuses there would surely be little mathematics in them.”

A serious evaluation of the ‘success’ or ‘failure’ of school mathematics would address complicated questions about the variety of different (and possibly even conflicting) aims of school mathematics. Rather than addressing these, Lave argues (without any evidence) that the main aim of school mathematics is to *replace* ‘everyday’ with ‘school’ methods:

There is a very general first assumption that it is the responsibility of schooling to replace the (presumably) faulty and inefficient mathematical knowledge acquired by people in the real world. This project is viewed as especially urgent in a highly technical world. (Lave, Smith, and Butler, 1989, p. 67)

Lave seems to suggest that schools not only teach certain algorithms, but that they also transmit the view (or ideology?) that these algorithms are the *only* permissible procedure in *all* circumstances. In other words, Lave sees schools as teaching a prohibition of ‘informal’ or ‘everyday’ procedures throughout all areas of life. This view of schools seems to be not uncommon, for example, both Taylor (1991, p. 110) and Masingila (1994, p. 430) cite Lave approvingly.

We find this view rather implausible. However, we can get a better understanding of what Lave might have in mind by looking at perhaps the only detailed description of

“math practice in school classrooms” that we could find in her corpus (Lave, 1990, pp. 320-323; reproduced in Lave, 1992, pp. 81-84). This study (conducted by Michael Hass) analysed a three-week unit of multiplication and division in a bilingual Spanish/English third-grade classroom and focused “on a group of 11 children, the ‘upper’ math group” (p. 320). Lave provides the following summary:

[...] in the three-week period the children were deeply engaged in math work during individual work time (about 75% of class time), but invested minimal attention and involvement in ongoing activity during the teacher’s instruction sessions (about 25% of class time). During the three weeks the children gave no evidence of having adopted *any* of the specific strategies demonstrated by the teacher during periods of general instruction. (p. 321; emphasis in original)

For Lave, the study showed a clear contrast between methods and strategies taught by the teacher and those used by children, i.e., between ‘formal’ methods (that schools aim to teach) and ‘informal’ methods (that schools supposedly aim to replace). However, note that these methods again differ not in the *results* they produce, but in the *ways* of arriving at these results. In other words, pupils were able to solve the problems given to them by the teacher, since “[e]ach of the 11 turned in nearly errorless daily practice assignments” (p. 321). According to Lave the children did not use the teacher’s methods to *solve* the problems, but instead used “problem solving methods they invented or brought with them to the classroom” (p. 321). The only example of a ‘pupil’ procedure given by Lave is this:

They discovered that the multiplication table printed in their book could be used to solve division problems, an opportunity for mathematical discussion of which the teacher was unaware. (p. 321)

We might wonder in which sense this ‘pupil’ method qualifies as a radically different (or even ‘conflicting’) method from that taught by the teacher. In our view, even the most minimal understanding of multiplication and division involves understanding that each operation is the inverse of the other, and it is not this that the children presumably discovered entirely autonomously. What the children perhaps discovered, and what the teacher was perhaps unaware of, was that there was a multiplication table in the textbook that they could use. In other words, the children discovered that for these problems using the multiplication table in the book was an easier method than using the method proposed by the teacher.

Since the children’s ‘discovered’ method and the (unspecified, but unused) teacher’s method are mathematically equivalent, we wonder whether Lave generally neglects the fact that mathematics frequently provides a *plurality* of *equivalent* procedures for solving any problem. For example, shoppers in supermarkets may not use certain paper-and-pencil place-holding arithmetic procedures, but this does not necessarily imply that they do not calculate in the way that they have been taught in school, since an understanding of the *equivalence* of many arithmetic procedures is an integral part of what they have been taught (see also Lynch, 1995, p. 599, endnote 20). Using a multiplication table ‘in reverse’ is no ‘outlaw’ mathematical technique, but a perfectly sound one. There is certainly nothing mathematically invalid about using a multiplication table to solve division problems. Of course, this technique may be temporarily prohibited in a classroom exercise, for example for *educational* purposes if the teacher wants pupils to move from familiar techniques to unfamiliar ones.

