Supplementary Materials for QQ Models: Joint Modeling for Quantitative and Qualitative Quality Responses in Manufacturing Systems

Xinwei Deng and Ran Jin*

*Department of Statistics and Grado Department of Industrial and Systems Engineering
Virginia Tech*

A. The Description of Quadratic Approximation of $l(\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)})$ in (2.6)

We start to detail the quadratic approximation with respect to the parameters $\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}$. Note that the log-likelihood function $l(\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)})$ in (2.6) can be written as

$$l(\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}) = g(\boldsymbol{\eta}) + h(\boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}), \quad (1)$$

where

$$g(\boldsymbol{\eta}) = \sum_{i=1}^n \{z_i \mathbf{x}_i' \boldsymbol{\eta} - \log(1 + \exp(\mathbf{x}_i' \boldsymbol{\eta}))\}. \quad (2)$$

It is known that

$$\begin{aligned} \frac{\partial g(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} &= \sum_{i=1}^n \mathbf{x}_i (z_i - p(\mathbf{x}_i, \boldsymbol{\eta})) = \mathbf{X}^T (\mathbf{z} - \mathbf{p}), \\ \frac{\partial^2 g(\boldsymbol{\eta})}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}^T} &= - \sum_{i=1}^n \mathbf{x}_i \mathbf{x}_i^T p(\mathbf{x}_i, \boldsymbol{\eta}) (1 - p(\mathbf{x}_i, \boldsymbol{\eta})) = -\mathbf{X}^T \mathbf{W} \mathbf{X}, \end{aligned}$$

where $p(\mathbf{x}_i, \boldsymbol{\eta}) = \frac{\exp(\mathbf{x}_i' \boldsymbol{\eta})}{1 + \exp(\mathbf{x}_i' \boldsymbol{\eta})}$ and the n -dimensional vector $\mathbf{p} = (p(\mathbf{x}_1, \boldsymbol{\eta}), \dots, p(\mathbf{x}_n, \boldsymbol{\eta}))'$. Here $\mathbf{W}$ is a weight matrix as $\mathbf{W} = \text{diag}(p(\mathbf{x}_1, \boldsymbol{\eta})(1 - p(\mathbf{x}_1, \boldsymbol{\eta})), \dots, p(\mathbf{x}_n, \boldsymbol{\eta})(1 - p(\mathbf{x}_n, \boldsymbol{\eta})))$, which

*Address for correspondence: Ran Jin, Grado Department of Industrial and Systems Engineering, Virginia Tech, Blacksburg, VA 24061 (E-mail: jran5@vt.edu).

is an $n \times n$ diagonal matrix. Given an current estimate $\boldsymbol{\eta}_c$, a quadratic approximation can be written as

$$\begin{aligned} g(\boldsymbol{\eta}) &= g(\boldsymbol{\eta}_c) + (\boldsymbol{\eta} - \boldsymbol{\eta}_c)^T \mathbf{X}^T (\mathbf{z} - \mathbf{p}) - \frac{1}{2} (\boldsymbol{\eta} - \boldsymbol{\eta}_c)^T \mathbf{X}^T \mathbf{W} \mathbf{X} (\boldsymbol{\eta} - \boldsymbol{\eta}_c) \\ &= C_1 - \frac{1}{2} \boldsymbol{\eta}^T \mathbf{X}^T \mathbf{W} \mathbf{X} \boldsymbol{\eta} + \boldsymbol{\eta}^T \mathbf{X}^T \mathbf{W} [\mathbf{X} \boldsymbol{\eta}_c + \mathbf{W}^{-1} (\mathbf{z} - \mathbf{p})] \\ &= C_2 - \frac{1}{2} (\tilde{\mathbf{z}} - \mathbf{X} \boldsymbol{\eta})^T \mathbf{W} (\tilde{\mathbf{z}} - \mathbf{X} \boldsymbol{\eta}), \end{aligned}$$

where $\tilde{\mathbf{z}} = \mathbf{X} \boldsymbol{\eta}_c + \mathbf{W}^{-1} (\mathbf{z} - \mathbf{p})$ with $\mathbf{W} = \text{diag}(p(\mathbf{x}_1, \boldsymbol{\eta}_c)(1 - p(\mathbf{x}_1, \boldsymbol{\eta}_c)), \dots, p(\mathbf{x}_n, \boldsymbol{\eta}_c)(1 - p(\mathbf{x}_n, \boldsymbol{\eta}_c)))$, and $\mathbf{p} = (p(\mathbf{x}_1, \boldsymbol{\eta}_c), \dots, p(\mathbf{x}_n, \boldsymbol{\eta}_c))'$.

For the log-likelihood function $h(\boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)})$ of the continuous response y , it is straightforward to have

$$h(\boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}) = -\frac{n}{2} \log(\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^n \{z_i (y_i - \mathbf{x}_i' \boldsymbol{\beta}^{(1)})^2 + (1 - z_i) (y_i - \mathbf{x}_i' \boldsymbol{\beta}^{(2)})^2\} \quad (3)$$

Without loss of the generality, we assume that the first n_1 observations with binary response $z = 1$, and the remaining n_2 observations with response $z = 0$, where $n = n_1 + n_2$. Then, we can define $\mathbf{y}_{(1)} = (y_1, \dots, y_{n_1})^T$ and $\mathbf{y}_{(2)} = (y_{n_1+1}, \dots, y_n)^T$. Similarly, we can correspondingly define $\mathbf{X}_{(1)}$ and $\mathbf{X}_{(2)}$, respectively. Therefore, we can rewrite $h(\boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)})$ in (3) as

$$-\frac{n}{2} \log(\sigma^2) - \frac{1}{2\sigma^2} [(\mathbf{y}_{(1)} - \mathbf{X}_{(1)} \boldsymbol{\beta}^{(1)})^T (\mathbf{y}_{(1)} - \mathbf{X}_{(1)} \boldsymbol{\beta}^{(1)}) + (\mathbf{y}_{(2)} - \mathbf{X}_{(2)} \boldsymbol{\beta}^{(2)})^T (\mathbf{y}_{(2)} - \mathbf{X}_{(2)} \boldsymbol{\beta}^{(2)})] \quad (4)$$

