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Rotor vibration measurements using laser Doppler vibrometry: essential post-processing for resolution of radial and pitch/yaw vibrations

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ABSTRACT

Background: Laser Doppler Vibrometry is now a well established technique enabling non-contact vibration measurements in the most challenging of environments. Rotor vibration measurements are often highlighted as a major application of Laser Vibrometers due to their non-contact operation and inherent immunity to shaft run-out.

Method of Approach: In such measurements, resolution of the individual axial and torsional vibration components is possible via particular arrangement of the laser beam(s). Resolution of the radial or pitch / yaw vibration components, however, can only be achieved by essential post-processing of the data from simultaneous orthogonal measurements.

Results: This paper describes the principle and rigorous examination of a novel, dedicated resolution algorithm enabling, for the first time, real-time post-processing of the outputs from standard commercial instruments.

Conclusions: The system performed well, even in the presence of noise and other typical measurement errors, and was implemented successfully in an engine vibration study.

KEYWORDS: Laser Doppler Vibrometry, vibration measurement, rotating machinery.

**ROTOR VIBRATION MEASUREMENTS USING
LASER DOPPLER VIBROMETRY:
ESSENTIAL POST-PROCESSING FOR RESOLUTION OF
RADIAL AND PITCH / YAW VIBRATIONS**

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1. INTRODUCTION

The principle of Laser Doppler Vibrometry (LDV) relies on the detection of a Doppler shift in the frequency of coherent light scattered by a moving target, from which a time-resolved measurement of the target velocity is obtained. The Laser Vibrometer is now well established as an effective non-contact alternative to the use of a traditional contacting vibration transducer. Laser Vibrometers are technically well suited to general application but offer special benefits where certain measurement constraints are imposed, for example by the context, which may demand high frequency operation, high spatial resolution or remote transducer operation, or by the structure itself, which may be hot, light or rotating.

Measurements directly from rotating targets have often been highlighted as a major application of Laser Vibrometers because of their non-contact operation and inherent immunity to shaft run-out (shape). Indeed, one of the first reported LDV applications, from 1969, was for axial vibration measurement directly from a rotating turbine blade [1]. Investigations of magnetic discs [2,3], bladed discs [4,5], spindle vibrations [6] and modal analysis on rotating discs [7] are typical and more recent examples of translational rotor vibration measurements that can be made using a single probe laser beam. Parallel beam arrangements, for angular vibration measurements, have also been used in rotor applications, for example for assessment of torsional damper health [8] and crankshaft bending vibration [9]. Single beam scanning arrangements [10-12] are also now commonplace with applications in modal analysis [13, 14] and for axial rotor vibrations [15-17].

In fact, much of this previous work has concentrated on axial and torsional vibration measurements which are relatively straightforward. A second feature, especially in the earliest work, was prediction of acceptable performance but only in the presence of a single

vibration component, neglecting the effects of other components present in the more complex motions likely to be encountered in practice. It is now known that in radial and pitch / yaw vibration measurements on rotors, which are the focus of this paper, the presence of a velocity component due to the rotation itself has an important effect on the measured velocity. The effect was first reported for radial [18] and then for pitch / yaw vibration measurements [9]; significant sensitivity is found to both speed fluctuation (including torsional oscillation) and motion perpendicular to the intended measurement. Even these were still, at the time, only special cases of the full velocity sensitivity [19] when a laser beam is incident in an arbitrary direction on a rotating target requiring three translational and three rotational coordinates to describe its vibratory motion. This paper begins with a short review of this comprehensive velocity sensitivity model.

This novelty of this paper lies in the introduction of the new and essential post-processing algorithm for resolution of these cross-sensitivities, both in the theoretical basis of the algorithm and in its practical implementation. The algorithm is the core of a LabVIEW-based system that controls the whole measurement task from acquisition of raw data to presentation of resolved spectra. The performance of the algorithm in the presence of typical measurement noise and other likely measurement errors is explored in detail to demonstrate its robustness and the paper concludes with a short engine crankshaft vibration survey to confirm the suitability of the system for real engineering measurements.

2. COMPREHENSIVE VELOCITY SENSITIVITY MODEL

2.1. TOTAL VELOCITY MEASURED AT A POINT BY A LASER VIBROMETER

With reference to Figure a, the case considered is that of an axial element of a shaft of arbitrary cross-section, rotating about its spin axis whilst undergoing arbitrary, six degree-of-

freedom vibration. A translating reference frame, xyz , has its origin, O , fixed to a point on the shaft spin axis with the undeflected shaft rotation axis defining the direction and position of the z axis. The time dependent unit vector $\hat{z}_R$ defines the changing direction of the spin axis, which deviates from the z axis as the shaft undergoes angular vibration. P is the instantaneous point of incidence of the laser beam on the shaft and is identified by the time dependent position vector $\vec{r}_p$.

The direction of the incident laser beam is described by the angles β and α as shown in Figure 1b. The convention used is that the laser beam orientation is initially defined by $\hat{x}$, followed by rotation by an angle β around $\hat{y}$, then by an angle α around $\hat{z}$. Clearly, the choice of orientation of the reference frame, xyz , relative to the structure lies with the user. Since β and α are finite rotations, their order must be maintained.

