Online Supplement to

Pricing Decisions in a Strategic Single Retailer/Dual Suppliers Setting under Order Size Constraints

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1. Retailer's Optimal Ordering Policy

- $Q'_1 \leq C_1, Q'_2 \leq C_2$: Without loss of generality, we assume that $p_1 \leq p_2$. If the retailer chooses to order from both suppliers, the amount ordered from Supplier 1 (Q_1^j) has to be equal to C_1 since $p_1 \leq p_2$ ¹. Then, the annual cost of the joint-order policy is going to be:

$$\begin{aligned} G_3(Q_1^j, Q_2^j) &= \frac{K_3\lambda}{C_1 + Q_2^j} + \frac{\lambda}{C_1 + Q_2^j}(p_1 C_1 + p_2 Q_2^j) + \frac{p_1 C_1 + p_2 Q_2^j}{2} I \\ &\geq \frac{K_1\lambda}{C_1 + Q_2^j} + \frac{\lambda}{C_1 + Q_2^j} p_1 (C_1 + Q_2^j) + \frac{p_1 (C_1 + Q_2^j)}{2} I \\ &> G_1(Q'_1) \end{aligned}$$

Therefore, joint-order policy is not preferable.

- $Q'_1 > C_1, Q'_2 \leq C_2$ (or equivalently, $Q'_1 \leq C_1, Q'_2 > C_2$): We analyze this setting in two cases:
 - $p_1 \leq p_2$: In case of a joint order, Q_1^j will be equal to C_1 , and the cost of optimal joint-order policy is found by taking the derivative of the following function with respect to Q_2 :

$$G_3(C_1, Q_2) = \frac{K_3\lambda}{C_1 + Q_2} + \frac{\lambda}{C_1 + Q_2}(p_1 C_1 + p_2 Q_2) + \frac{p_1 C_1 + p_2 Q_2}{2} I \quad (1)$$

¹Otherwise, we would have order sizes Q_1^j and Q_2^j such that $Q_1^j < C_1$. In this case, if $\Delta := C_1 - Q_1^j \leq Q_2^j$, then the retailer can reduce annual cost of ordering by ordering $Q_1^j + \Delta = C_1$ units from Supplier 1 and $Q_2^j - \Delta$ units from Supplier 2. The annual fixed ordering cost does not change, but the annual variable ordering cost and annual holding cost decrease in the new solution. If $\Delta > Q_2^j$, then the retailer can still reduce annual cost of ordering by ordering $Q_1^j + Q_2^j$ units from Supplier 1 and zero units from Supplier 2. However, in this case joint-order policy cannot be optimal.

Then, we obtain $Q_2^j = \min\{C_2, \max\{0, \sqrt{\frac{2\lambda(K_3+p_1C_1-p_2C_1)}{p_2I}} - C_1\}\}$.

– $p_1 > p_2$: Similar to $Q_1' \leq C_1, Q_2' \leq C_2$ case, one can show that ordering from Supplier 2 only is always better than the joint-order policy.

- $Q_1' > C_1, Q_2' > C_2$: If $p_1 \leq p_2$, we have $Q_1^j = C_1$. Optimal solution for the joint-order policy is found by differentiating the function in Equation (1) with respect to Q_2 . Hence, $Q_2^j = \min\{C_2, \max\{0, \sqrt{\frac{2\lambda(K_3+p_1C_1-p_2C_1)}{p_2I}} - C_1\}\}$.

2. Asymmetric Information Case

2.1. Threshold-Pricing Strategy

We first develop a model to find the best pricing strategy so that the retailer orders only from Supplier 1. The solution of the following model makes sure that the retailer prefers ordering from Supplier 1 over Supplier 2 and placing a joint order while maximizing Supplier 1's revenue.

$$(M1): \text{Maximize} \quad \lambda p_1 \quad (2)$$

$$\text{subject to} \quad \min\{G_2^*, G_3^*\} \geq \frac{K_1\lambda}{Q_1} + \lambda p_1 + \frac{Q_1 I p_1}{2} \quad (3)$$

$$0 \leq Q_1 \leq C_1 \quad (4)$$

$$0 \leq p_1 \quad (5)$$

In M1, G_2^* is the cost of optimal ordering policy from Supplier 2 only, i.e., $G_2^* = G_2(Q_2^s)$, and G_3^* is the annual cost of optimal joint-order policy which depends on p_1 . The decision variables are Q_1 , which is the order quantity from Supplier 1, and p_1 , which is the unit price offered by Supplier 1. Note that Q_1 is the retailer's decision, not the supplier's decision, but it depends on the price (p_1) set by Supplier 1. As discussed above, Q_1 is a complicated function of p_1 . Hence, with a slight abuse of notation, we keep Q_1 in the model M1 as a decision variable to provide Supplier 1's pricing problem in a compact form. Let Q_1^* and p_1^* be the optimal solution of the above model. Although we do not require Q_1 to be the optimal order quantity for the retailer in model M1, using the convexity of the function on the right hand side of Constraint (3) with respect to Q_1 , one can easily show that there is always an optimal solution to M1 where Q_1^* is the optimal order quantity for the retailer given p_1^* .