The negative log-likelihood function can be further written as

$$\begin{aligned} -2l(\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}) &= (\tilde{\mathbf{z}} - \mathbf{X} \boldsymbol{\eta})^T \mathbf{W} (\tilde{\mathbf{z}} - \mathbf{X} \boldsymbol{\eta}) + n \log(\sigma^2) \\ &\quad + \frac{1}{\sigma^2} [(\mathbf{y}_{(1)} - \mathbf{X}_{(1)} \boldsymbol{\beta}^{(1)})^T (\mathbf{y}_{(1)} - \mathbf{X}_{(1)} \boldsymbol{\beta}^{(1)}) + (\mathbf{y}_{(2)} - \mathbf{X}_{(2)} \boldsymbol{\beta}^{(2)})^T (\mathbf{y}_{(2)} - \mathbf{X}_{(2)} \boldsymbol{\beta}^{(2)})], \end{aligned} \quad (5)$$

given an current estimation of $\boldsymbol{\eta}_c$. Then we use the nonnegative garrote approach for re-parameterizing and estimating the parameters $\boldsymbol{\eta}, \boldsymbol{\beta}^{(1)}, \boldsymbol{\beta}^{(2)}$. Specifically, we approximate the

original constrained optimization in (2.6) by

$$\begin{aligned}
& \min \left\{ (\tilde{\mathbf{z}} - \mathbf{X}\boldsymbol{\eta})^T \mathbf{W}(\tilde{\mathbf{z}} - \mathbf{X}\boldsymbol{\eta}) + n \log(\sigma^2) \right. \\
& \quad \left. + \frac{1}{\sigma^2} [(\mathbf{y}_{(1)} - \mathbf{X}_{(1)}\boldsymbol{\beta}^{(1)})^T (\mathbf{y}_{(1)} - \mathbf{X}_{(1)}\boldsymbol{\beta}^{(1)}) + (\mathbf{y}_{(2)} - \mathbf{X}_{(2)}\boldsymbol{\beta}^{(2)})^T (\mathbf{y}_{(2)} - \mathbf{X}_{(2)}\boldsymbol{\beta}^{(2)})] \right\} \\
& s.t. \quad \beta_k^{(1)} = \theta_k^{(1)} \tilde{\beta}_k^{(1)}, \beta_k^{(2)} = \theta_k^{(2)} \tilde{\beta}_k^{(2)}, \eta_k = \tau_k \tilde{\eta}_k; \\
& \quad \theta_k^{(1)} + \theta_k^{(2)} + \tau_k \leq t_k, k = 1, \dots, p; \\
& \quad \tau_k \leq \theta_k^{(1)} + \theta_k^{(2)}, k = 1, \dots, p; \\
& \quad \theta_k^{(1)} \geq 0, \theta_k^{(2)} \geq 0, \tau_k \geq 0, k = 1, \dots, p; \\
& \quad \sum_{k=1}^p t_k \leq M, t_k \geq 0.
\end{aligned} \tag{6}$$

