Product Allocation Problem for an AS/RS with Multiple in-the-Aisle Pick Positions

Faraz Ramtin & Jennifer A. Pazour

Department of Industrial Engineering and Management Systems,

University of Central Florida, Orlando, FL, 32816, USA

E-mail: faraz.ramtin@ucf.edu; jennifer.pazour@ucf.edu

A. Optimal m for CR Policy

$E(W_m)$ is calculated as equations (A.1) and (A.2) for two cases of b ($0 < b < 1/2$ and $1/2 \leq b \leq 1$). Equations (A.1) and (A.2) are continuous piecewise functions. To find the optimal m , we follow this procedure for each range of b .

STEP 1) Find m^* for each range of m by setting $\frac{dE(W_m)}{dm} = 0$, then solve for m and calculate the value of $E(W_{m^*})$ if the m^* is in the given range.

STEP 2) Calculate the value of $E(W_m)$ for the beginning and ending of each range of m .

STEP 3) Compare the results from STEP 1 and 2 to find the m^* .

The results of both ranges of b ($0 < b < 1/2$ and $1/2 \leq b \leq 1$) are shown in Table A1 and A2, respectively. For both cases, for the first and third range of m , the calculated m^* is not in the range; therefore, we calculate the $E(W_{m^*})$ only for the second range of m , and calculate the boundaries value for $E(W_m)$ for the first and third ranges. Finally, we compare the results obtained in the previous steps. The minimum $E(W_m)$ is equal to $b^2/3 + 1/4$, corresponding to $m^* = 1/2$.

Table A1. Results for calculating m^* for $0 < b < 1/2$

Interval	m^*		Boundaries value for $E(W_m)$		$E(W_{m^*})$
$0 \leq m < b$	$m^* = \sqrt{2b} - b$ (not in range)	$m^* = -\sqrt{2b} - b$ (not in range)	$E(W_0) = +\infty$	$E(W_b) = 5b^2/6 + (1-b)^2/2$	—
$b \leq m < 1-b$	$m^* = 1/2$		$E(W_b) = 5b^2/6 + (1-b)^2/2$	$E(W_{1-b}) = 5b^2/6 + (1-b)^2/2$	$E(W_{0.5}) = b^2/3 + 1/4$
$1-b \leq m < 1$	$m^* = 1 - \sqrt{2b} + b$ (not in range)	$m^* = 1 + \sqrt{2b} + b$ (not in range)	$E(W_{1-b}) = 5b^2/6 + (1-b)^2/2$	$E(W_1) = b^2/6 + 1/2$	—

Table A2. Results for calculating m^* for $1/2 \leq b \leq 1$

Interval	m^*		Boundaries value for $E(W_m)$		$E(W_{m^*})$
$0 \leq m < 1 - b$	$m^* = \sqrt{2b} - b$ (not in range)	$m^* = -\sqrt{2b} - b$ (not in range)	$E(W_0)$ $= +\infty$	$E(W_{1-b})$ $= 5b^2/6$ $+ (1-b)^2/2$	—
$1 - b \leq m < b$	$m^* = 1/2$		$E(W_{1-b})$ $= 5b^2/6$ $+ (1-b)^2/2$	$E(W_b)$ $= 5b^2/6$ $+ (1-b)^2/2$	$E(W_{0.5})$ $= b^2/3 + 1/4$
$b \leq m < 1$	$m^* = 1 - \sqrt{2b} + b$ (not in range)	$m^* = 1 + \sqrt{2b} + b$ (not in range)	$E(W_b)$ $= 5b^2/6$ $+ (1-b)^2/2$	$E(W_1)$ $= b^2/6 + 1/2$	—

B. Proof of $D_{OPT}(m, s)$ is a Valid Probability Distribution Function (PDF)

Any continuous PDF, $f(x)$, must satisfy the two following conditions. $f(x) \geq 0 \forall x \in \mathbb{R}$, and $\int_{-\infty}^{+\infty} f(x)dx = 1$ (Mendenhall & Sincich, 2006). We show that both conditions hold for $D_{OPT}(m, s)$ as follows.

1) $D_{OPT}(m, s) \geq 0$ for $0 \leq m \leq 1$ and $0 \leq s \leq 1$.

2) $\int_{-\infty}^{+\infty} D_{OPT}(m, s)dx = \int_0^{0.5} s(1-2m)^{s-1} dm + \int_{0.5}^1 s(2m-1)^{s-1} dm = 1/2 + 1/2 = 1$.

References

Mendenhall, W., & Sincich, T. (2006). *Statistics for Engineering and the Sciences* (5 edition). Pearson, Upper Saddle River, NJ.