In the usual configuration, where the target beam is collected in direct backscatter, a Laser Vibrometer measures target velocity in the direction of the incident laser beam. Provided that the illuminated axial element of the shaft can be assumed to be of rigid cross-section, the velocity measured by a laser beam incident on the shaft surface, U_m , is given by [19]:

$$\begin{aligned}
 U_m = & \cos \beta \cos \alpha [\dot{x} + (\dot{\theta}_z + \Omega)y - (\dot{\theta}_y - \Omega\theta_x)z] \\
 & + \cos \beta \sin \alpha [\dot{y} - (\dot{\theta}_z + \Omega)x + (\dot{\theta}_x + \Omega\theta_y)z] \\
 & - \sin \beta [\dot{z} - (\dot{\theta}_x + \Omega\theta_y)y + (\dot{\theta}_y - \Omega\theta_x)x] \\
 & - (y_0 \sin \beta + z_0 \cos \beta \sin \alpha) [\dot{\theta}_x + \Omega\theta_y] \\
 & + (z_0 \cos \beta \cos \alpha + x_0 \sin \beta) [\dot{\theta}_y - \Omega\theta_x] \\
 & + (x_0 \cos \beta \sin \alpha - y_0 \cos \beta \cos \alpha) [\dot{\theta}_z + \Omega]
 \end{aligned} \tag{1}$$

where $\dot{x}$, $\dot{y}$, $\dot{z}$ and x , y , z are the translational vibration velocities and displacements of the origin, O , in the x , y , z directions, $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$ and θ_x , θ_y , (θ_z) are the angular vibration

velocities and displacements of the shaft around the x , y , z axes (referred to as pitch, yaw and roll, respectively), Ω is the angular velocity of the axial shaft element (combining rotation speed and any torsional oscillation), and (x_0, y_0, z_0) is the position of an arbitrary known point that lies along the line of the incident beam. Equation (1) shows that the total measured velocity is the sum of six terms, each a product of a combination of geometric parameters, relating to the laser beam orientation, and an inseparable combination of motion parameters – the vibration “set”.

2.2. ISOLATION OF INDIVIDUAL VIBRATION SETS

Since the first description of the cross-sensitivity problem [18], there has been discussion about whether a particular arrangement of laser beams or a particular variation of the arrangement, for example by scanning the laser beam, might enable resolution of individual motion components. For rotating targets, equation (1) shows how pure axial, z , vibration measurements are obtained by careful alignment of the laser beam with the rotation axis, other terms in the axial vibration set are products of vibration components and, therefore, small. Provided consideration is given for the laser speckle effect [20], this measurement can be obtained in the same way as for a similar measurement on a non-rotating target. Similarly, direct measurement of the rotational angular velocity, Ω , is possible, accepting that the torsional oscillation and roll motion are indistinguishable. Conversely, direct measurement of pure radial, $\dot{x}$ and $\dot{y}$, or pitch / yaw, $\dot{\theta}_x$ and $\dot{\theta}_y$, vibration is not possible because the measurement will always show a significant sensitivity to the product of rotation speed and the orthogonal vibration displacement. Although equation (1) shows other, smaller cross-sensitivities, it is this product of rotation speed and the orthogonal vibration displacement that constitutes the major limitation to the use of Laser Vibrometers for rotor vibration measurements. Resolution is essential and is the purpose of the study presented here.

2.2.1. Translational Vibration Sets

With reference to Figure 1a, measurement of the x radial vibration set requires that the Laser Vibrometer is aligned so that it passes through the centre of the shaft and along the x axis, i.e. $\alpha = \beta = 0^\circ$ and the alignment offset, y_0 , is zero. Substituting these values into equation (1) and setting $z_0 = 0$ so that the plane of the origin of the xyz reference frame and the “measurement plane” are coincident, since this is merely a matter of definition, results in:

$$U_m = \left[\dot{x} + (\dot{\theta}_z + \Omega)y - (\dot{\theta}_y - \Omega\theta_x)z \right] \quad (2a)$$

Similarly, alignment through the centre of the shaft and along the y axis, i.e. $\alpha = 90^\circ$, $\beta = 0^\circ$ and the alignment offset, x_0 , is zero, enables the y radial vibration set to be isolated:

$$U_m = \left[\dot{y} - (\dot{\theta}_z + \Omega)x + (\dot{\theta}_x + \Omega\theta_y)z \right] \quad (2b)$$

Since the third set of terms in equations (2a&b), $(\dot{\theta}_y - \Omega\theta_x)z$ and $(\dot{\theta}_x + \Omega\theta_y)z$, are products of vibration parameters, they are typically an order of magnitude smaller than the first two terms and simulation has confirmed that they are insignificant. In addition, the shaft rotation and roll motion are generally difficult to distinguish and because later experimentation uses a Laser Rotational Vibrometer for angular velocity measurement, it is convenient to consider total angular velocity, $\Omega_T = \dot{\theta}_z + \Omega$. Finally, it is not possible to align the laser beams perfectly and so it is necessary to include non-zero x_0 and y_0 . The x and y radial vibration set measurements, U_x and U_y , are therefore given to an acceptable approximation by:

$$U_x = \dot{x} + \Omega_T(y - y_0) \quad (3a)$$

and

$$U_y = \dot{y} - \Omega_T(x - x_0) \quad (3b)$$

2.2.2. Rotational Vibration Sets

Isolation of each rotational vibration set is impossible with a single laser beam and is best achieved using a parallel beam arrangement. Repeated application of equation (1) reveals a number of possible combinations of the geometric parameters that enable isolation of the pitch and yaw vibration sets [21]. For incidence on the side of the shaft element, a good option is to have one pair of beams parallel with the x -axis and in a plane parallel to the xz plane, for the yaw vibration set, with the second pair of beams parallel with the y -axis and in a plane parallel to the yz plane, for the pitch vibration set. An important alternative when optical access is limited to the face of the shaft element is to have both pairs of beams parallel with the z -axis with one in a plane parallel to the xz plane, for the yaw vibration set, and the second in a plane parallel to the yz plane, for the pitch vibration set. Again acknowledging the difficulty in distinguishing rotational angular velocity and roll, the pitch and yaw vibration set measurements, Ω_x and Ω_y , are given to an acceptable approximation by:

$$\Omega_x = \dot{\theta}_x + \Omega_T \theta_y \quad (4a)$$

and

$$\Omega_y = \dot{\theta}_y - \Omega_T \theta_x \quad (4b)$$

Since equations (3a&b) and (4a&b) have the same form, the radial and pitch / yaw cross-sensitivity problems are equivalent and a solution for resolution of radial vibrations will be immediately applicable to resolution of pitch / yaw vibrations. For brevity, this paper is set out in terms of radial vibration measurements only. Full experimental validations of equations (3a&b) and (4a&b) have been presented previously [9, 18, 19].