B. Simulation Results in Section 4

Table 1: Averages and Standard Errors of Number of False Positives from 50 Simulation Runs.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	1.74 (0.15)	2.44 (0.13)	5.31 (0.24)	1.92 (0.16)
		BIC_{lm}	0.84 (0.16)	0.32 (0.07)	0.60 (0.14)	0.24 (0.07)
		Add_{lm}	0.82 (0.16)	0.38 (0.07)	0.60 (0.14)	0.28 (0.07)
		QQ_{logit}	0.00 (0.00)	0.00 (0.00)	0.06 (0.03)	0.02 (0.02)
		BIC_{logit}	4.78 (0.40)	0.64 (0.13)	4.02 (0.36)	0.88 (0.12)
		Add_{logit}	5.46 (0.41)	0.90 (0.16)	4.70 (0.40)	1.24 (0.12)
1	50	QQ_{lm}	2.07 (0.35)	0.09 (0.04)	10.98 (0.55)	1.02 (0.18)
		BIC_{lm}	3.08 (0.40)	1.34 (0.18)	2.86 (0.34)	1.66 (0.22)
		Add_{lm}	3.04 (0.39)	1.34 (0.18)	2.90 (0.35)	1.88 (0.22)
		QQ_{logit}	0.00 (0.00)	0.08 (0.04)	0.14 (0.05)	0.02 (0.02)
		BIC_{logit}	2.36 (0.26)	1.36 (0.18)	3.22 (0.28)	1.90 (0.19)
		Add_{logit}	2.52 (0.27)	1.72 (0.15)	3.68 (0.32)	1.78 (0.18)
2	20	QQ_{lm}	1.95 (0.15)	0.98 (0.13)	0.33 (0.08)	0.17 (0.05)
		BIC_{lm}	3.14 (0.11)	3.28 (0.12)	3.51 (0.13)	3.26 (0.16)
		Add_{lm}	3.11 (0.11)	3.27 (0.12)	3.51 (0.14)	3.28 (0.15)
		QQ_{logit}	1.04 (0.12)	1.18 (0.13)	0.74 (0.14)	0.70 (0.11)
		BIC_{logit}	4.42 (0.41)	0.86 (0.12)	4.80 (0.37)	1.00 (0.12)
		Add_{logit}	5.08 (0.46)	1.16 (0.17)	5.14 (0.39)	1.34 (0.17)
2	50	QQ_{lm}	0.60 (0.19)	0.11 (0.04)	0.19 (0.06)	0.98 (0.18)
		BIC_{lm}	7.36 (0.45)	5.57 (0.28)	8.13 (0.37)	6.00 (0.27)
		Add_{lm}	7.10 (0.45)	5.40 (0.29)	7.77 (0.37)	5.91 (0.27)
		QQ_{logit}	0.40 (0.10)	0.88 (0.12)	1.12 (0.12)	0.88 (0.12)
		BIC_{logit}	2.40 (0.24)	1.30 (0.16)	3.20 (0.26)	1.94 (0.17)
		Add_{logit}	2.44 (0.25)	1.66 (0.18)	3.52 (0.24)	1.90 (0.20)
3	20	QQ_{lm}	0.12 (0.06)	0.01 (0.01)	0.51 (0.12)	0.00 (0.00)
		BIC_{lm}	0.72 (0.11)	0.34 (0.09)	0.80 (0.14)	0.48 (0.08)
		Add_{lm}	0.84 (0.14)	0.32 (0.09)	0.74 (0.15)	0.40 (0.09)
		QQ_{logit}	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
		BIC_{logit}	4.42 (0.41)	0.78 (0.14)	4.48 (0.41)	0.54 (0.09)
		Add_{logit}	4.98 (0.43)	1.22 (0.19)	4.78 (0.39)	0.70 (0.11)
3	50	QQ_{lm}	1.55 (0.34)	0.07 (0.04)	0.74 (0.15)	0.08 (0.04)
		BIC_{lm}	3.28 (0.38)	2.20 (0.26)	4.04 (0.45)	2.22 (0.21)
		Add_{lm}	3.32 (0.39)	2.32 (0.28)	4.26 (0.46)	2.12 (0.21)
		QQ_{logit}	0.04 (0.03)	0.08 (0.04)	0.02 (0.02)	0.04 (0.03)
		BIC_{logit}	2.86 (0.24)	1.42 (0.17)	3.50 (0.28)	1.64 (0.16)
		Add_{logit}	2.80 (0.22)	1.58 (0.17)	3.60 (0.31)	2.00 (0.21)
4	20	QQ_{lm}	1.96 (0.16)	0.24 (0.06)	0.06 (0.04)	2.21 (0.14)
		BIC_{lm}	3.05 (0.15)	2.49 (0.14)	3.33 (0.14)	3.16 (0.13)
		Add_{lm}	3.01 (0.14)	2.41 (0.14)	3.31 (0.14)	3.15 (0.13)
		QQ_{logit}	0.94 (0.12)	0.32 (0.07)	0.28 (0.07)	1.46 (0.14)
		BIC_{logit}	4.36 (0.43)	0.82 (0.13)	3.82 (0.32)	0.94 (0.12)
		Add_{logit}	5.02 (0.43)	1.28 (0.17)	4.28 (0.36)	1.26 (0.13)
4	50	QQ_{lm}	1.52 (0.29)	0.12 (0.05)	0.08 (0.04)	0.34 (0.10)
		BIC_{lm}	7.86 (0.44)	4.23 (0.23)	8.25 (0.46)	4.99 (0.27)
		Add_{lm}	7.58 (0.39)	4.06 (0.22)	8.02 (0.40)	4.93 (0.25)
		QQ_{logit}	0.18 (0.07)	0.94 (0.13)	1.30 (0.15)	1.10 (0.13)
		BIC_{logit}	2.14 (0.25)	1.36 (0.17)	2.68 (0.27)	1.74 (0.18)
		Add_{logit}	2.74 (0.26)	1.60 (0.17)	3.06 (0.28)	1.90 (0.17)

