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SQUID-arrays coupled to on-chip integrated thin-film superconducting input coils operating coherently

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ABSTRACT

Recently, Superconducting Quantum Interference Device (SQUID) arrays operating in a coherent voltage-modulation state at 77 K showed a flux-noise 10 times lower than single-SQUIDs at similar temperatures. To exploit SQUID-arrays flux-noise superiority in applications, however, it is essential to preserve the coherent state while the magnetic field to be measured, B_z , is highly inhomogeneous along the array as being generated by thin film integrated superconducting input coils or flux-transformers located in close proximity. Indeed, the flux coupled to each individual SQUID may vary significantly along the array, leading to a rapid degradation in the coherency. Here we present several solutions to avoid that based on a methodology we developed to assess the efficiency of signal coupling to SQUID-arrays while maintaining a highly coherent state. As a proof of concept, we applied it to highly-integrated $\text{YBa}_2\text{Cu}_3\text{O}_7$ 800/770 SQUID-arrays inductively coupled to on-chip integrated thin film superconducting input coils. Each SQUID in the array is directly coupled to two individual flux focusers leading to an increase in the effective area for which we derived an analytical formula. Consequently, we achieved SQUID-like voltage oscillation amplitudes above 10 mV in the temperature range (75-83) K leading to a magnetic flux noise of $0.2 \mu\Phi_0/\text{Hz}^{1/2}$, consistent with an ultra-enhanced coherent operation reached. For the

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strongest coupling scheme implemented experimentally a current white noise of $S_I^{1/2}=32 \text{ pA/Hz}^{1/2}$ was measured. This scheme can be used as the input coil of a flux-transformer resulting in a SQUID-array-based magnetometer with an estimated field sensitivity of $13 \text{ fT/Hz}^{1/2}$.

The sensitivity of single-SQUID¹⁻³ based magnetometers/gradiometers can be improved by building a series SQUID-array of N non-interacting SQUIDs operating in a coherent voltage modulation state. Indeed, it has been predicted⁴⁻⁶ that in this case the maximum SQUID-like voltage oscillation amplitude ΔV_{max} linearly scales with N , their dynamic range increases as $N^{1/2}$, whereas white flux-noise $S_\Phi^{1/2}$ decreases as $1/N^{1/2}$. Consequently, the signal to noise ratio $\Delta V_{max}/S_\Phi^{1/2}$ improves as $N^{1/2}$. Much larger ΔV_{max} means their matching for room temperature readout is greatly simplified. Coherent SQUID-arrays also have an improved bandwidth and impedance matching as their impedance is N times larger relative to a single SQUID. Coherence in SQUID-arrays made of $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) operating at 77 K has been first achieved in relatively small arrays $N=30$ in the pioneering work⁷ and later in $N=10$ arrays connected to large area flux-focusers⁸. Earlier first attempts to operate large N (in the range 35-130) SQUID-arrays at 77 K did not show a coherent state throughout the entire array^{9,10}. Recently coherent operation has been achieved¹¹ in highly integrated large arrays ($N=484, 770$) at 77 K and above leading to an $1/N^{1/2}$ decrease for $S_\Phi^{1/2}$ to below values of optimized single-SQUIDs with similar microns-size junction width operating at 4.2 K. This result has two profound implications: first is the practicality of operation at 77 K; second is that, due to its fundamental significance, SQUID-array's flux noise superiority can potentially translate into an improved field, current or energy sensitivities. This is because SQUID operation is based on the flux-quantization in a superconducting ring¹ and

therefore SQUID's ultimate resolution as a sensor is fundamentally limited by the flux-noise $S_\Phi^{1/2}$ that determines to which extent the flux-quantization phenomenon is deteriorated¹². In¹¹ the SQUID-arrays were inductively coupled to a 3-dimensional (3D) multiturn copper coil located 10 mm above which is not practical in applications due to a very weak coupling. Indeed, in most applications¹³ B_z is applied via on-chip integrated 2D thin-film superconducting coils located in the near proximity, micrometres apart from the SQUIDs. Those signal coupling schemes/configurations¹³ were purposely developed for single-SQUIDs, which are small structures typically about 10 μm in size, and variations of B_z beyond this scale are irrelevant and have not been considered. However, B_z changes significantly on a scale of a few millimetres, generating a large gradient along SQUID-arrays, which are 2D structures distributed over several millimetres. Consequently, the flux Φ coupled to each individual SQUID may vary significantly along the array, potentially leading to a rapid degradation in the coherency. Here we address two fundamental questions of what impact a *large* gradient of B_z has on the degree of coherency in SQUID-arrays and how their response can be optimized. As a proof of concept of our approach/methodology two inductively signal coupling schemes implementing on-chip integrated superconducting thin-film input coils are considered theoretically and experimentally.