3. RESOLUTION ALGORITHM

Experimentally, two simultaneous, orthogonal Laser Vibrometer measurements are required, isolating the x and y radial vibration sets as described in section 2.2.1. These are combined

with a simultaneous rotational angular velocity measurement. The novel resolution algorithm presented here is the core of a resolution system that is, for the first time, able to complete the full processing task from acquisition of raw data to presentation of resolved spectra. Its performance will be investigated in considerable depth, in subsequent sections, from the perspective of real measurement conditions.

The full resolution algorithm is written in the National Instruments LabVIEW software and comprises two parts. A constant angular velocity resolution algorithm is followed by a correction algorithm specifically intended to correct the effects of time dependency in the rotational angular velocity such as torsional vibration or speed fluctuations. LabVIEW constitutes a powerful tool for this type of exercise since it is possible to design algorithms and then test them using simulated signals, for which the algorithm outputs can easily be compared with the desired outputs, before straightforwardly reconfiguring the code to make use of actual measured data.

3.1. RESOLUTION ALGORITHM DEVELOPMENT

By separating the rotational angular velocity term into mean and time dependent components,

$\Omega_T = \overline{\Omega_T} + \Delta\Omega_T$, and considering AC-coupled Vibrometer outputs, $\tilde{U}_x$ and $\tilde{U}_y$, equations

(3a&b) become:

$$\tilde{U}_x = \dot{x} + \overline{\Omega_T}y + \Delta\Omega_T(y - y_0) \quad (5a)$$

and

$$\tilde{U}_y = \dot{y} - \overline{\Omega_T}x - \Delta\Omega_T(x - x_0) \quad (5b)$$

Integrating equation (5b), substituting for y in equation (5a) and neglecting DC terms leads to the following integro-differential equation describing the intended measurement parameter, in this case x -radial vibration:

$$\dot{x} + \overline{\Omega_T}^2 \int_0^t x dt = \left(\tilde{U}_x - \overline{\Omega_T} \int_0^t \tilde{U}_y dt \right) - \left(\Delta\Omega_T y + \overline{\Omega_T} \int_0^t \Delta\Omega_T x dt \right) + \left(\Delta\Omega_T y_0 + \overline{\Omega_T} x_0 \int_0^t \Delta\Omega_T dt \right) \quad (6)$$

The right hand side of equation (6) is shown as the sum of three bracketed terms. The first is readily formulated from the two Laser Vibrometer outputs and the rotational angular velocity measurement. The second and third bracketed terms are related to the products of the time dependent component of rotational angular velocity, $\Delta\Omega_T$, and, respectively, the radial vibration displacements, x and y , and the alignment offsets, x_0 and y_0 .

It has proved convenient to proceed in the frequency domain, where quantities can be straightforwardly integrated or differentiated at any given frequency:

$$FT \left[\dot{x} + \overline{\Omega_T}^2 \int_0^t x dt \right] = \dot{X}(\omega) + \overline{\Omega_T}^2 \frac{X(\omega)}{j\omega} = \dot{X}(\omega) \left(1 - \frac{\overline{\Omega_T}^2}{\omega^2} \right) \quad (7)$$

where $X(\omega)$ and $\dot{X}(\omega)$ are the Fourier Transforms of vibration displacement and velocity respectively. Making use of equation (7) in equation (6) enables $\dot{X}(\omega)$ to be written as:

$$\begin{aligned} \dot{X}(\omega) = W(\omega) FT \left[\left(\tilde{U}_x - \overline{\Omega_T} \int_0^t \tilde{U}_y dt \right) \right] \\ - W(\omega) FT \left[\left(\Delta\Omega_T y + \overline{\Omega_T} \int_0^t \Delta\Omega_T x dt \right) \right] + W(\omega) FT \left[\left(\Delta\Omega_T y_0 + \overline{\Omega_T} x_0 \int_0^t \Delta\Omega_T dt \right) \right] \quad (8a) \end{aligned}$$

The equivalent expression for the y radial vibration is obtained in the same fashion:

$$\begin{aligned} \dot{Y}(\omega) = W(\omega) FT \left[\left(\tilde{U}_y + \overline{\Omega_T} \int_0^t \tilde{U}_x dt \right) \right] \\ + W(\omega) FT \left[\left(\Delta\Omega_T x - \overline{\Omega_T} \int_0^t \Delta\Omega_T y dt \right) \right] - W(\omega) FT \left[\left(\Delta\Omega_T x_0 - \overline{\Omega_T} y_0 \int_0^t \Delta\Omega_T dt \right) \right] \quad (8b) \end{aligned}$$