Table 2: Averages and Standard Errors of Number of False Negatives from 50 Simulation Runs.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
		BIC_{lm}	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
		Add_{lm}	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
		QQ_{logit}	0.50 (0.10)	1.52 (0.14)	1.78 (0.12)	3.72 (0.20)
		BIC_{logit}	1.08 (0.12)	4.16 (0.17)	1.54 (0.13)	6.16 (0.17)
		Add_{logit}	1.04 (0.14)	4.68 (0.26)	1.70 (0.16)	6.98 (0.24)
1	50	QQ_{lm}	0.81 (0.13)	20.70 (0.53)	0.36 (0.09)	6.26 (0.27)
		BIC_{lm}	0.00 (0.00)	0.38 (0.07)	0.00 (0.00)	1.60 (0.13)
		Add_{lm}	0.00 (0.00)	0.38 (0.07)	0.00 (0.00)	1.56 (0.13)
		QQ_{logit}	8.04 (0.17)	16.64 (0.31)	5.60 (0.21)	20.48 (0.34)
		BIC_{logit}	6.76 (0.19)	21.76 (0.27)	7.82 (0.17)	24.90 (0.21)
		Add_{logit}	6.94 (0.25)	23.76 (0.45)	8.88 (0.28)	26.20 (0.38)
2	20	QQ_{lm}	0.00 (0.00)	0.78 (0.06)	0.49 (0.01)	1.43 (0.05)
		BIC_{lm}	1.26 (0.09)	3.10 (0.15)	1.05 (0.09)	3.54 (0.17)
		Add_{lm}	1.27 (0.09)	3.15 (0.16)	1.03 (0.09)	3.62 (0.16)
		QQ_{logit}	2.50 (0.13)	4.00 (0.16)	2.86 (0.12)	6.32 (0.20)
		BIC_{logit}	1.16 (0.14)	4.10 (0.21)	1.32 (0.14)	5.78 (0.19)
		Add_{logit}	1.36 (0.14)	4.54 (0.20)	1.52 (0.15)	6.28 (0.23)
2	50	QQ_{lm}	0.52 (0.10)	24.13 (0.43)	2.09 (0.15)	5.48 (0.34)
		BIC_{lm}	6.66 (0.28)	11.23 (0.45)	6.01 (0.23)	11.96 (0.38)
		Add_{lm}	7.00 (0.30)	11.98 (0.47)	6.59 (0.24)	12.25 (0.41)
		QQ_{logit}	10.16 (0.22)	20.08 (0.31)	9.38 (0.22)	21.60 (0.33)
		BIC_{logit}	6.76 (0.16)	21.10 (0.31)	7.88 (0.15)	24.72 (0.23)
		Add_{logit}	6.84 (0.16)	22.28 (0.42)	8.24 (0.18)	25.44 (0.34)
3	20	QQ_{lm}	0.04 (0.03)	0.80 (0.04)	0.47 (0.05)	4.11 (0.09)
		BIC_{lm}	0.00 (0.00)	0.66 (0.08)	0.34 (0.07)	2.54 (0.10)
		Add_{lm}	0.00 (0.00)	0.68 (0.08)	0.36 (0.07)	2.44 (0.10)
		QQ_{logit}	1.78 (0.15)	7.14 (0.24)	1.70 (0.12)	7.22 (0.17)
		BIC_{logit}	1.12 (0.11)	4.04 (0.20)	1.38 (0.15)	5.70 (0.19)
		Add_{logit}	1.56 (0.16)	4.76 (0.26)	1.50 (0.15)	6.04 (0.23)
3	50	QQ_{lm}	0.86 (0.13)	23.75 (0.43)	2.04 (0.17)	16.98 (0.36)
		BIC_{lm}	0.00 0.00	6.68 (0.38)	1.04 (0.13)	8.86 (0.30)
		Add_{lm}	0.04 (0.03)	6.60 (0.39)	1.18 (0.14)	9.08 (0.32)
		QQ_{logit}	6.78 (0.27)	17.50 (0.32)	7.18 (0.20)	15.96 (0.31)
		BIC_{logit}	7.02 (0.20)	22.12 (0.22)	7.74 (0.20)	24.28 (0.21)
		Add_{logit}	7.54 (0.30)	23.28 (0.38)	8.28 (0.27)	26.04 (0.40)
4	20	QQ_{lm}	0.00 (0.00)	0.11 (0.03)	0.03 (0.02)	0.02 (0.01)
		BIC_{lm}	1.49 (0.11)	3.21 (0.17)	1.23 (0.10)	2.24 (0.14)
		Add_{lm}	1.53 (0.10)	3.55 (0.19)	1.23 (0.10)	2.25 (0.14)
		QQ_{logit}	1.44 (0.14)	6.04 (0.26)	3.22 (0.10)	3.54 (0.20)
		BIC_{logit}	1.02 (0.12)	4.16 (0.18)	1.48 (0.13)	6.38 (0.17)
		Add_{logit}	1.10 (0.15)	4.92 (0.24)	1.64 (0.15)	6.98 (0.22)
4	50	QQ_{lm}	1.17 (0.11)	19.36 (0.39)	4.89 (0.16)	11.21 (0.39)
		BIC_{lm}	6.34 (0.26)	16.81 (0.49)	7.17 (0.25)	16.67 (0.35)
		Add_{lm}	7.02 (0.23)	17.92 (0.43)	7.32 (0.24)	16.95 (0.36)
		QQ_{logit}	10.40 (0.26)	18.48 (0.35)	9.66 (0.26)	19.24 (0.28)
		BIC_{logit}	6.56 (0.18)	22.12 (0.24)	7.66 (0.21)	24.24 (0.25)
		Add_{logit}	7.28 (0.22)	23.24 (0.37)	7.92 (0.23)	25.24 (0.35)

Table 3: Averages and Standard Errors of RMSPEs (or MEs) from 50 Simulation Runs for the error of Chi-squares.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	0.71 (0.01)	0.75 (0.01)	0.71 (0.01)	0.71 (0.01)
		BIC_{lm}	0.69 (0.01)	0.72 (0.01)	0.71 (0.01)	0.69 (0.01)
		QQ_{logit}	0.11 (0.01)	0.12 (0.01)	0.09 (0.00)	0.10 (0.00)
		BIC_{logit}	0.14 (0.01)	0.14 (0.01)	0.12 (0.01)	0.12 (0.01)
1	50	QQ_{lm}	0.85 (0.03)	5.31 (0.60)	0.94 (0.03)	2.19 (0.27)
		BIC_{lm}	0.79 (0.01)	0.97 (0.02)	0.81 (0.01)	1.00 (0.02)
		QQ_{logit}	0.16 (0.01)	0.24 (0.01)	0.18 (0.01)	0.20 (0.01)
		BIC_{logit}	0.17 (0.01)	0.27 (0.01)	0.18 (0.01)	0.22 (0.01)
2	20	QQ_{lm}	0.81 (0.02)	0.86 (0.01)	0.75 (0.01)	0.91 (0.02)
		BIC_{lm}	3.44 (0.04)	3.93 (0.05)	3.21 (0.04)	4.15 (0.06)
		QQ_{logit}	0.24 (0.01)	0.12 (0.01)	0.11 (0.01)	0.14 (0.01)
		BIC_{logit}	0.14 (0.01)	0.14 (0.01)	0.13 (0.01)	0.12 (0.01)
2	50	QQ_{lm}	3.37 (0.14)	11.25 (0.33)	3.46 (0.19)	10.65 (0.59)
		BIC_{lm}	6.97 (0.11)	10.00 (0.17)	7.77 (0.12)	9.51 (0.14)
		QQ_{logit}	0.23 (0.01)	0.25 (0.01)	0.31 (0.01)	0.20 (0.01)
		BIC_{logit}	0.18 (0.01)	0.25 (0.01)	0.16 (0.01)	0.24 (0.01)
3	20	QQ_{lm}	0.73 (0.01)	0.93 (0.03)	0.76 (0.01)	0.84 (0.01)
		BIC_{lm}	2.34 (0.03)	3.70 (0.05)	2.22 (0.03)	2.94 (0.04)
		QQ_{logit}	0.10 (0.01)	0.14 (0.01)	0.09 (0.01)	0.10 (0.01)
		BIC_{logit}	0.14 (0.01)	0.12 (0.01)	0.12 (0.01)	0.13 (0.01)
3	50	QQ_{lm}	1.73 (0.08)	10.65 (0.31)	1.58 (0.09)	8.24 (0.56)
		BIC_{lm}	3.25 (0.05)	7.98 (0.16)	4.22 (0.06)	6.42 (0.10)
		QQ_{logit}	0.16 (0.01)	0.22 (0.01)	0.16 (0.01)	0.20 (0.01)
		BIC_{logit}	0.17 (0.01)	0.25 (0.01)	0.17 (0.01)	0.24 (0.01)
4	20	QQ_{lm}	0.80 (0.02)	1.43 (0.13)	0.75 (0.01)	0.90 (0.03)
		BIC_{lm}	4.18 (0.05)	5.25 (0.07)	4.48 (0.04)	4.63 (0.06)
		QQ_{logit}	0.11 (0.00)	0.17 (0.01)	0.10 (0.00)	0.11 (0.01)
		BIC_{logit}	0.14 (0.01)	0.14 (0.01)	0.13 (0.01)	0.12 (0.01)
4	50	QQ_{lm}	4.14 (0.27)	11.05 (0.26)	2.69 (0.13)	11.13 (0.53)
		BIC_{lm}	7.96 (0.12)	13.06 (0.19)	6.93 (0.11)	10.90 (0.13)
		QQ_{logit}	0.27 (0.01)	0.28 (0.01)	0.28 (0.01)	0.20 (0.01)
		BIC_{logit}	0.18 (0.01)	0.27 (0.01)	0.17 (0.01)	0.24 (0.01)

