RISK ANALYSIS OF COMBINED RAINWATER DETENTION AND PUMPING SYSTEM

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1 Supplementary Material. Analytical derivation of Risk Models

In a conceptualized SSS, detention is achieved by a storage capacity (SU) and a pumping station (PS) ensures flow continuity to the final destination, because gravity drainage is not possible (Figure 1). Two set-ups are examined here: above-ground SU downstream of the PS (Figure 1a) and SU upstream of it (Figure 1b).

1.1 Risk of overflow of combined PS and above-ground SU

Assuming that, when rain begins to fall, the PS discharge capacity is $i Q_p$ because $n + r - i$ pumps have failed, the probability that the inflow discharge to the SU exceeds the pumps discharge capacity ($\varphi S h \tau^{-1} > i Q_p$) is:

$$R_{1a} = \int_0^\infty f_h \int_0^{\frac{\varphi S h}{i Q_p}} f_\tau d\tau dh = \int_0^\infty f_\tau \int_{\frac{i Q_p \tau}{\varphi S h}}^\infty f_h dh d\tau \quad (1)$$

Although $\tau > \varphi S h (i Q_p)^{-1}$ or equivalently $h < i Q_p \tau (\varphi S)^{-1}$, so that no failure occurs at the PS, the system shown in Figure 1a, may overflow

Figure 1: Conceptual model of low impact structural measures for storm sewer system (SSS). SU =storage unit, PS=pumping station.

even at SU. Assuming that the SU is full at the end of the first of two consecutive rain events, if their inter-arrival time is: $t > V_s \cdot (Q_0)^{-1}$, the SU is empty before the second rainfall event and overflow occurs if $i Q_p \tau > V_s$. Conversely, if $t < V_s \cdot (Q_0)^{-1}$, the residual storage capacity available for flood control is $t \cdot Q_0 < V_s$ and overflow occurs if $i Q_p \tau > Q_0 t$. Thus, the risk of SU overflow, provided that the PS did not undergo any ($Q < i Q_p$) is:

$$R_{2a}(i) = \int_{IETD}^{\frac{V_s}{Q_0}} f_t \int_{\frac{Q_0 t}{i Q_p}}^{\infty} f_{\tau} \int_0^{\frac{i Q_p \tau}{\varphi S}} f_h dh d\tau dt + \int_{\frac{V_s}{Q_0}}^{\infty} f_t \int_{\frac{V_s}{i Q_p}}^{\infty} f_{\tau} \int_0^{\frac{i Q_p \tau}{\varphi S}} f_h dh d\tau dt \quad (2)$$

R_{1a} and R_{2a} may be written as a function of four dimensionless groups: k_{ζ} , k_{λ} , $k_{\psi,0}$ and $k_{T,a}$

$$R_{1a}(i) = \frac{k_{\lambda}}{k_{\zeta} + k_{\lambda}} \quad (3)$$

$$R_{2a}(i) = \frac{k_{\psi,0} \cdot e^{-K_{T,a}(k_{\lambda}+k_{\psi,0})} - k_{\lambda} \cdot e^{-(k_{\lambda}+k_{\psi,0})}}{k_{\lambda} + k_{\psi,0}} + \frac{k_{\lambda} \cdot \left[e^{-k_{T,a}(k_{\zeta}+k_{\lambda}+k_{\psi,0})} - e^{-(k_{\zeta}+k_{\lambda}+k_{\psi,0})} \right]}{k_{\zeta} + k_{\lambda} + k_{\psi,0}} \quad (4)$$

where

$$k_\zeta = \frac{\zeta V_s}{\varphi S} \quad (5)$$

$$k_\lambda = \frac{\lambda V_s}{i Q_p} \quad (6)$$

$$k_{\psi,0} = \frac{\psi V_s}{Q_0} \quad (7)$$

$$k_{T,a} = \frac{IETD Q_0}{V_s} \quad (8)$$

Combining equations (3) and (4), the risk of overflow for a SSS with PS and above ground SU is:

$$R_a = R_{1a}(i) + R_{2a}(i) \quad (9)$$

Under the non-conservative assumption that the SU is empty at the onset of each rain event, the long inter-arrival time risk of SU overflow is:

$$R'_{2a}(i) = \int_{\frac{V_s}{iQ_p}}^{\infty} f_\tau \int_0^{\frac{iQ_p \tau}{\varphi S}} f_h dh d\tau = e^{-k_\lambda} - \frac{k_\lambda}{k_\zeta + k_\lambda} e^{-(k_\zeta + k_\lambda)} \quad (10)$$

Equation(10) corresponds to the lower limit of equation (4) for small $k_{\psi,0}$ and $k_{T,a}$.

1.2 Risk of overflow of combined PS and below-ground SU

The PS is equipped with $n + r$ pumps each one having discharge capacity Q_p . When rain starts to fall, $n + r - i$ pumps are not working.