Similarly, but in this case making use of equations (4a&b), the pitch and yaw vibration velocities can be calculated:

$$\dot{\Theta}_x(\omega) = W(\omega)FT\left[\left(\Omega_x - \overline{\Omega_T} \int_0^t \Omega_y dt\right)\right] - W(\omega)FT\left[\left(\Delta\Omega_T \theta_y + \overline{\Omega_T} \int_0^t \Delta\Omega_T \theta_x dt\right)\right] \quad (8c)$$

$$\dot{\Theta}_y(\omega) = W(\omega)FT\left[\left(\Omega_y + \Omega_T \int_0^t \Omega_x dt\right)\right] + W(\omega)FT\left[\left(\Delta\Omega_T \theta_x - \overline{\Omega_T} \int_0^t \Delta\Omega_T \theta_y dt\right)\right] \quad (8d)$$

where

$$W(\omega) = \omega^2 / (\omega^2 - \overline{\Omega_T}^2) \quad (8e)$$

This need for this weighting function is a further reason to utilise frequency domain processing. The first bracketed term on the right hand side of equations (8a) to (8d), referred to as the Vibrometer outputs term, is a function of the three channels of measured data, namely the two Vibrometer outputs and the mean rotational angular velocity. The second bracketed term in equations (8a) to (8d), referred to as the oscillation-displacement term, is dependent on the rotational angular velocity, which can be measured, and the vibration displacements, x and y , which are obviously unknown because they are the intended measurements. The third bracketed term on the right hand side of equations (8a) and (8b) is dependent on the rotational angular velocity, which can be measured, and the offsets, x_0 and y_0 , which cannot be measured. This term, which is referred to as the oscillation-offset term, can be minimised by careful alignment of the single beam Vibrometers with the target rotation axis. This minimisation is not a concern for pitch / yaw vibrations since the oscillation-offset term is not present in equations (8c) and (8d).

3.2 CONSTANT ANGULAR VELOCITY RESOLUTION ALGORITHM

If the angular velocity variation is small relative to the mean angular velocity, $\Delta\Omega_T \approx 0$, equation (8a) simplifies to:

$$\dot{X}(\omega) = W(\omega)FT\left[\left(\tilde{U}_x - \overline{\Omega_T} \int_0^t \tilde{U}_y dt\right)\right] \quad (9)$$

Continuing in the frequency domain, it is possible to re-express equation (9) (and its equivalent for $\dot{y}$) in terms of the real, $\Re[]$, and imaginary, $\Im[]$, parts of the Fourier Transforms as follows:

$$\dot{X}(\omega) = W(\omega) \left(\left\{ \Re[\tilde{U}_x] - \frac{\overline{\Omega_T}}{\omega} \Im[\tilde{U}_y] \right\} + j \left\{ \Im[\tilde{U}_x] + \frac{\overline{\Omega_T}}{\omega} \Re[\tilde{U}_y] \right\} \right) \quad (10a)$$

and

$$\dot{Y}(\omega) = W(\omega) \left(\left\{ \Re[\tilde{U}_y] + \frac{\overline{\Omega_T}}{\omega} \Im[\tilde{U}_x] \right\} + j \left\{ \Im[\tilde{U}_y] - \frac{\overline{\Omega_T}}{\omega} \Re[\tilde{U}_x] \right\} \right) \quad (10b)$$

The block diagram representation of this constant angular velocity resolution algorithm, as illustrated in Figure 2a, was used to develop a LabVIEW based version of this essential resolution algorithm.

3.2.1 Synchronous Vibration Measurements

It can be seen from equation (8e) that the frequency dependent weighting function, $W(\omega)$, becomes infinite at synchronous frequency. At the same time it can be shown that the Vibrometer outputs terms in equations (8a) to (8d) become zero at synchronous frequency and it is not possible to establish a meaningful solution for the synchronous vibration component. This is not a limitation of the resolution algorithm but a fundamental feature of the synchronous velocity component sensed by the laser beam. As a consequence, all spectra presented have a gap at the synchronous frequency. As a further consequence, it may not always be meaningful to return from the frequency domain to the time domain since the synchronous component will be missing. For rotating machinery diagnostics, the inability to resolve the synchronous vibration component is a serious limitation to the use of Laser Vibrometers for radial and pitch / yaw vibration measurement that must be acknowledged.

Note, however, that there is no such issue for measurement of axial or torsional / roll vibration.

It is also important to recognise that $W(\omega)$ is very large close to synchronous frequency and any additional content, such as measurement noise, will be amplified. In some situations, the level of this amplified spectral content may be equivalent to the genuine vibration velocity and may make the resolved output misleading. In such a situation it may be necessary to disregard a number of spectral components adjacent to the synchronous frequency component and this will be discussed in more detail in section 4.

3.3. CORRECTION ALGORITHM

Of course, in the presence of torsional oscillations, the accuracy of the constant angular velocity resolution algorithm will be compromised. Nonetheless, the amplitude of the oscillation-displacement term will still be small and the output of the constant angular velocity resolution algorithm will constitute a good vibration velocity estimate (at all non-synchronous frequencies). It is then possible to use the integral of the first vibration velocity estimates and the rotational angular velocity and torsional oscillation measurements to create a first estimate of the oscillation-displacement term. Subtracting this from the initial vibration velocity estimate results in an improved estimate of the true vibration velocity. Repeating this procedure, it is possible to make increasingly accurate estimates of the true vibration velocity. If the output of the constant angular velocity resolution algorithm is the first frequency domain estimate of vibration velocity, $\dot{X}_{est1}(\omega)$, the $(n+1)^{th}$ estimate, $\dot{X}_{estn+1}(\omega)$, generated after n iterations through the correction algorithm, can be written:

$$\dot{X}_{estn+1}(\omega) = \dot{X}_{est1}(\omega) - W(\omega)FT \left[\Delta \Omega_T y_{estn} + \overline{\Omega_T} \int_0^t \Delta \Omega x_{estn} dt \right] \quad (11)$$