We consider a series array of N independent SQUIDs and implement the resistively-shunted-junction model¹⁴ assuming that all SQUIDs in the array have identical Josephson junctions (of negligible width W_{JJ} and characterized by identical Josephson critical currents I_c and normal resistances R_N) and inductances L , while the screening parameter $\beta_L = 2LI_c/\Phi_0$ is negligible (Φ_0 is the flux quantum). By generalizing the theory of single-SQUIDs¹⁴, the total voltage response V of the SQUID-array to a magnetic field to be measured applied perpendicular to its planar structure, B_z , is a sum of all individual SQUID voltage oscillations:

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$$V/\Delta V_{\text{SQUID}} = \sum_{j=1}^N \sqrt{\frac{I}{2I_c} - \left| \cos\left(\pi \frac{\Phi_j}{\Phi_0}\right) \right|^2} \quad (1)$$

Here $\Delta V_{\text{SQUID}} = I_c R_N$ is the amplitude of voltage oscillations of one SQUID, I , the dc bias current,

Fig.1. Layouts of the SQUID-arrays inductively coupled to on-chip integrated thin-film superconducting coils. Upper-left design: a 3-turns coil coupled to an 800 SQUID array placed inside the coil. Lower-left design: a 50-turns coil coupled via its outside edge to a 770 SQUID-array located outside the coil; Right: enlarged view of a small central part of the 800 SQUID array showing 9 SQUIDs. Each SQUID has two Josephson junctions (seen as narrow bridges crossing the bicrystal boundary) and is directly connected to two large area narrow flux focusers (highlighted in red) having width W_{ff} and length L_{ff} .

and Φ_j , the flux coupled to each individual SQUID, j , in the array. The effect on V of a (5-10) % spread in the values of I_c or R_N has been theoretically considered elsewhere¹⁵⁻¹⁷. The SQUID-arrays distributed along an axis parallel to x axis (see Fig.1) have been fabricated with a record level of integration of 100 SQUIDs/mm using standard bicrystal technology as described in¹¹. Each SQUID has two identical Josephson junctions connected in parallel in a rectangular loop configuration. The SQUIDs are connected in series in a serpentine path along the bicrystal grain boundary shown by the red dotted line. Two different designs (see Table 1) were fabricated and measured: an 800 SQUID-array and a 770-SQUID array. Each SQUID in the arrays is directly coupled to two long rectangular-shaped flux-focusers (ff) of length L_{ff} and width W_{ff} that

Table 1. SQUID-array fabrication and design parameters, electric transport measurements at $T_{\max}$ (where the modulation amplitude reaches its maximum $\Delta V = \Delta V_{\max}$).