Table 4: Averages and Standard Errors of Number of False Selections from 50 Simulation Runs for the error of Chi-squares.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	0.05 (0.04)	0.55 (0.13)	1.04 (0.12)	1.06 (0.18)
		BIC_{lm}	0.72 (0.10)	0.64 (0.15)	0.94 (0.13)	0.40 (0.11)
		QQ_{logit}	2.40 (0.11)	3.32 (0.21)	1.20 (0.12)	3.90 (0.21)
		BIC_{logit}	5.72 (0.57)	5.08 (0.35)	5.70 (0.48)	6.76 (0.30)
1	50	QQ_{lm}	7.02 (0.71)	8.35 (0.72)	4.94 (0.48)	5.84 (0.55)
		BIC_{lm}	2.20 (0.28)	2.14 (0.26)	3.80 (0.41)	1.86 (0.25)
		QQ_{logit}	5.34 (0.28)	16.12 (0.39)	6.88 (0.22)	17.42 (0.52)
		BIC_{logit}	8.42 (0.38)	23.10 (0.46)	11.94 (0.50)	25.68 (0.40)
2	20	QQ_{lm}	0.86 (0.03)	2.16 (0.20)	0.79 (0.12)	1.83 (0.18)
		BIC_{lm}	5.46 (0.25)	5.74 (0.30)	5.20 (0.28)	6.94 (0.29)
		QQ_{logit}	4.76 (0.14)	4.16 (0.29)	3.46 (0.28)	5.06 (0.29)
		BIC_{logit}	6.16 (0.61)	5.16 (0.32)	5.96 (0.54)	6.64 (0.27)
2	50	QQ_{lm}	5.69 (0.25)	22.67 (0.50)	3.15 (0.22)	23.72 (0.52)
		BIC_{lm}	14.32 (0.67)	18.88 (0.74)	15.66 (0.68)	19.24 (0.61)
		QQ_{logit}	9.10 (0.34)	20.52 (0.47)	11.72 (0.34)	20.96 (0.50)
		BIC_{logit}	9.26 (0.42)	23.24 (0.40)	10.52 (0.48)	26.42 (0.41)
3	20	QQ_{lm}	1.51 (0.14)	1.42 (0.13)	2.71 (0.21)	0.73 (0.15)
		BIC_{lm}	0.58 (0.13)	2.80 (0.22)	1.18 (0.21)	1.24 (0.16)
		QQ_{logit}	0.52 (0.10)	3.20 (0.20)	0.98 (0.12)	2.98 (0.22)
		BIC_{logit}	5.32 (0.54)	4.28 (0.28)	6.04 (0.42)	6.80 (0.32)
3	50	QQ_{lm}	2.58 (0.42)	18.22 (0.51)	1.40 (0.25)	20.93 (0.45)
		BIC_{lm}	4.08 (0.47)	8.10 (0.59)	4.20 (0.43)	7.98 (0.53)
		QQ_{logit}	6.52 (0.25)	15.24 (0.37)	7.22 (0.23)	16.72 (0.35)
		BIC_{logit}	9.12 (0.45)	22.20 (0.40)	11.22 (0.51)	26.16 (0.35)
4	20	QQ_{lm}	1.25 (0.15)	1.24 (0.12)	1.96 (0.15)	2.59 (0.16)
		BIC_{lm}	4.86 (0.29)	7.10 (0.32)	5.38 (0.29)	5.98 (0.31)
		QQ_{logit}	2.20 (0.26)	5.22 (0.36)	3.40 (0.23)	4.54 (0.33)
		BIC_{logit}	5.32 (0.52)	4.74 (0.33)	6.08 (0.50)	6.96 (0.32)
4	50	QQ_{lm}	8.12 (0.27)	24.03 (0.42)	3.92 (0.17)	17.65 (0.47)
		BIC_{lm}	13.38 (0.74)	22.68 (0.87)	14.16 (0.72)	21.44 (0.60)
		QQ_{logit}	9.86 (0.40)	21.62 (0.48)	11.40 (0.31)	19.74 (0.48)
		BIC_{logit}	9.28 (0.45)	23.10 (0.43)	10.82 (0.52)	25.94 (0.38)