As in section 3.2, it is possible to re-express equation (11) (and its equivalent for $\dot{y}$) as follows:

$$\dot{X}_{estn+1}(\omega) = \dot{X}_{est1}(\omega) - W(\omega) \left(\left\{ \Re[\Delta\Omega_T y_{estn}] + \frac{\overline{\Omega_T}}{\omega} \Im[\Delta\Omega_T x_{estn}] \right\} + j \left\{ \Im[\Delta\Omega_T y_{estn}] - \frac{\overline{\Omega_T}}{\omega} \Re[\Delta\Omega_T x_{estn}] \right\} \right) \quad (12a)$$

and

$$\dot{Y}_{estn+1}(\omega) = \dot{Y}_{est1}(\omega) + W(\omega) \left(\left\{ \Re[\Delta\Omega_T x_{estn}] - \frac{\overline{\Omega_T}}{\omega} \Im[\Delta\Omega_T y_{estn}] \right\} + j \left\{ \Im[\Delta\Omega_T x_{estn}] + \frac{\overline{\Omega_T}}{\omega} \Re[\Delta\Omega_T y_{estn}] \right\} \right) \quad (12b)$$

where the n^{th} estimates of the x and y vibration displacement, x_{estn} and y_{estn} , are given by:

$$x_{estn} = FT^{-1}[X_{estn}(\omega)] = FT^{-1} \left[\frac{1}{\omega} (\Im[\dot{X}_{estn}(\omega)] - j\Re[\dot{X}_{estn}(\omega)]) \right] \quad (13a)$$

and

$$y_{estn} = FT^{-1}[Y_{estn}(\omega)] = FT^{-1} \left[\frac{1}{\omega} (\Im[\dot{Y}_{estn}(\omega)] - j\Re[\dot{Y}_{estn}(\omega)]) \right]. \quad (13b)$$

Figure 2b shows the block diagram representation of equations (12a&b). As for the constant angular velocity resolution algorithm, a LabVIEW based version of this correction algorithm was developed.

4. FULL RESOLUTION ALGORITHM PERFORMANCE

This section contains a detailed examination of the performance of the constant angular velocity resolution algorithm and the correction algorithm by use of simulated data. In all simulations, the vibration velocities shown in Figures 3a&b, in the time domain, and Figures 3c&d, in the frequency domain, were used as the basis of the simulated Vibrometer outputs, generated according to equations (3a&b). The time and frequency axes used to present all simulated data have been non-dimensionalised in order to generalise the conclusions drawn. The vibration velocities used represent medium/high severity vibrations for typical rotor

systems [22]. The amplitude, phase and, in particular, frequency of the vibration velocities have been deliberately made distinct as have the alignment offsets, x_0 and y_0 , so that the effects of the cross-sensitivity and the performance of the resolution algorithm can be explored more easily. Clearly, the x and y vibration frequencies are likely to be the same in practice. The simulations are presented in terms of radial vibrations but the same principles apply to pitch / yaw vibration data.

4.1. CONSTANT ANGULAR VELOCITY RESOLUTION ALGORITHM

In the first simulation, the AC-coupled Laser Vibrometer outputs in Figures 4a&b show how cross-sensitivity can result in a significant difference between the measured and true vibration velocities (see figure 3). In this simple example, it is relatively straightforward to observe the effect of the cross-sensitivity in the time domain (Figures 4a&b) because the genuine x and y vibrations have been chosen to have unequal amplitude components with different frequency and phase. For more complex vibrations, however, the frequency domain representation will be more useful. In this case, it is clear from the frequency spectra in Figures 4c&d that each signal contains a genuine vibration peak, with the correct amplitude and phase, due to the intended measurement parameter and an additional peak due to cross-sensitivity.

Figures 4e&f shows the resolution algorithm outputs in which the peaks due to cross-sensitivity are completely eliminated and the genuine peaks have identical frequency, amplitude and phase (not shown) to the true vibration velocity, as required. This simple but important result is intended to assist the reader in understanding the function of the resolution algorithm and also confirms that the resolution algorithm is performing the correct manipulations. More complex, simulated vibrations (same frequency, multiple frequency and broadband frequencies) have also been used to verify this performance. For subsequent

simulations, discussion will be confined to spectra and to a single resolved vibration component to maintain brevity in the figures. Resolved spectra from each simulation can be compared with the spectrum in Figure 4e which might be regarded as the desired resolved spectrum.

In a real Laser Vibrometer measurement, random noise will be present and, in a measurement on a rotor, pseudo-random noise associated with the laser speckle effect [20] will also be present in the output signal. For a target rotating at constant angular velocity, measurement experience has shown that the combination of these noise sources typically results in noise with spectral content of approximately 0.1mm/s at integer multiples of rotation frequency with an underlying broadband noise floor of approximately 0.015mm/s/ $\sqrt{\text{Hz}}$. The second simulation examines the effect of this inevitable measurement noise on the resolution algorithm. Figure 5 shows a simulated Laser Vibrometer output equivalent to that shown in Figure 4c but with this additional noise content. The simulated rotational angular velocity measurement (not shown) also contains noise of this nature because a Laser Rotational Vibrometer was used in the real experiments discussed later. In this simulation (not shown), the resolved vibration velocity peaks showed identical frequency, amplitude that is correct to within 1% and phase that is correct to within 10mrad of the true vibration velocity and the cross-sensitivity peaks were reduced to insignificant levels.