SQUID-array	800 SQUID-arrays	770 SQUID-array
SrTiO ₃ bicrystals YBCO/Au films thickness deposition method	30° symmetric [001] tilt 200 nm YBCO/300 nm Au reactive co-evaporation	24° symmetric [001] tilt 100 nm YBCO/200 nm Au pulsed laser
Josephson junction's width, W_{JJ} SQUID holes (length, L_h , width, W_h) numerically calculated L_{SQUID} , W_{SQUID} calculated SQUID inductance, L flux-focusers dimensions: (L_{ff} , W_{ff} , s_{ff}) array length flux focusing factor $A_{eff,j}/A_{h,j}$	2 μm (8, 3) μm (19.7, 7) μm 21 pH (180, 17, 3) μm 8 mm 11.9	3 μm (13, 3) μm (34, 9) μm 30 pH (160, 21, 3) μm 9 mm 16.1
on-chip integrated input coil linewidth separation between turns rectangle dimensions calculated input coil inductance, L_i	3-turns rectangular spiral 100 μm 30 μm minimum: (L_{min}, W_{min})=(8.2, 5.2) mm 32 nH	50-turns rectangular spiral 10 μm 10 μm maximum: (L_{max}, W_{max})=(9.34, 3.24) mm 560 nH
$T_{\max}$	(81-83) K	(80-82) K
$\Delta V_{\max}$	10.1 mV	12.9 mV
I_c , R_N , β (average across the arrays)	44 μA , 1.1 Ω , 0.92	90 μA , 0.8 Ω , 2.6
R_{array}	0.44/0.22/0.11 k Ω	0.31 k Ω
flux noise, $S_{\Phi}^{1/2}$ current noise, $S_I^{1/2}$	Estimated*: 0.26 $\mu\Phi_0/\text{Hz}^{1/2}$ 5.8 nA /Hz ^{1/2}	Measured: 0.2 $\mu\Phi_0/\text{Hz}^{1/2}$ 32 pA /Hz ^{1/2}

* assuming a similar voltage white noise as measured for the 770 SQUID-array.

simultaneously ensure an increased field sensitivity and high levels of integration¹¹. There is a narrow slot of width s_{ff} between each pair of consecutive flux focusers strips. As for square-washer single-SQUIDS¹⁸⁻²⁰, for each individual SQUID in the array there are significant flux-focusing effects expected leading to an increase in their effective pickup area $A_{eff,j}$ relative to their respective rectangular hole area $A_{h,j}$ of length L_h and width W_h . The difference between the two cases is that now the square-washers flux focusers¹⁸⁻²⁰ are stretched out into long rectangles (strips) (see Fig. 1 and insets in Fig. 2). The main flux-focusing effect in SQUID-arrays can be understood within a model^{21,22} where two parallel thin film superconducting strips of width W separated by a slot of width s are placed in a perpendicular magnetic field B_z . In this geometry Meissner supercurrents concentrated near the edges of both strips (see red arrows in the left inset of Fig.2) will flow to screen the magnetic field B_z from the superconducting strips. As a result of the field expelled from the strips, the field in the slot B_{slot} is enhanced^{21,22}:

$$\frac{B_{slot}}{B_z} = \frac{\pi k}{2K(\sqrt{1 - 1/k^2})} \quad (2)$$

Here $k=(2W+s)/s$ and K is the complete elliptic integral of the first kind. In our designs (see Fig.1) each SQUID in the array is surrounded by two pairs of parallel superconducting strips separated by a slot. Pair (1,2) is located above the SQUID hole and pair (3,4) below (see right inset in Fig. 2). Consequently, the field in the two slots located in the proximity of each SQUID hole is enhanced by a factor B_{slot}/B_z given by Eq.(2) with $k=k_{ff}=(2W_{ff}+s_{ff})/s_{ff}$ calculated using flux-focuser dimensions. There is an additional flux focusing effect in SQUID-arrays that can be understood within a model^{18,23} where a thin film superconducting annular ring of outer and inner radii a and b is placed in a perpendicular magnetic field. In this geometry Meissner supercurrents concentrated near the hole and around the outside edge will flow to screen the magnetic field from the superconducting ring. As a result, the field inside the hole is enhanced^{18,19,23} by a factor

Fig.2. Increase in the normalized effective area $A_{eff,j}/A_{h,j}(k_{ff})$ given by Eq. (3). Left inset: the field is enhanced by B_{slot}/B_z in a slot between two parallel superconducting strips in a perpendicular magnetic field B_z . Right inset: in SQUID-arrays the field is enhanced by B_{slot}/B_z in the slot between each pair of flux-focuser strips (such as (1,2) or (3,4)); a further field enhancement B_{eff}/B_{slot} occurs inside the hole area of SQUID j , $A_{h,j}$.