Table 5: Averages and Standard Errors of RMSPEs (or MEs) from 50 Simulation Runs for the error of t distribution.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	0.51 (0.01)	0.52 (0.01)	0.50 (0.01)	0.52 (0.01)
		BIC_{lm}	0.55 (0.01)	0.62 (0.01)	0.55 (0.01)	0.65 (0.01)
		QQ_{logit}	0.09 (0.00)	0.17 (0.01)	0.23 (0.01)	0.10 (0.00)
		BIC_{logit}	0.14 (0.01)	0.13 (0.01)	0.13 (0.01)	0.11 (0.01)
1	50	QQ_{lm}	0.74 (0.03)	5.67 (0.68)	0.63 (0.02)	1.50 (0.32)
		BIC_{lm}	0.62 (0.01)	0.81 (0.01)	0.73 (0.02)	0.87 (0.02)
		QQ_{logit}	0.19 (0.01)	0.23 (0.01)	0.19 (0.01)	0.22 (0.01)
		BIC_{logit}	0.18 (0.01)	0.26 (0.01)	0.18 (0.01)	0.24 (0.01)
2	20	QQ_{lm}	0.63 (0.01)	0.72 (0.02)	0.56 (0.01)	0.80 (0.03)
		BIC_{lm}	4.03 (0.04)	4.25 (0.05)	3.70 (0.04)	4.41 (0.06)
		QQ_{logit}	0.11 (0.01)	0.19 (0.01)	0.10 (0.01)	0.18 (0.01)
		BIC_{logit}	0.14 (0.01)	0.13 (0.01)	0.13 (0.01)	0.12 (0.01)
2	50	QQ_{lm}	3.50 (0.21)	9.52 (0.29)	2.16 (0.15)	6.37 (0.44)
		BIC_{lm}	7.61 (0.10)	8.66 (0.14)	6.80 (0.13)	8.30 (0.11)
		QQ_{logit}	0.29 (0.01)	0.27 (0.01)	0.27 (0.01)	0.23 (0.00)
		BIC_{logit}	0.17 (0.01)	0.26 (0.01)	0.18 (0.01)	0.24 (0.01)
3	20	QQ_{lm}	0.56 (0.01)	0.66 (0.01)	0.52 (0.01)	0.63 (0.01)
		BIC_{lm}	2.05 (0.03)	3.28 (0.04)	1.25 (0.02)	3.12 (0.04)
		QQ_{logit}	0.10 (0.00)	0.13 (0.01)	0.11 (0.01)	0.09 (0.01)
		BIC_{logit}	0.12 (0.00)	0.12 (0.01)	0.13 (0.01)	0.11 (0.01)
3	50	QQ_{lm}	1.17 (0.10)	10.14 (0.31)	1.02 (0.06)	9.39 (0.63)
		BIC_{lm}	2.14 (0.03)	7.85 (0.13)	2.33 (0.04)	7.61 (0.12)
		QQ_{logit}	0.18 (0.01)	0.23 (0.01)	0.22 (0.01)	0.19 (0.01)
		BIC_{logit}	0.17 (0.01)	0.25 (0.01)	0.19 (0.01)	0.24 (0.01)
4	20	QQ_{lm}	0.60 (0.01)	1.22 (0.13)	0.59 (0.01)	1.44 (0.17)
		BIC_{lm}	4.17 (0.05)	5.79 (0.10)	3.34 (0.04)	5.80 (0.08)
		QQ_{logit}	0.11 (0.00)	0.15 (0.01)	0.10 (0.01)	0.11 (0.01)
		BIC_{logit}	0.15 (0.00)	0.14 (0.01)	0.13 (0.01)	0.12 (0.01)
4	50	QQ_{lm}	3.98 (0.22)	11.28 (0.26)	3.41 (0.17)	8.89 (0.55)
		BIC_{lm}	8.35 (0.10)	9.80 (0.14)	7.89 (0.13)	10.60 (0.17)
		QQ_{logit}	0.32 (0.01)	0.25 (0.01)	0.22 (0.01)	0.20 (0.01)
		BIC_{logit}	0.18 (0.01)	0.28 (0.01)	0.18 (0.01)	0.24 (0.01)

Table 6: Averages and Standard Errors of Number of False Selections from 50 Simulation Runs for the error of t-distribution.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	1.55 (0.17)	1.01 (0.12)	1.84 (0.17)	0.34 (0.10)
		BIC_{lm}	0.82 (0.13)	0.32 (0.08)	0.62 (0.12)	0.48 (0.11)
		QQ_{logit}	0.72 (0.09)	3.48 (0.20)	1.96 (0.12)	6.00 (0.35)
		BIC_{logit}	5.54 (0.51)	5.14 (0.32)	5.94 (0.49)	6.52 (0.30)
1	50	QQ_{lm}	4.66 (0.65)	16.99 (0.63)	4.35 (0.42)	5.47 (0.54)
		BIC_{lm}	2.38 (0.25)	2.54 (0.26)	5.56 (0.59)	1.80 (0.27)
		QQ_{logit}	6.38 (0.22)	15.88 (0.33)	8.28 (0.25)	20.14 (0.42)
		BIC_{logit}	9.56 (0.37)	23.62 (0.38)	10.82 (0.50)	26.32 (0.39)
2	20	QQ_{lm}	1.92 (0.17)	2.21 (0.13)	0.89 (0.12)	2.02 (0.08)
		BIC_{lm}	4.38 (0.26)	7.50 (0.30)	4.74 (0.25)	7.08 (0.27)
		QQ_{logit}	2.54 (0.21)	7.82 (0.38)	3.98 (0.27)	9.40 (0.26)
		BIC_{logit}	5.28 (0.61)	4.86 (0.33)	5.84 (0.48)	6.50 (0.30)
2	50	QQ_{lm}	2.25 (0.37)	24.72 (0.47)	2.90 (0.22)	19.69 (0.42)
		BIC_{lm}	14.38 (0.75)	18.88 (0.79)	15.28 (0.64)	19.46 (0.51)
		QQ_{logit}	10.90 (0.38)	23.54 (0.41)	10.46 (0.31)	24.02 (0.53)
		BIC_{logit}	8.94 (0.41)	23.02 (0.35)	10.72 (0.54)	27.00 (0.39)
3	20	QQ_{lm}	0.82 (0.13)	2.44 (0.17)	1.65 (0.20)	0.32 (0.08)
		BIC_{lm}	0.52 (0.10)	0.28 (0.07)	0.56 (0.14)	0.48 (0.13)
		QQ_{logit}	1.80 (0.19)	1.70 (0.15)	0.78 (0.11)	5.24 (0.37)
		BIC_{logit}	4.96 (0.50)	4.52 (0.26)	5.76 (0.60)	6.90 (0.29)
3	50	QQ_{lm}	3.73 (0.54)	16.03 (0.53)	3.16 (0.45)	14.68 (0.51)
		BIC_{lm}	3.82 (0.38)	7.08 (0.53)	3.46 (0.50)	8.00 (0.61)
		QQ_{logit}	6.08 (0.33)	15.64 (0.40)	7.66 (0.27)	15.32 (0.36)
		BIC_{logit}	9.42 (0.41)	22.84 (0.38)	11.40 (0.50)	26.52 (0.43)
4	20	QQ_{lm}	0.35 (0.10)	2.43 (0.16)	0.47 (0.13)	0.67 (0.11)
		BIC_{lm}	4.60 (0.28)	6.50 (0.44)	4.76 (0.24)	5.66 (0.32)
		QQ_{logit}	2.94 (0.22)	4.20 (0.31)	3.26 (0.26)	5.30 (0.38)
		BIC_{logit}	5.74 (0.60)	5.12 (0.32)	6.46 (0.46)	6.48 (0.40)
4	50	QQ_{lm}	5.59 (0.22)	19.29 (0.49)	3.72 (0.21)	14.22 (0.42)
		BIC_{lm}	14.14 (0.62)	16.14 (0.78)	15.12 (0.63)	18.40 (0.66)
		QQ_{logit}	9.36 (0.35)	18.12 (0.43)	10.78 (0.32)	19.36 (0.47)
		BIC_{logit}	9.64 (0.42)	23.14 (0.37)	11.70 (0.50)	26.44 (0.44)