In situations where genuine vibrations are expected in regions close to synchronous frequency, small errors in the independent measurement of $\overline{\Omega_T}$ will result in large errors in $W(\omega)$ and, hence, in the resolved velocity. Figure 6a shows a resolved output for the third simulation in which, in addition to the noise in the second simulation, there is a -2% (very large) error in the measurement of $\overline{\Omega_T}$. Here the genuine vibration velocity peaks in the

resolved outputs have amplitude and phase errors of the order of 10% and 10mrad and the cross-sensitivity peaks are reduced to very low but still noticeable levels. Errors are obviously larger with proximity of the vibration components to the synchronous frequency. Random noise in the Vibrometer outputs results in relatively high spectral levels around synchronous frequency. As shown in Figure 6a, apparent levels in excess of 1mm/s are seen when only the spectral line at synchronous frequency is eliminated but, as suggested in section 3.2.1, it can be less misleading to disregard additional frequencies around synchronous provided that important vibrations are not expected to occur in that region. In this particular example, if all data points within the frequency range $\overline{\Omega_T} \pm 0.1\overline{\Omega_T}$ are eliminated, the errors appearing in the regions adjacent to synchronous frequency are limited to the order of 0.1mm/s, as shown in Figure 6b. Since noise and/or speed errors such as these will always occur in this type of Laser Vibrometer measurement, all resolved outputs examined in the remainder of this paper will have data points in the frequency range $\overline{\Omega_T} \pm 0.1\overline{\Omega_T}$ eliminated from the presented data. (The data remains within the calculations performed in the resolution algorithm). At all frequencies away from synchronous, the noise levels are similar to those in the original, simulated Laser Vibrometer outputs and, importantly, are not amplified by the resolution algorithm.

The simulations presented in this subsection have shown that the constant angular velocity resolution algorithm provides good estimates of the true vibration velocities, under circumstances where the original Vibrometer outputs show unacceptable cross-sensitivities. Provided measurements are taken within the constraints that no information is available at synchronous frequency and that additional measurement content around synchronous will be amplified by the weighting function, the performance of the resolution algorithm is excellent and the technique may be implemented to generate valuable data when the rotor angular

velocity is constant. When the rotor angular velocity is not constant, however, the oscillation-displacement and oscillation-offset terms, as described in section 3.1, may adversely affect the resolved outputs. For this reason, the correction algorithm is used to process all data to reduce the impact of the oscillation-displacement term and its performance is the subject of this next subsection.

4.2. CORRECTION ALGORITHM

For the fourth simulation, the alignment offsets, x_0 and y_0 , are zero and the angular velocity contains a broadband torsional oscillation/speed fluctuation with RMS amplitude of 210mrad/s (12deg/s) from 0 to $6\overline{\Omega_T}$, equivalent to a significant torsional vibration [9]. Figure 7 shows a resulting measurement in which cross-sensitivity is apparent. Additional measurement content due to the product of the torsional oscillation and the (in-plane) vibration displacement, $\Delta\Omega_T y$, can also be observed but at a low level.

Figure 8a shows the first estimate of the vibration velocity that is obtained simply by applying the constant angular velocity resolution algorithm. As expected, the algorithm reduces the main cross-sensitivity peak significantly, in this case to a level that is less than $\Delta\Omega_T y$. The outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 1% and phase that is correct to within 10mrad of the true vibration velocity. As discussed in section 3.2.1, the weighting function amplifies the level of any additional signal content around synchronous frequency; in this example $\Delta\Omega_T y$ is amplified to within two orders of magnitude of the genuine vibration peak. As in the previous section, spectral lines at $\overline{\Omega_T} \pm 0.1\overline{\Omega_T}$ have been eliminated.

The errors in the resolved vibration velocity after one iteration through the correction algorithm are typically significantly reduced, as shown in Figure 8b. Here, the outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 0.01% and phase that is correct to within 100 μ rad. After only three correction iterations, the errors may be considered insignificant as they are, in this case, reduced (by approximately three orders of magnitude) to at least five orders of magnitude lower than the genuine vibration peaks, as shown in Figure 8c. Here, the outputs show vibration velocity peaks with identical frequency, amplitude that is correct to within 0.001% and phase that is correct to within 1 μ rad! Further correction iterations show increasingly accurate outputs confirming that the correction algorithm is performing the correct manipulations and, further to this, more complex vibration schemes (same frequency, multiple frequency and broadband frequencies) have been used to verify this performance.

The correction algorithm is formulated specifically to counter the effects of torsional oscillations and cannot, of course, correct for speckle noise or other noise present in the measurements or for the effects of alignment offsets. Consideration of the apparent velocity levels encountered reveals that both speckle noise and oscillation-offset errors introduce signal content that is, typically, at least one order of magnitude higher than the oscillation-displacement errors. In the case of alignment offsets, this is easy to anticipate because these offsets, which cannot currently be measured, are likely to be an order of magnitude greater than the vibration displacements. Under such circumstances, no improvement through use of the correction algorithm is expected, nor is any seen. Of great importance, however, is that no amplification of these errors occurs as a result of using the correction algorithm. In reality, the engineer will not know how much each source of error is contributing to measured data. The performance of the correction algorithm is such that all data can be processed through it

and at frequencies where the oscillation-displacement error dominates improvement will be found and at frequencies where other error sources dominate no change will occur.

The final simulation presented considers a challenging but realistic scenario in which measured data contains oscillation-offset errors, oscillation-displacement errors and broadband noise (as target oscillation increases the pseudo-random nature of speckle noise shown in the second and third simulations is less pronounced and the noise becomes more random with a very marginally increased rms level). The difficulties in resolution are compounded by speed measurement error. Figure 9 shows the simulated Vibrometer output and Figure 10a shows how the constant angular velocity resolution algorithm performs well in eliminating the cross-sensitivity. Figure 10b shows how the correction algorithm results in neither a further improvement nor in amplification of noise in the resolved data. As in previous simulations, the weighting function in the resolution algorithm amplifies the additional content to significant levels around the synchronous frequency and data points around this frequency have been eliminated to prevent data misinterpretation. The vibration velocity peaks contain amplitude errors of the order of 10% and phase errors of the order of 10mrad.