Lave seems to give little attention to what teachers are specifically trying to do when they teach a new method. As already mentioned, according to Lave schools are trying to ‘replace’ (informal) children’s methods by (formal) teachers’ methods:

[It] is a widespread assumption that it is the responsibility of schooling to *replace* the (presumably) faulty and inefficient mathematical knowledge acquired by people in the real world. (p. 318; emphasis in original)

However, if one gives up the notion that there can only be *one* method, then one can see that schools are not necessarily trying to ‘replace’ existing methods (if they produce mathematically correct results, e.g., addition using fingers), but rather are trying to *expand the range* of methods. Lave’s case may actually represent a readily recognisable and commonplace teaching situation, one in which teachers face the problem of leveraging the learning of a new technique, which at first may be more difficult and cumbersome than methods already learned. Therefore the pupils may find it preferable to use existing methods rather than struggle with the new one. It may be for that reason that already learned methods are temporarily prohibited (without thereby implying that the previous methods are not mathematically sound).

One might make an analogy with practice sessions of a basketball team. The coach might, as an educational exercise, prohibit players from using their strong hand to dribble in order to improve dribbling with the weaker hand. Alternatively, the coach might, in order to improve passing the ball and to teach players to be in a position to receive a pass, have a training match where no dribbling is allowed. In both of these cases, players know that dribbling-with-your-weak-hand and playing-without-dribbling are not techniques that are supposed to replace their existing ones, but are ‘training’ techniques (since players would not play a ‘real’ match without dribbling).

Lave seems to be puzzled by the fact that neither dieters nor shoppers, who have all been to school and therefore have supposedly learned ‘school algorithms’, use these algorithms to solve the arithmetic problems they encounter in their everyday lives. Why is this surprising? Because she further assumes that “[s]chool algorithms should be more powerful and accurate than quick, informal procedures (that’s why they are taught in school)” (Lave, 1988, p. 57). However, this seems to be a mischaracterisation of school algorithms, which – on this we agree with Lave – are ‘over-powerful’ for many practical affairs.

We would argue that school algorithms are not taught in order to improve the arithmetic calculations that pupils can already perform (e.g., as part of playing cards or checking the change in a shop), but to expand their computational resources (which, incidentally, implies that not every pupil will actually use these new resources outside school – but then a lot of what is taught in school is potentially ‘unsuccessful’ in that sense). Place-holding algorithms are not more ‘powerful’ in every situation for every problem, but rather are more powerful in the range of problems that they can be used for. Try to calculate in your head how much money you will have in five years if you save 50 pounds every month at an interest rate of 4.6 per cent (not forgetting compound interest). Paper-and-pencil algorithms are more powerful in the sense that they can be applied to that kind of problem, and if one was looking to see whether people ever used such methods, supermarket shopping and diet portion preparation would not be the places to look for them.

In sum, it seems to us that Lave’s way of characterising the observed practices is misleading, since it may suggest that the way the teacher wanted children to solve the problem and the way that children actually solved them (or the school-taught place-

holding algorithm and the kind of mental arithmetic employed by shoppers) are in an important sense in conflict or contradiction (see Greiffenhagen and Sharrock, 2006, for a related discussion with respect to mathematical relativism). We have tried to show that rather than assuming that school mathematics aims to replace one arithmetic procedure with another, we could see school mathematics as teaching, *inter alia*, the equivalence of different methods (e.g., using pen-and-paper algorithms to check mental computations).

What is ‘real’ mathematics?

While we have argued that Lave’s findings do not point to the existence of two different kinds of mathematics (‘school’ and ‘everyday’), this leaves the possibility that schools may teach a different *attitude* towards mathematics than that acquired as part of everyday practices. Lave’s studies of everyday mathematics seem to follow in the footsteps of neo-Marxist critical theories (cf., Pea 1991, p. 488; Gourlay, 1999, p. 481; Contu and Willmott, 2003, p. 285; Henning, 2004, p. 153) in assuming that people may be ideologically confused about their own abilities and needs (here: their mathematical competences). In other words, it may be through the ‘hidden curriculum’ (Illich, 1971) of school mathematics that some children are socialised into compliance with socially subordinate statuses. Studies of everyday mathematics can then be seen as attempts to ‘enlighten’ everyday users of their own competences.