Since G_3^* depends on p_1 , solving M1 directly is challenging. Hence, we propose solving it in an iterative way. That is, we know that the minimum value for p_1 is zero. Starting from that value, we calculate G_3^* and replace left-hand side of Constraint (3) by T where $T = \min\{G_2^*, G_3^*\}$. We denote this model by M1'. If model M1' is infeasible, then there is no p_1 value which makes ordering only from Supplier 1 preferable for the retailer. Otherwise, we use the new value of p_1 (found after solving M1') to recalculate G_3^* and resolve M1' after updating T . Let G_1^* denote the optimal cost of ordering from Supplier 1 only based on the unit

price p_1 found after solving M1'. Then, G_1^* is equal to the value of the expression on the right-hand side of Constraint (3) at the optimal solution of M1'. We terminate this iterative procedure when $G_3^* \geq G_2^*$ or the difference between G_3^* and G_1^* is small enough. Note that G_1^* increases at a rate higher than G_3^* when p_1 increases. Furthermore, we obtained convergence in all the cases tested in less than 100 iterations where the termination gap between G_3^* and G_1^* is set to 10^{-6} .

Next, we explain how to solve M1'. The optimal solution $(\bar{p}_1, \bar{Q}_1)$ to model M1' is as follows:

- (i) If $T < \frac{K_1\lambda}{C_1}$, then M1' is infeasible.
- (ii) If $T \geq \frac{K_1\lambda}{C_1}$ and $TIC_1^2 - 2K_1\lambda^2 - 2K_1C_1\lambda I < 0$, then $(\bar{p}_1, \bar{Q}_1) = (\frac{2TC_1 - 2K_1\lambda}{C_1^2I + 2\lambda C_1}, C_1)$.
- (iii) If $T \geq \frac{K_1\lambda}{C_1}$ and $TIC_1^2 - 2K_1\lambda^2 - 2K_1C_1\lambda I \geq 0$, then
 $(\bar{p}_1, \bar{Q}_1) = (\frac{T^2}{T\lambda + K_1\lambda I + \sqrt{K_1^2\lambda^2I^2 + 2TIK_1\lambda^2}}, \frac{K_1\lambda I + \sqrt{K_1^2\lambda^2I^2 + 2TIK_1\lambda^2}}{TI})$.

Case (i) is trivial since the cost of ordering from Supplier 1 only is greater than or equal to $\frac{K_1\lambda}{C_1}$. To show cases (ii) and (iii), we use Karush-Kuhn-Tucker (KKT) conditions for M1'. Using the KKT conditions, the necessary conditions for a local optima are as follows:

- (i) $0 \leq Q_1 \leq C_1$
- (ii) $0 \leq p_1$
- (iii) $Q_1^2Ip_1 + 2\lambda p_1Q_1 + 2K_1\lambda - 2TQ_1 \leq 0$
- (iv) $\mu_1(Q_1^2Ip_1 + 2\lambda Q_1p_1 + 2K_1\lambda - 2TQ_1) = 0$
- (v) $\mu_2(Q_1 - C_1) = 0$
- (vi) $-\lambda + \mu_1(Q_1^2I + 2\lambda Q_1) = 0$
- (vii) $\mu_1(2Q_1Ip_1 + 2\lambda p_1 - 2T) + \mu_2 = 0$
- (viii) $\mu_1, \mu_2 \geq 0$

From the above conditions, there are two alternatives for μ_2 : (a) $\mu_2 > 0$, and (b) $\mu_2 = 0$.

- (a) $\mu_2 > 0$: In this case, to satisfy condition (v) we have to have $Q_1 = C_1$. Moreover, we have $\mu_1 \neq 0$ due to condition (vi). Condition (iv) implies that $Q_1^2Ip_1 + 2\lambda Q_1p_1 + 2K_1\lambda - 2TQ_1 = 0$. Then, we have

$$p_1 = \frac{2TC_1 - 2K_1\lambda}{C_1^2I + 2\lambda C_1} \quad [p_1 \geq 0 \text{ since } T \geq \frac{K_1\lambda}{C_1}] \quad (6)$$

Finally, in order to satisfy $\mu_2 > 0$, we have $2Q_1Ip_1 + 2\lambda p_1 - 2T < 0$ which implies $TIC_1^2 - 2K_1\lambda^2 - 2K_1C_1\lambda I < 0$.