5. ENGINE VIBRATION MEASUREMENTS

In this section, a series of Laser Vibrometer measurements on the crankshaft pulley of a 4-stroke, 4-cylinder, 2-litre petrol engine are presented. The full resolution algorithm was implemented to obtain, for the first time, real-time resolved radial and pitch / yaw vibration measurements. Each set of measurements was obtained using a pair of Polytec OFV4000 Laser Rotational Vibrometers, either configured to measure translational vibration velocity

(by reflecting one beam directly back into the instrument with a “single point adapter” lens cap) or, using both beams, to measure angular vibration velocity. In addition to this, the resolution technique requires an independent measurement of the target rotational angular velocity which in this work was obtained directly from the engine crankshaft pulley using a Brüel & Kjær Torsional Vibration Meter Type 2523.

The type of experimental arrangement used is shown, with exaggerated laser beam paths, in Figure 11. The positions of the instruments were arranged carefully using tripods such that alignment offsets were minimised. As examples of the data obtainable, Figures 12a&b show, in waterfall format, a series of x radial and yaw measurements, respectively, obtained with the engine operating at full throttle across a range of speeds, i.e. under varying load. Three iterations around the correction algorithm have been applied to these data.

As expected for this type of engine, significant vibration occurs mostly at $\frac{1}{2}x$ engine orders with larger peaks at $2x$ engine orders. In the radial measurement, it is apparent that the $2x$ engine speed vibration component, the amplitude of which increases steadily with engine speed, is generally the most significant. Of the other $2x$ harmonic components, both figures show that vibration levels are generally the highest when the engine order coincides with the crankshaft natural frequencies (around 400Hz in this case). Such detailed information is difficult to obtain by other means, emphasising the potential of Laser Vibrometry to provide valuable data under challenging operating conditions.

Features of importance or interest in these resolved outputs include the fact that data has been eliminated in each spectrum in the region around synchronous frequency, resulting in the presence of a flat (zero level) in the spectra. The characteristic shape of $W(\omega)$ is not easily

observed in Figure 12a because the vibration velocities are high relative to noise levels but it is particularly evident in Figure 12b where vibration velocities are relatively lower and the relatively high noise levels become evident around synchronous frequency.

6. CONCLUSIONS

This paper has set out the theoretical basis of a novel and essential post-processing algorithm intended to resolve the cross-sensitivities encountered when Laser Vibrometer measurements of radial and pitch / yaw vibrations are taken directly from rotors. The full resolution algorithm, which processes all data in the frequency domain, comprises the fundamental resolution algorithm, based on an assumption of constant rotational angular velocity, followed by a correction algorithm to account for any time-dependency in angular velocity. The algorithm functions perfectly under ideal conditions with small but acceptable errors apparent when common problems of measurement noise, principally speckle noise, and errors in rotational angular velocity measurement are included. The data presented have also highlighted the need for reduction of alignment offsets and speckle noise levels as the next technological challenges to lower the minimum measurable velocities for this application.

Features of the resolved data include a gap in the data at the synchronous frequency, where it is fundamentally impossible to resolve the data, and increased noise levels around the synchronous frequency. The increased noise levels have led the authors to adopt the practice of eliminating data points at frequencies within 10% of synchronous to prevent presentation of data that might mislead. Within this limitation, simulation and real measured data has indicated that the engineer can expect a minimum measurable velocity below 0.5mm/s close to synchronous frequency and between 10-100 μ m/s elsewhere. Equivalent figures for angular vibrations are from 5-10deg/s around synchronous frequency down to 0.1-1deg/s elsewhere.

These are respectable figures and, because the whole process is transparent to the engineer who is simply presented with resolved data and thus needs no specialist knowledge of the data manipulation, this system has tremendous potential in rotating machinery diagnostics.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support of the UK Engineering and Physical Sciences Research Council who funded this project.

Nomenclature

t	Time
U_m	Translational velocity measured by a laser beam
U_x, U_y	x and y radial vibration set measurements
$\tilde{U}_x, \tilde{U}_y$	AC-coupled x and y radial vibration set measurements
$W(\omega)$	Frequency dependent weighting function, see equation (8e)
$X(\omega), Y(\omega)$	Fourier Transforms of x and y vibration displacement
$X_{estn}(\omega), Y_{estn}(\omega)$	n^{th} estimate of $X(\omega)$ and $Y(\omega)$
$\dot{X}(\omega), \dot{Y}(\omega)$	Fourier Transforms of x and y vibration velocity.
$\dot{X}_{estn}(\omega), \dot{Y}_{estn}(\omega)$	n^{th} estimate of $\dot{X}(\omega)$ and $\dot{Y}(\omega)$
$\dot{x}, \dot{y}, \dot{z}$	Translational vibration velocity components
x, y, z	Translational vibration displacement components
x_{estn}, y_{estn}	n^{th} estimates of x and y
(x_0, y_0, z_0)	An arbitrary known point along the line of the incident laser beam. In this context, x_0 , and y_0 are alignment offsets.
α, β	Laser beam orientation angles defined in Figure 1b

$\dot{\Theta}_x(\omega), \dot{\Theta}_y(\omega)$	Fourier Transforms of $\dot{\theta}_x$ and $\dot{\theta}_y$ vibration velocity
$\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z$	Angular vibration velocity components around the x, y, z axes
θ_x, θ_y	Angular vibration displacement components around the x, y, z axes
ω	Angular frequency
Ω	Angular velocity of the axial shaft element
Ω_x, Ω_y	Pitch and yaw vibration set measurements,
Ω_T	$\dot{\theta}_z + \Omega$
$\overline{\Omega_T}$	Mean component of Ω_T
$\Delta\Omega_T$	Oscillating component of Ω_T