Table 7: Averages and Standard Errors of RMSPEs (or MEs) from 50 Simulation Runs for the Nonlinear Models.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	465.34 (24.44)	3553.20 (152.56)	392.47 (21.27)	4292.52 (192.56)
		BIC_{lm}	491.37 (21.34)	3591.43 (149.28)	429.06 (20.14)	4623.38 (180.40)
		QQ_{logit}	0.13 (0.01)	0.21 (0.01)	0.11 (0.01)	0.20 (0.01)
		BIC_{logit}	0.14 (0.01)	0.14 (0.01)	0.14 (0.01)	0.12 (0.01)
1	50	QQ_{lm}	3313.35 (139.05)	9337.41 (555.86)	3883.74 (186.84)	32002.64 (1666.36)
		BIC_{lm}	3282.35 (116.56)	9757.60 (520.23)	3898.09 (151.11)	34150.83 (1293.14)
		QQ_{logit}	0.25 (0.01)	0.40 (0.01)	0.23 (0.01)	0.48 (0.01)
		BIC_{logit}	0.17 (0.01)	0.28 (0.01)	0.18 (0.01)	0.24 (0.01)
2	20	QQ_{lm}	509.70 (21.25)	2541.49 (125.98)	1558.53 (69.78)	4415.13 (211.45)
		BIC_{lm}	638.47 (18.91)	2905.41 (109.29)	1891.07 (62.16)	4752.12 (193.98)
		QQ_{logit}	0.13 (0.01)	0.20 (0.01)	0.11 (0.00)	0.19 (0.01)
		BIC_{logit}	0.13 (0.01)	0.14 (0.01)	0.12 (0.01)	0.11 (0.01)
2	50	QQ_{lm}	2944.76 (130.44)	9289.79 (422.27)	7714.07 (462.53)	35282.07 (1486.29)
		BIC_{lm}	3498.73 (122.61)	10049.66 (357.14)	8890.38 (409.72)	36948.96 (1161.63)
		QQ_{logit}	0.27 (0.01)	0.40 (0.01)	0.34 (0.01)	0.48 (0.01)
		BIC_{logit}	0.16 (0.01)	0.26 (0.01)	0.19 (0.01)	0.25 (0.01)
3	20	QQ_{lm}	309.00 (13.95)	3064.07 (138.68)	1261.34 (71.66)	4585.99 (192.82)
		BIC_{lm}	337.26 (12.11)	3438.73 (121.48)	1404.58 (66.18)	4944.75 (158.44)
		QQ_{logit}	0.13 (0.01)	0.19 (0.01)	0.13 (0.01)	0.19 (0.01)
		BIC_{logit}	0.14 (0.01)	0.13 (0.01)	0.13 (0.01)	0.12 (0.01)
3	50	QQ_{lm}	3130.30 (142.76)	12794.76 (591.95)	5695.97 (298.82)	36815.33 (1763.67)
		BIC_{lm}	3220.17 (116.43)	14220.74 (523.92)	5845.06 (254.49)	39289.07 (1414.08)
		QQ_{logit}	0.26 (0.01)	0.43 (0.01)	0.28 (0.01)	0.49 (0.01)
		BIC_{logit}	0.17 (0.01)	0.25 (0.01)	0.17 (0.01)	0.24 (0.01)
4	20	QQ_{lm}	858.99 (42.12)	1823.84 (100.41)	1424.29 (59.58)	5882.14 (352.76)
		BIC_{lm}	1141.31 (39.29)	2130.90 (90.58)	1740.13 (54.66)	6385.26 (303.59)
		QQ_{logit}	0.15 (0.01)	0.15 (0.01)	0.14 (0.01)	0.22 (0.01)
		BIC_{logit}	0.13 (0.01)	0.14 (0.01)	0.14 (0.01)	0.11 (0.01)
4	50	QQ_{lm}	2739.04 (120.82)	12054.76 (561.63)	2761.94 (126.10)	28668.37 (1919.03)
		BIC_{lm}	3290.43 (101.10)	13775.22 (565.53)	3417.18 (120.52)	33745.47 (1509.08)
		QQ_{logit}	0.25 (0.01)	0.41 (0.01)	0.23 (0.01)	0.46 (0.01)
		BIC_{logit}	0.17 (0.01)	0.27 (0.01)	0.18 (0.01)	0.24 (0.01)

Table 8: Averages and Standard Errors of Number of False Selections from 50 Simulation Runs for the Nonlinear Models.