We base this view on Lave’s discussion of the shoppers’ self-characterisation of their use of arithmetic in the supermarket. Shoppers apologised for not doing ‘real math’ – which puzzles Lave, since it seems to contradict the fact that shoppers were extremely successful at ‘supermarket math’ (remember: “virtually error free”):

Researchers in the Adult Math Project discovered that *all* participants had poor opinions of their arithmetic practices in everyday settings. They apologized for not doing what they called ‘real maths’ – the math taught in school. This is especially interesting in the face of their extraordinary arithmetic efficacy in kitchen and supermarket. (Lave, 1985, p. 174; emphasis in original)

Lave seems to think that shoppers’ self-image results from school ideology placing undue emphasis on mathematics, thereby instilling a belief that people are unable to do mathematics:

Jpfs [just plain folks] who are not mathematicians appear to take away from school the belief that they don’t know and are unable to do ‘real mathematics’. They are unaware of their own abilities. (Lave *et al.*, 1989, p. 68)⁵

This claim is cited and propagated in the literature. For example, Millroy (1992, p. 3) cites it and reports that “the carpenters did not consider their work to have much to do with ‘real’ mathematics” (p. 158). Lave’s and Millroy’s argument seems to be something like this: schools place exclusive emphasis on one kind of arithmetic procedure (place-holding algorithms), which is thereby taken to define ‘real’ mathematics. Since many people have problems with this kind of procedure in school, they believe that they are unable to do mathematics – despite the fact that they *are* able to solve the arithmetic problems they encounter in everyday life. Lave sees in

⁵ See also Lave et al. (1982, p. 45), Lave (1986a, p. 104), Lave (1986b, p. 147), and Lave (1988, p. 183).

this a demonstration of a ‘false consciousness’: “They are unaware of their own abilities.”⁶

Lave finds further evidence for her view in the aforementioned study of “math practice in school classrooms”. Since for Lave there is a clear contrast between children’s ‘own’ methods (that they have discovered in order to solve the problem) and the teacher’s ‘official’ method (to which children have to conform), children experience the difference between ‘real’ and ‘other’ math already in school:

[...] by working out answers using their own techniques and then translating them into acceptable classroom form on their worksheets, the children generated a powerful categorical distinction for themselves between ‘real’ and ‘other’ math. It is not necessary to search beyond the classroom for the generation of this distinction. (Lave, 1990, p. 322)

Lave’s way of characterising the situation implies a strong opposition between children’s ‘own’ method and the teacher’s ‘official’ method. Since schools are seen to

⁶ There is an interesting tension on this point in Lave’s work. On the one hand, Lave wants to argue that ‘just plain folks’ are extremely efficacious in the conduct of their everyday lives; on the other hand, Lave treats them as incompetent in knowing what they are doing and often rejects the self-characterisations of ‘just plain folks’, since for Lave these only show that ‘just plain folks’ are victims of ideology and therefore unaware of the true extent of their own abilities. Ordinary people, for Lave, are smarter than her academic and professional opponents acknowledge (since they solve all kinds of everyday problems), but they are not smart enough to recognise for themselves what Lave’s studies attempt to show, namely, that they are smarter than *they* themselves believe. As Lave (1988, p. 191, endnote 1) puts it: “The term ‘just plain folks’ (jpf) will be used throughout the text. A double irony is intended: on the colonialist’s distance and condescension that plagues psychology only slightly more subtly than anthropology [...] *and on the belief of jpf that the rubric is appropriate*” (our emphasis).

aim to replace the former by the latter, it seems that children are alienated in schools: it is only the teacher's method that counts as 'real' mathematics, while the children's method is being dismissed.

We have already expressed our doubts whether schools really try to 'replace' one kind of procedure with another, as well as whether the children's methods are 'their own' in the strong sense Lave implies, but here want to give an alternative interpretation of why shoppers and carpenters apologize for not doing 'real' mathematics. Far from being an ideologically distorted underestimation of their own arithmetic abilities, we could see this as simply a way of saying that grocery shopping and carpentry involves very little mathematics (as demonstrated in Murtaugh's tables) and that the few calculations that occur are relatively simple in comparison to some of the arithmetic problems encountered in school. While Lave translates shoppers' denials that they do 'real' mathematics into a denial that they do *any* mathematics, the carpenters' response to Millroy's question gives us a different way of interpreting this denial:

I inquired, "Don't you do any maths in this workshop?" The response was quick. "Oh no! Maybe a bit of adding and multiplying, but that's not real maths!" and he walked off. (Millroy, 1992, p. 101)