REFERENCES

- [1] Davis, Q.V. and Kulczyk, W.K., 1969 “Vibrations of Turbine Blades Measured by Means of a Laser,” *Nature* 222, pp. 475-476.
- [2] Riener, T.A., Goding, A.C. and Talke, F.E., 1988 “Measurement of Head / Disc Spacing Modulation using a Two Channel Fiber Optic Laser Doppler Vibrometer,” *IEEE Transactions on Magnetics*, 24(6), pp. 2745-2747..
- [3] Wlezein, R.W., Miu, D.K. and Kibens, V., 1984 “Characterization of Rotating Flexible Disks using a Laser Doppler Vibrometer,” *Optical Engineering*, 24(4), pp. 436-442.
- [4] Cookson, R.A. and Bandyopadhyay, P., 1980 “A Fiber-Optics Laser Doppler Probe for Vibration Analysis of Rotating Machines,” *ASME J. Engineering for Power*, 102, pp. 607-612.
- [5] Reinhardt, A.K., Kadambi, J.R. and Quinn, R.D., 1995 “Laser Vibrometry Measurements on Rotating Blade Vibrations,” *ASME J. Engineering for Gas Turbines and Power*, 117, pp. 484-488.

- [6] Rantatalo, M., Tatar, K. and Norman, P., “Laser Doppler Vibrometry Measurements of a Rotating Milling Machine Spindle,” *Proc. of Institution of Mechanical Engineers - 8th International Conference on Vibrations in Rotating Machinery*, 2004 Swansea, UK, pp. 231-240.
- [7] Stanbridge, A.B. and Ewins, D.J., 1995, “Modal Testing of Rotating Discs using a Scanning LDV,” *Proc. of ASME Design Engineering Technical Conference, Boston, USA* **3(B)**, pp. 1207-1213.
- [8] Halliwell, N.A., 1996, “The Laser Torsional Vibrometer: A Step Forward in Rotating Machinery Diagnostics,” *J. Sound and Vibration*, 190(3), pp. 399-418.
- [9] Miles, T.J., Lucas, M., Halliwell, N.A. and Rothberg, S.J., 1999, *Journal of Sound and Vibration* **226(3)**, 441-467. Torsional and Bending Vibration Measurement on Rotors using Laser Technology.
- [10] Stoffregen, B. and Felske, A., 1985, “Scanning Laser Doppler Vibration Analysis System,” *Transactions of SAE – Technical Paper Series 850327*, pp. 934-940.
- [11] Junge, B., 1994, “Experiences with Scanning Laser Vibrometry in Automotive Industries,” *Proc. SPIE – First International Conference on Vibration Measurements by Laser Techniques*, Ancona, Italy, 2358, pp. 377-382.
- [12] Hancox, J., Staples, B. C. and Parker, R. J., 1995, “The Application of Scanning Laser Doppler Vibrometry in Aero-Engine Development,” *Proceedings of IMechE – Journal of Aerospace Engineering*, 209, pp. 35-42.
- [13] Sriram, P., Craig, J. I. and Hanagud, S., 1990, “A Scanning Laser Doppler Vibrometer for Modal Testing,” *The International Journal of Analytical and Experimental Modal Analysis*, 5(3), pp. 155-167.
- [14] Stanbridge, A. B. and Ewins, D. J., 1999, “Modal testing using a scanning laser Doppler Vibrometer,” *Mechanical Systems and Signal Processing*, 13(2), pp. 255-270.

- [15] Castellini, P. and Paone, N., 2000, “Development of the Tracking Laser Vibrometer: Performance and Uncertainty Analysis,” *Review of Scientific Instruments*, 71(12), pp. 4639-4647.
- [16] Halkon, B.J. and Rothberg, S.J., 2003, “Vibration Measurements Using Continuous Scanning Laser Doppler Vibrometry: Theoretical Velocity Sensitivity Analysis with Applications,” *Measurement Science and Technology*, 14, pp. 382-393.
- [17] Halkon, B.J., Frizzel, S.R. and Rothberg, S.J., 2003, “Vibration Measurements Using Continuous Scanning Laser Vibrometry: Velocity Sensitivity Model Experimental Validation,” *Measurement Science and Technology*, 14, pp. 773-783.
- [18] Rothberg, S.J. and Halliwell, N.A., 1994, “Vibration Measurements on Rotating Machinery using Laser Doppler Velocimetry,” *ASME J. Vibration and Acoustics*, 116, pp. 326-331.
- [19] Bell, J.R. and Rothberg, S.J., 2000, “Laser Vibrometers and Contacting Transducers, Target Rotation and Six Degree-of-Freedom Vibration: What Do We Really Measure?,” *J. Sound and Vibration*, 237(2), pp. 245-261.
- [20] Rothberg, S.J., Baker, J.R. and Halliwell, N.A., 1989, “Laser Vibrometry: Pseudo-Vibrations,” *J. Sound and Vibration*, 135(3), pp. 516-522.
- [21] Bell, J.R. and Rothberg, S.J., 2000, “Rotational Vibration Measurements using Laser Vibrometry: Comprehensive Theory and Practical Application,” *J. Sound and Vibration*, 238(4), pp. 673-690.
- [22] ISO 10816-1:1995 “Mechanical Vibration – Evaluation of Machine Vibration by Measurements on Non-Rotating Parts – Part 1 General Guidelines.”

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Figure 7 – Simulated AC coupled Laser Vibrometer output for a measurement in the x direction in the presence of broadband torsional oscillation

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$$\overline{\Omega}_T = 100\pi \text{ rad/s}, \Delta\overline{\Omega}_T = 0.67 \times 10^{-3} \overline{\Omega}_T \text{ RMS (arbitrary phase)}, x_0 = 0.25 \text{ mm}, y_0 = 0.5 \text{ mm})$$

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