approximately equal to a/b (see Fig. 9 in²³). We performed numerical simulations²⁴ based on the theory developed in²³ and found that for rectangular shaped loops (SQUIDS) the field enhancement factor B_{eff}/B_{slot} is equal to $(A_{SQUID,j}/A_{h,j})^{0.5}$. Here $A_{SQUID,j}$ is the SQUID area rectangle (marked by dotted line in the right inset of Fig. 2) having a width $W_{SQUID} = 2 \times W_{JJ} + W_h$ and a length L_{SQUID} determined from numerical simulations²⁴ (see Table 1). This result is similar to the case of square-washer SQUIDS^{18,19}. Combining the two flux-focusing effects it follows that the effective pickup area $A_{eff,j}$ of each SQUID, j , in the array increases as:

$$\frac{A_{eff,j}}{A_{h,j}} = \sqrt{\left(\frac{A_{SQUID,j}}{A_{h,j}}\right)} \frac{\pi k_{ff}}{2K\left(\sqrt{1 - 1/k_{ff}^2}\right)} \quad (3)$$

$A_{eff,j}/A_{h,j}(k_{ff})$ given by Eq.(3) with $(A_{SQUID,j}/A_{h,j})^{0.5}=2.8$ is plotted in Fig.2. Using data from Table 1 one finds $(k_{ff}, (A_{SQUID,j}/A_{h,j})^{0.5})=(12.3,2.4)$ and $(15,2.8)$ leading to an increase for $A_{eff,j}/A_{h,j}$ of 11.9

and 16.1 for the 800 and 770 SQUID arrays, respectively. Φ_j can now be calculated by integrating $B_z(x,y,z)$ over $A_{eff,j}$ area in the (x,y) plane:

$$\Phi_j = \iint_{A_{eff,j}} B_z(x,y,z) dA \quad (4)$$

A SQUID-array operates in a perfectly coherent voltage-modulation state when Φ_j given by Eq.(4) is constant along the array, ($\Phi_j = \Phi_{const}$ for all SQUIDs j). In this case the SQUIDs-arrays are at their best against single-SQUIDs with their voltage oscillation amplitude ΔV being N times larger in Eq. (1): $V/\Delta V_{SQUID} = N\sqrt{I/2I_c - |\cos(\pi\Phi_{const}/\Phi_0)|^2}$.

We consider two inductive signal coupling schemes implementing on-chip integrated spiral rectangular superconducting thin-film input coils of inductance L_i (see Fig. 1 and Table 1). First scheme is a 3-turns input coil coupled to an 800 SQUID-array placed inside. In this configuration the SQUIDs are located at *different* distances away from the coil with the first/last SQUIDs in the array being the closest at 50 μm . Second scheme is a 50-turns input coil strongly coupled via its outside edge to a 770 SQUID-array located outside the coil. Consequently, all SQUIDs in the array are located at the *same* distance from one edge of the coil with their centres at 180 μm and their upper flux focusers edges at 10 μm . For the 800 SQUID-array V was measured along the entire array or at intermediate contacts across smaller parts of the arrays in multiple sets of 200 SQUIDs. Accordingly, in the calculations, we varied N as well: $N=200, 400, 800$.

To calculate the z component of the magnetic field perpendicular to the SQUID-array plane B_z generated by the current I_B flowing through the input coils in the two coupling schemes considered we implement the Biot-Savart law for rectangular loops²⁵. One can approximate a rectangular M -turns spiral coil as superposed multiple M concentric rectangular loops with

increasing length L_m and width W_m . B_z can then be derived from fields superposition generated by loop currents I_B in each of the rectangular loops:

$$B_z(x, y, z) = \frac{\mu I_B}{4\pi} \sum_{m=1}^M \sum_{\alpha=1}^4 \frac{1}{r_\alpha} \left[\frac{(-1)^\alpha y_\alpha}{r_\alpha + (-1)^\alpha x_\alpha} + \frac{x_\alpha}{r_\alpha + y_\alpha} \right] \quad (5)$$

where $r_1 = \sqrt{(x + L_m/2)^2 + (y + W_m/2)^2 + z^2}$, $r_2 = \sqrt{(x - L_m/2)^2 + (y + W_m/2)^2 + z^2}$,

$r_3 = \sqrt{(x - L_m/2)^2 + (y - W_m/2)^2 + z^2}$, $r_4 = \sqrt{(x + L_m/2)^2 + (y - W_m/2)^2 + z^2}$,

$-x_1 = x_4 = x + L_m/2$, $x_2 = -x_3 = x - L_m/2$, $y_1 = y_2 = y + W_m/2$, and $y_3 = y_4 = y -$

$W_m/2$.