In our view, the shoppers and carpenters are 'apologizing' to the researcher for the fact that the researcher will not be able to observe many instances of what they, the shoppers, imagine to be the 'real' mathematics the researcher is hoping to see. However, such a characterisation is not based on shoppers' or carpenters' conviction that they do *no* mathematics (which could be countered by showing that they *do* perform some mathematics), but rather on a distinction between 'pretty basic' perfunctory mathematics (that pretty much anyone can perform, often without going to school) and more 'difficult' and 'elaborate' mathematics (that is taught in later

schooling and everyone finds harder to master). Lave effectively portrays the shoppers as involved in an ideologically induced self-abasement (writing off their own manifest competence in mathematics), but it is equally plausible to allow shoppers a perfectly accurate understanding of their situation: the shoppers acknowledge that grocery shopping is – despite the role of money – only marginally a ‘mathematical’ operation and that the few calculations that occur are normally very basic and of the sort that pretty much anyone can do (involving only the most minimal mathematical ‘workings out’). The status ‘not real’ mathematics does therefore not imply the denial of *any* mathematical status to, for example, a bit of adding and multiplying, but rather locates it as marginal relative to heavy-duty practice (denying the status ‘real football’ to some idle kicking around of a ball does not deny that, in a basic sense, it can be called football).

As mentioned above, Lave’s aim seems to ‘enlighten’ shoppers:

We might relieve those in charge of mathematics problem-solving instruction in school of the burden of teaching mathematics as preparation for their everyday lives. In so doing we might relieve pupils of the belief that what they do in everyday situations is of no value. If they were able to recognize the inventiveness of their own practice, it might also help to mitigate the belief that only geniuses can discover and invent mathematics. (Lave *et al.*, 1989, pp. 68-69)

We think that Lave, despite her emphasis on studying the ‘academic hinterland’ (Lave, 1988, p. 3), and on complementing the intelligence of ‘just plains folks’, nonetheless treats them as ‘dopes’ (Garfinkel, 1967, p. 68). In her data we find no evidence that shoppers have a problem with their arithmetic ability *in the supermarket*. That people regard their grocery shopping as routine and the calculations that they do there as not real mathematics requires a considerable stretch

to be converted into the sweepingly generalised conviction that “what they do in everyday situations is of no value”.

We also think that it is misleading to characterise the shoppers or tailors as having ‘discovered’ or ‘invented’ new mathematics. It is one thing to counter the view that only people who are good at formal (school) mathematics are intelligent (i.e., to use mathematics as the sole indicator of rationality), but is quite another to argue that shoppers and carpenters are as good as professional mathematicians at *doing mathematics*. People can be said to be ingenious and inventive in finding practical solutions (‘my rice cup’) for the problems they encounter, but this does not mean that their solutions are *that* ingenious (unless you start off with the view that people are astonishingly dim) or constitute a contribution to mathematics. There is a world of difference between making variably inventive applications of basic standard systems of computation (such as elementary arithmetic) and developing new procedures of computation.

Conclusion

We have tried, using the exhibited materials taken from a series of classic studies of everyday mathematics, to raise some questions about the concept of ‘everyday mathematics’ and have tried to show why it is necessary to separate a variety of differences. ‘Everyday mathematics’ as it is found in Lave’s studies and, we would argue, in other comparable studies, appears to postulate two different kinds of reasoning, those of a ‘formal’ sort which are taught in school, and those of an ‘informal’ sort which grow up wild (to borrow from Levi-Strauss). We hold that this postulate is stretched, if not invalid, depending on a very narrow interpretation of ‘what is taught in school’ contrasting with a very liberal one of what is ‘developed

independently of school mathematics'. We do not deny the differences between the two situations, for example, between doing formal school tests (at one pole) and supermarket shopping or carpentry (at the other), but argue that the differences are precisely there: *in the situations*. The examples illustrate that the demands for computation in practical affairs such as supermarket shopping are few, limited and, often, inconsequential (e.g., that the sums involved are not economically very significant to the shoppers).

The school/everyday contrast imagines (it does not demonstrate) that the pencil-and-paper place-holding algorithm is taught in schools as if to absolutely prohibit all other practical decision making considerations (showing the danger of identifying teaching the solving of specific mathematical problems with teaching a particular method as *the* method of problem solving). It is at least possible to think that the place-holder method is taught as a *fall back* method, and one which, like other problem solving methods, can be used in a 'good sense' way, one which does not necessarily or automatically prioritise exacting numerical determinations over the situation's requirements for, e.g., a rough approximation. In sum, whilst acknowledging the existence of different procedures to solve arithmetic problems, we have questioned whether the differences between them are mathematical differences, whether the latter should really be seen as an autonomous system, and whether schools really teach the exclusive use of the former while everyday situations are the sole proprietors of the latter.

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