Example	p	Method	$\rho = 0$		$\rho = 0.5$	
			$s = 20\%$	$s = 50\%$	$s = 20\%$	$s = 50\%$
1	20	QQ_{lm}	5.16 (0.41)	7.19 (0.55)	6.59 (0.43)	6.95 (0.40)
		BIC_{lm}	0.84 (0.19)	2.86 (0.36)	1.50 (0.25)	7.20 (0.31)
		QQ_{logit}	7.26 (0.36)	9.26 (0.49)	7.74 (0.40)	9.86 (0.45)
		BIC_{logit}	5.04 (0.50)	4.96 (0.30)	6.24 (0.49)	6.42 (0.24)
1	50	QQ_{lm}	16.50 (0.86)	28.03 (0.78)	17.34 (0.81)	28.16 (0.76)
		BIC_{lm}	7.94 (0.64)	19.86 (0.70)	9.84 (0.58)	24.56 (0.67)
		QQ_{logit}	15.68 (0.81)	32.48 (0.66)	14.36 (0.71)	34.84 (0.11)
		BIC_{logit}	9.00 (0.40)	22.92 (0.46)	10.48 (0.40)	25.94 (0.35)
2	20	QQ_{lm}	4.22 (0.37)	7.19 (0.47)	4.57 (0.39)	6.32 (0.41)
		BIC_{lm}	4.90 (0.32)	8.58 (0.34)	5.12 (0.32)	9.14 (0.33)
		QQ_{logit}	6.68 (0.38)	8.12 (0.41)	6.46 (0.38)	9.94 (0.48)
		BIC_{logit}	5.04 (0.56)	5.34 (0.30)	6.14 (0.56)	6.88 (0.31)
2	50	QQ_{lm}	17.07 (0.85)	27.63 (0.71)	15.11 (0.68)	28.59 (0.58)
		BIC_{lm}	15.20 (0.68)	25.68 (0.78)	16.18 (0.69)	26.30 (0.73)
		QQ_{logit}	15.74 (0.83)	31.86 (0.53)	14.70 (0.53)	34.80 (0.06)
		BIC_{logit}	8.80 (0.41)	23.32 (0.35)	11.42 (0.47)	26.28 (0.41)
3	20	QQ_{lm}	5.24 (0.40)	6.88 (0.45)	5.42 (0.40)	8.32 (0.43)
		BIC_{lm}	1.18 (0.23)	5.14 (0.37)	1.62 (0.29)	6.46 (0.33)
		QQ_{logit}	6.36 (0.33)	8.30 (0.52)	7.46 (0.42)	10.22 (0.43)
		BIC_{logit}	4.76 (0.53)	5.06 (0.33)	5.82 (0.48)	6.68 (0.29)
3	50	QQ_{lm}	17.34 (0.88)	28.26 (0.72)	15.33 (0.75)	29.00 (0.72)
		BIC_{lm}	7.38 (0.66)	22.24 (0.80)	10.98 (0.66)	25.20 (0.70)
		QQ_{logit}	16.16 (0.81)	33.64 (0.31)	15.16 (0.72)	34.86 (0.08)
		BIC_{logit}	9.20 (0.45)	22.90 (0.37)	10.60 (0.42)	26.16 (0.40)
4	20	QQ_{lm}	4.86 (0.38)	7.04 (0.44)	4.59 (0.42)	8.61 (0.39)
		BIC_{lm}	4.84 (0.32)	8.24 (0.41)	5.18 (0.35)	8.26 (0.34)
		QQ_{logit}	5.98 (0.41)	5.98 (0.38)	6.72 (0.43)	10.36 (0.45)
		BIC_{logit}	5.78 (0.56)	5.36 (0.34)	5.26 (0.45)	6.68 (0.30)
4	50	QQ_{lm}	16.92 (0.74)	28.01 (0.82)	17.74 (0.88)	29.08 (0.64)
		BIC_{lm}	15.66 (0.73)	25.72 (0.80)	16.26 (0.71)	27.10 (0.57)
		QQ_{logit}	15.94 (0.74)	32.76 (0.48)	16.48 (0.79)	34.66 (0.12)
		BIC_{logit}	9.22 (0.46)	22.96 (0.41)	11.74 (0.49)	26.48 (0.39)

C. List of Variables and Analysis Results in Section 5.

Table 9: Measured Predictor Variables and Quality Responses in the Lapping Process.

Type	Name	Definition
Lapping Process Variables	x_1 : Pressure (N/m^2)	The high pressure of the upper to lower plate
	x_2 : Rotation (Rpm)	The rotation speed
	x_3 : LPTime (Sec.)	The time for low pressure
	x_4 : HPTime (Sec.)	The time for high pressure
Quality Covariates before Lapping	x_5 : CTHK0 (μm)	Central thickness of wafers
	x_6 : TTV0 (μm)	Total thickness variation of wafers
	x_7 : TIR0 (μm)	Total indicator reading of wafers
	x_8 : STIR0 (μm)	Site total indicator reading (STIR) of wafers
	x_9 : BOW0 (μm)	Deviation of local warp at the center of wafers
	x_{10} : WARP0 (μm)	Maximum of local warp of wafers
QQ Responses	y : TTV (μm)	Continuous total thickness variation of wafers
	z : STIR indicator (0 or 1)	Binary indicator for the conformity of STIR

Figure 1: The QQ normal plots of the residuals from the QQ models for the Lapping Process: (a) the QQ normal plot for the model $y|z = 1, \mathbf{x}$; (b) the QQ normal plot for the model $y|z = 0, \mathbf{x}$.

Figure 2: The residual plots from the QQ Models for the Lapping Process: (a) the residual plot for the model $y|z = 1, \mathbf{x}$; (b) the residual plot for the model $y|z = 0, \mathbf{x}$.