Here x , y , and z are the distances from the centre of the coils in the plane of the SQUID-array along both x and y axis, or in a direction perpendicular to the SQUID-array plane, respectively (see Figs.3a and 3b). Although in our experiments $z=0$ always, to understand how coherency changes with z , its value could be changed in the calculations. In Fig.3a $B_z(x, y, z)$ is plotted for the 3-turns coil for various z as approximated by Eq.(5) with $M=3$ rectangular loops with dimensions $(L_m, W_m) = (7.94 + m \times 0.26, 4.94 + m \times 0.26)$ mm and $m=1, 2, 3$. For $z=0$ $B_z(x, y=0, z=0)$ is highly inhomogeneous along the 800 SQUID-array increasing exponentially with x from about 5 at $x=0$ (middle of array) to about 40 at $x=4$ (end of array; for better clarity the full range of this curve is not shown in Fig.3a). For $z=0.5$ and 1 $B_z(x, y=0, z=0.5)$ has a maximum, while for $z=2, 3$, and 4 it decreases monotonically. In Fig.3b $B_z(x=0, y, z=0)$ is plotted for the 50-turns coil as approximated by Eq.(5) with $M=50$ rectangular loops with dimensions $(L_m, W_m) = (9.380 - m \times 0.04, 3.280 - m \times 0.04)$ mm and $m=1, 2, \dots, 50$. We found that $B_z(x, y=1.79, z=0)$ along the 770 SQUID-array axis is highly homogenous (1.79 mm is the y coordinate of the centre of all SQUIDs). However, $B_z(x=0, y, z=0)$ is highly inhomogeneous along every single SQUID unit (consisting of the SQUID itself and the two flux-focusers) decreasing exponentially with y (see Fig. 3b). By integrating $B_z(x, y, z)$ given by

Eq. (5) using Eq.(4), Φ_j can be calculated for each individual SQUID, j , in the array located at (x_j, y_j) . We have done this for both coupling schemes considered. Averaging the values of Φ_j over the entire arrays $\langle \Phi_j \rangle_{array}$ we found that the flux couples more efficiently into the 770 SQUID-

Fig.3. a) Normalized $B_z(x, y=0, z)$ along x -axis produced by a current I_B through the 3-turns coil calculated with Eq. (5) for various z (the distance between the coil and the SQUID-array plane) in the range (0, 4). b) Normalized $B_z(x=0, y, z=0)$ along y -axis produced by a current I_B through the 50-turns input coil calculated with Eq.(5). Inset shows (not to scale) the schematics of a flux-transformer consisting of a single turn, large area A_p pickup coil of inductance L_p and the 50-turn input coil of inductance L_i inductively coupled to the SQUID array.

array by a factor of $110 = \langle \Phi_j \rangle_{770} / \langle \Phi_j \rangle_{800}$. Φ_j values calculated above can finally be used in Eq.(1) to get the voltage response $V(I_B) / \Delta V_{SQUID}$ of the 800 SQUID-array (see Figs.4a, 4b). These calculations reveal several very interesting features as follow. The periodicity of $V / \Delta V_{SQUID}$ with I_B monotonically increases with z (due to B_z decreasing with z), translating into a decrease in the device's signal sensitivity with z (see Fig.4a). ΔV and its rate of decay with I_B strongly depends on z with an optimum response reached for $z_{opt} = 1.5$ mm as in this case $B_z(x, y, z)$ has the highest degree of homogeneity along the array. At $z=0$ although the periodicity with I_B is smallest (i.e., the sensitivity to I_B is highest), the degree of coherence is worst, leading to a rapid decay of ΔV with I_B . Fig. 4b shows the effect N has on the voltage response of the 800 SQUID-array coupled to the 3-turns coil when $z=0$. ΔV across the full array ($N=800$) decreases fast with increasing I_B and, at its best, is only about 300 times larger than that of a single-SQUID. This is far less than 800, the expected value for a perfectly coherent array. The voltage response across the middle 400 SQUIDS is very similar even though there are half as many SQUIDS. This is because the combined voltage response of the first 200 and the last 200 SQUIDS in the array is highly incoherent: those SQUIDS are distributed between $(-4, -2)$ mm and $(2, 4)$ mm in a direction parallel to x axis, where the inhomogeneity of B_z is largest (see Fig. 3a). Consequently, $V(I_B) / \Delta V_{SQUID}$ has small oscillation amplitudes that are strongly modulated with I_B while its periodicity with I_B is relatively smaller. Consequently, their contribution does not interfere constructively with the response of the middle 400 SQUIDS. In fact, the voltage response of the middle 200 SQUIDS is almost as good as that of the full array for a similar reason. Decreasing N further below 200 leads to an approximately linear decrease in the oscillation amplitude with N , as expected for coherent operation. Qualitatively similar results were obtained for non-coplanar coils ($z \neq 0$). Since $B_z(x=0, y, z)$ decreases