Examining two consecutive rain events, and assuming that the tank is full at the end of the first event, if the event inter-arrival time is: $t > V_s \cdot (iQ_p)^{-1}$, the tank is empty at the beginning of the second rain event and no overflow can occur if $\varphi S h < V_s$. Conversely, if the inter-arrival time of two consecutive rainfall events is shorter than the draw down time of V_s , i.e.: $t < V_s \cdot (iQ_p)^{-1}$, the available storage volume at the onset of the second rainfall event is $i Q_p t < V_s$. The probability R_b of an overflow is:

$$R_b(i) = \int_{IETD}^{\frac{V_s}{i Q_p}} f_t \int_{\frac{i Q_p t}{\varphi S}}^{\infty} f_h dh dt + \int_{\frac{V_s}{i Q_p}}^{\infty} f_t \int_{\frac{V_s}{\varphi S}}^{\infty} f_h dh dt \quad (11)$$

R_b may be written as a function of four dimensionless groups: k_ζ , k_λ , $k_{T,b}$ and k_ψ .

$$R_b = \frac{k_\psi}{k_\psi + k_\zeta} \cdot e^{-k_{T,b}(k_\psi + k_\zeta)} + \frac{k_\zeta}{k_\psi + k_\zeta} \cdot e^{-(k_\psi + k_\zeta)} \quad (12)$$

where

$$k_{T,b} = \frac{IETD i Q_p}{V_s} \quad (13)$$

and

$$k_\psi = \frac{\psi V_s}{i Q_p} \quad (14)$$

For $Q_p = Q_0 n^{-1}$,

$$k_\psi = \frac{n}{i} k_{\psi,0} \quad (15)$$

Assuming that the tank is empty at the onset of each rain event, the long-inter-arrival-time limit of the risk of overflow is:

$$R'_b(i) = \int_{\frac{V_s}{\varphi S}}^{\infty} f_h dh = e^{-k_\zeta} \quad (16)$$

Equation (16) corresponds to the lower limit of equation (12) and may be derived for $k_\psi \ll 1$.

1.3 Risk of insufficient water quality control

The risk of insufficient detention time, i.e. the probability that the detention time of the second of two consecutive rain events, with inter-arrival t , is less than the design detention time T_d , is evaluated under the assumption that the tank is full after the first of the two consecutive rain events. In this way, risk analysis of water quality control is consistent with the estimate of the overflow risk, even though the former is not conservative. The risks of insufficient water treatment are:

$$R_{T_d,a} = \int_{IETD}^{\frac{V_s}{Q_0}} f_t \int_0^{\frac{Q_0(T_d+t)-V_s}{iQ_p}} f_\tau d\tau dt + \int_{\frac{V_s}{Q_0}}^{\infty} f_t \int_0^{\frac{Q_0T_d}{iQ_p}} f_\tau d\tau dt \quad (17)$$

$$R_{T_d,b} = \int_{IETD}^{\frac{V_s}{iQ_p}} f_t \int_0^{\frac{iQ_p(T_d+t)-V_s}{\varphi S}} f_h dh dt + \int_{\frac{V_s}{iQ_p}}^{\infty} f_t \int_0^{\frac{iQ_pT_d}{\varphi S}} f_h dh dt \quad (18)$$

for SSS with above- and below-ground SU respectively.

Integration of equations (17) and (18) gives:

$$R_{T_d,a} = e^{-k_{T,a}k_{\psi 0}} - \frac{k_\lambda e^{-(k_{\lambda,T_d}+k_{\psi 0})} + k_{\psi 0} e^{[-k_{\lambda,T_d}-k_{T,a}k_{\psi 0}-k_\lambda(k_{T,a}-1)]}}{k_\lambda + k_{\psi 0}} \quad (19)$$

$$R_{T_d,b} = e^{-k_{T,b}k_\psi} - \frac{k_\zeta e^{-(k_{\zeta,T_d}+k_\psi)} + k_\psi e^{[-k_{\zeta,T_d}+k_{T,b}k_\psi-k_\zeta(k_{T,b}-1)]}}{k_\zeta + k_\psi} \quad (20)$$

where

$$k_{\lambda,T_d} = \frac{\lambda Q_0 T_d}{iQ_p} \quad (21)$$

and

$$k_{\zeta,T_d} = \frac{\zeta iQ_p T_d}{\varphi S} \quad (22)$$

Assuming that the SU is empty at the beginning of each rain event, the risks of insufficient detention time for SSS with above and below ground SU become:

$$R'_{T_d,a} = \int_0^{\frac{Q_0T_d}{iQ_p}} f_\tau d\tau = 1 - e^{-k_{\lambda,T_d}} \quad (23)$$

$$R'_{T_d,b} = \int_0^{\frac{iQ_pT_d}{\varphi S}} f_h dh = 1 - e^{-k_{\zeta,T_d}} \quad (24)$$