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Fig.4. $V(I_B)/\Delta V_{SQUID}$ calculated with Eq.(1) (using Eq. (4) for Φ_j and Eq. (5) for B_z) and $I=2I_c$ for: a) the 800 SQUID-array coupled to the 3-turns coil and various z (for clarity, the curves were shifted by multiples of 800); (b) the full array or various parts of it: middle 400 or middle 200 SQUIDs, as well as, the combined response of the first 200 & last 200 SQUIDs in the array (for clarity, the curves were shifted vertically); (c) the 770 SQUID-array coupled to the 50-turns coil. The red curve in (a) is for a variable area $A_{eff,j}$ 800 SQUID-array to compensate for the highly inhomogeneous B_z along the array. In this case a perfectly coherent operation is achieved with $\Delta V_{max}/\Delta V_{SQUID} = 800$.

monotonically with z , a single-SQUID would be most sensitive if located at $z=0$ where B_z is maximum. In contrast, a SQUID-array performs best if located not necessarily where B_z is maximum but where a trade-off between a maximum for B_z and a highest homogeneity for B_z along the array is reached. To avoid any degradation in coherency in case B_z is highly inhomogeneous along the array, one solution is to fabricate SQUID arrays with *variable* $A_{eff,j}$ defined by Eq.(3). One should, however, avoid variations in either $A_{SQUID,j}$ or $A_{h,j}$ along the array as this leads to variations of β_L , which are known to suppresses coherency¹¹. Instead one should vary the flux focuser's parameter $k_{ff}=(2W_{ff}+s_{ff})/s_{ff}$ along the array in accordance to Eq. (3) so that Φ_j , given by Eq.(4) (with $B_z(x,y,z)$ given by Eq.(5)), stays constant along the entire array. Thus, wider flux focusers or/and narrower slots are required in areas where B_z is small.

One can avoid dealing with a highly inhomogeneous B_z by implementing the other coupling scheme considered here. Indeed, one can place the SQUID-array outside a multi-turn rectangular spiral coil at $z=0$ in its proximity, with the array's axis parallel to one of the largest sides of the coil (see Fig.1). In this case, although B_z decays exponentially along the y -axis (see Fig. 3b), it is highly homogeneous along the array. As before, for each SQUID j located at (x_j,y_j) of the 770 SQUID-array we first use $B_z(x,y,z)$ given by Eq.(5) into Eq.(4) to calculate Φ_j and then use Eq.(1) to get $V(I_B)/\Delta V_{SQUID}$. Thus, amplitudes of SQUID-like voltage oscillations as large as 97% of its perfectly coherent state value of 770 are produced (see Fig. 4c). Their magnitude decreases only slightly with increasing I_B consistent with an ultra-enhanced coherent operation.

Families of dc current-voltage characteristics (IVC 's) were measured for the 800 SQUID array at various temperatures in the range (70-89) K and multiple I_B . From such families of IVC 's scanned over I_B , the dependence of the voltage modulation $V(I_B)$ for various bias currents I could be constructed. Typical sets of measurements are shown in Figs.5a and 5b with maximum

amplitudes ΔV_{max} of voltage oscillations around 10 mV. Remarkably, the amplitude of voltage oscillations of the middle 200 SQUIDS in the array ($\Delta V_{max}=10.1$ mV in Fig.5b) was not at all suppressed relative to the full array. In fact, the oscillation amplitude of $V(I_B)$ decreases only

Fig.5. (a) 800 SQUID-array: a set of 121 consecutive IVC's at 83 K for various B_z induced by I_B (applied via a 3-turns superconducting coil) changed in steps of 1 mA. Left/right insets: SQUID-like oscillations of $V(I_B)$ for 5 different positive/negative bias currents I in the range $(\pm 90, \pm 110)$ μ A. (b) 200 SQUID-array: $V(I_B)$ at 83 K for 5 different I in the range (90, 110) μ A. (c) 770 SQUID-array: $V(I_B)$ at 80 K with I_B (applied via a 50-turns superconducting coil) changed in steps of 5 μ A for 9 different I in the range (180, 220) μ A.

slightly with increasing I_B suggesting a superior coherency reached relative to the full array. In this case the ratio $\Delta V_{max}/N=10.1\text{mV}/200=50\text{ }\mu\text{V}/\text{SQUID}$ gets its largest value ever reported for SQUID-arrays operating above 77 K. In contrast, the oscillation amplitude of $V(I_B)$ of the first 200 SQUIDs and the last 200 SQUIDs in the array were strongly suppressed to below 1 mV. $\Delta V_{max}=9.7\text{ mV}$ was measured for the middle 400 SQUIDs in the array at 83 K, a value slightly larger relative to the full array. Qualitatively, this behaviour agrees well with our theoretical model (see Fig.4b).

Preliminary measurements of the 770 SQUID-array inductively coupled to a 3D copper coil located at $z=10\text{ mm}$ above showed¹¹ SQUID-like oscillations with $\Delta V_{max}=6.8\text{ mV}$ at 83 K leading to a measured white voltage noise $S_V^{1/2}=(8-9)\text{ nV}/\text{Hz}^{1/2}$. We have re-measured the 770 SQUID-array inductively coupled to the 50-turns superconducting coil (see Fig.1). The highest value for ΔV_{max} of 12.9 mV was obtained at a temperature of 80 K (see Fig.5c) leading to a magnetic flux noise $S_\Phi^{1/2}=S_V^{1/2}/(\pi\Delta V_{max}/\Phi_0)=0.2\text{ }\mu\Phi_0/\text{Hz}^{1/2}$. This increase in ΔV_{max} relative to previous report¹¹ is the consequence of an enhanced coherence state reached due to an improved degree of B_z homogeneity along the array, in accordance with the theory (see Figs.4a and 4c).

Due to approximations made in our model there are quantitative differences between experiments and simulations: a drift/asymmetry of the measured $V(I_B)$ in Fig. 5 b and 5c, as well as an enhanced amplitude modulation in the insets of Fig. 5a. In a more advanced model, one should consider non-zero values for both W_{JJ} and β_L , a superimposed remnant earth magnetic field in the cryostat, and an inhomogeneous distribution of Josephson critical currents along the array due to the non-uniformity of the bicrystal boundary. In this case there is no easy-to-use analytical solution, and the problem should be solved numerically.

The periodicity $(\Delta I_B)_{\Phi_0}$ of the measured $V(I_B)$ corresponds to an additional flux quantum Φ_0 coupled to each of the SQUIDs in the array (see Figs.5a-5c). Consequently, the average mutual

inductance $\langle M_j \rangle$ between the input coil and each individual SQUID j can be calculated as $\langle M_j \rangle = \Phi_0 / (\Delta I_B)_{\Phi_0}$. For the 3-turn input coil coupled to the 800 SQUID array, $(\Delta I_B)_{\Phi_0} = 20$ mA and $\langle M_j \rangle_{800} = 0.1$ pH while for the 50-turns input coil coupled to the 770 SQUID-array $(\Delta I_B)_{\Phi_0} = 160$ μ A and $\langle M_j \rangle_{770} = 12.9$ pH. The experimental value for the ratio $\langle M_j \rangle_{770} / \langle M_j \rangle_{800} = 129$ agrees well with the theoretical value found earlier for the ratio of flux efficiency coupling $\langle \Phi_j \rangle_{770} / \langle \Phi_j \rangle_{800} = 110$. This agreement further confirms the validity of our theoretical approach. A current noise sensitivity $S_I^{1/2} = S_\Phi^{1/2} / \langle M_j \rangle_{770} = 32$ pA/Hz^{1/2} is obtained for the 770-SQUID array at 83 K which is about a factor 3 larger than for single-SQUIDs operated at 68 K²⁶. To build a sensitive SQUID-array-based magnetometer, as for single-SQUIDs^{13,18-20}, an inductively coupled flux transformer would be required to increase the effective area well beyond the flux focusing effects given by Eq.(3). This can be done by connecting the 50-turn input coil of inductance L_i and area A_i in a closed loop configuration to a single turn, large area A_p square pickup loop of inductance L_p fabricated on a separate substrate and placed on top of the SQUID-array substrate (see inset in Fig. 3b). We found that, similarly to single-SQUID magnetometers²⁷, the total effective area of each SQUID j operating as a magnetometer is:

$$A_{mag,j} = A_{eff,j} + (A_p - A_i) \frac{\langle M_j \rangle_{770}}{L_p + L_i} \quad (6)$$

where the first term $A_{eff,j}$ (given by Eq.(3)) is the result of direct flux focused through the SQUID j hole, while the second term represents the indirect flux due to the induced current in the input coil. To allow a direct comparison to single-SQUIDs from²⁸ inductively coupled to flux-transformers, we choose similar parameters for the pickup coil: $A_p = 40 \times 40$ mm² = 1.6×10^{-3} m² and a 2 mm linewidth. Using formulas from¹³ one gets: $L_p = 80$ nH and $L_i = 560$ nH. Finally, using Eqs. (6) and (3) one finds $A_{mag,j} = 33 \times 10^{-3}$ mm² which translates into an estimated field noise sensitivity

for the 700 SQUID-array of $S_B^{1/2} = S_\Phi^{1/2}/A_{mag,j} = 13 \text{ fT/Hz}^{1/2}$. This is more than a factor of two better relative to single-SQUIDs from²⁸ having similar values for their SQUID inductance (20-40 pH). Relative to the best reported values of $6 \text{ fT/Hz}^{1/2}$ achieved in single SQUIDs with similar microns-size junction width inductively coupled to flux transformers and operating at 77K ²⁹, our result is about a factor of two worse (note that in²⁹ the SQUID inductance is larger while A_p is smaller). Since experimentally¹¹ $S_\Phi^{1/2}$ improves as $1/N^{1/2}$ we predict that, by increasing $\langle M_j \rangle$ to a few hundred pH, SQUID-arrays will outperform the best single-SQUID in terms of both $S_I^{1/2}$ and $S_B^{1/2}$. One possibility to increase $\langle M_j \rangle$ significantly is to place the input coil closer in a parallel plane located at a distance of $(0.5\text{-}3) \text{ }\mu\text{m}$ on top of the SQUID-array chip in a flip-chip configuration^{8,13,20,26-29} (rather than in the same plane sideways: see inset of Fig. 3b).

In conclusion we have developed a methodology based on analytical solutions to assess and optimize the degree of voltage modulation coherency in SQUID-arrays inductively coupled to 2D on-chip integrated thin film superconducting input coils located in the near proximity that usually produce a highly inhomogeneous magnetic field along the array. Several solutions are presented: a) doubling the minimum size L_{min} of a coplanar ($z=0$) spiral coil relative to that of a SQUID-array placed inside in the area of maximum B_z homogeneity; b) placing the SQUID-array outside a rectangular spiral coil in the near proximity (see Fig. 3b); c) placing a non-coplanar ($z \neq 0$) coil parallel to the SQUID-array plane at the optimal height z_{opt} above (see Fig. 3a); d) coupling a coil located at $z=0$ to SQUID-arrays with *variable* effective pickup area $A_{eff,j}$ within the arrays. Consequently, SQUID-like voltage oscillation amplitudes above 10 mV were measured in the temperature range (75-83) K leading to magnetic flux noises $S_\Phi^{1/2} = 0.2 \text{ }\mu\text{fT/Hz}^{1/2}$. Our thin-film signal coupling schemes pave the way to fabricate ultra-sensitive SQUID-arrays-based magnetometers using inductively coupled flip-chip flux transformers.

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Data availability. The data that support the findings of this study are available from the corresponding author upon reasonable request.

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