- (b) $\mu_2 = 0$: In this case, we have $\mu_1 \neq 0$ due to condition (vi). Then, in order to satisfy condition (vii), we have

$$p_1 = \frac{T}{\lambda + Q_1 I} \quad (7)$$

Moreover, from condition (iv) we have $Q_1^2 I p_1 + 2\lambda Q_1 p_1 + 2K_1 \lambda - 2TQ_1 = 0$. Replacing p_1 by its equivalent, we get

$$(Q_1^2 I + 2\lambda Q_1) \frac{T}{\lambda + Q_1 I} + 2K_1 \lambda - 2TQ_1 = 0 \quad (8)$$

After solving (8) for Q_1 , we obtain

$$Q_1 = \frac{K_1 \lambda I + \sqrt{K_1^2 \lambda^2 I^2 + 2T I K_1 \lambda^2}}{T I} \quad (9)$$

In order to satisfy condition (i), we have $Q_1 \leq C_1$ which implies $TIC_1^2 - 2K_1 \lambda^2 - 2K_1 C_1 \lambda I \geq 0$. $\square$

2.2. Joint-Order Pricing Strategy

Next, we explain how to find the best pricing scheme assuming that the joint-order policy is the best option for the retailer. To do this, first we solve the following model to find the initial value for the optimal unit price for Supplier 1.

$$(M2): \text{Maximize} \quad \frac{\lambda}{Q_1 + Q_2} Q_1 p_1 \quad (10)$$

$$\text{subject to} \quad G_2^* \geq G_3^*(p_1) \quad (11)$$

$$0 \leq Q_1 \leq C_1 \quad (12)$$

$$0 \leq Q_2 \leq C_2 \quad (13)$$

$$0 \leq p_1 \quad (14)$$

In this model, with a slight abuse of notation we use $G_3^*(p_1)$ to denote the total cost of best joint-order policy from the retailer's perspective as a function of the price offered by Supplier 1 (p_1).

Assuming Q_1^* , Q_2^* and p_1^* are the optimal values found after solving model M2, we determine the best ordering policy from Supplier 1 only from the retailer's perspective. Let $G_1^*(p_1^*)$ be the cost of this policy. If $G_1^*(p_1^*) < G_3^*(p_1^*)$, then ordering from Supplier 1 only is better from the retailer's perspective. Moreover, this is also a better solution for Supplier 1 because in that case the revenue of Supplier 1 will be λp_1^* instead of $\frac{\lambda}{Q_1^* + Q_2^*} Q_1^* p_1^*$. Therefore, in this case we conclude that the best pricing scheme which encourages the retailer to order from Supplier 1 only is better than the pricing scheme which encourages retailer to give a joint order. However, if ordering from Supplier 1 only is more costly than joint-order policy, then we conclude that p_1^* is the best pricing scheme for the joint-order policy.

We propose solving M2 using KKT conditions. We analyze the problem by comparing the prices offered

by each supplier: (i) $p_1 \leq p_2$, and (ii) $p_2 \leq p_1$.

(i) $p_1 \leq p_2$: In this case, we have $Q_1 = C_1$. Then, given the value of p_1 we can calculate Q_2 as follows:

$$Q_2 = \min\{C_2, \max\{0, \sqrt{\frac{2\lambda(K_3 + p_1C_1 - p_2C_1)}{p_2I}} - C_1\}\} \quad (15)$$

In this case, we have three scenarios for Q_2 :

- (a) $Q_2 = C_2$: Using (11), (14) and (15), we have the following conditions on p_1 : $\max\{A_1, 0\} \leq p_1 \leq \min\{A_2, p_2\}$ where $A_1 = \frac{(C_1+C_2)^2 p_2 I + 2\lambda p_2 C_1 - 2\lambda K_3}{2\lambda C_1}$ and $A_2 = \frac{G_2^* - \frac{K_3 \lambda}{C_1+C_2} - \frac{\lambda C_2 p_2}{C_1+C_2} - \frac{C_2 p_2 I}{2}}{\frac{\lambda C_1}{C_1+C_2} + \frac{C_1 I}{2}}$. Therefore, if $\max\{A_1, 0\} > \min\{A_2, p_2\}$, then no solution exists. Otherwise, $p_1 = \min\{A_2, p_2\}$.
- (b) $Q_2 = 0$: In this case, ordering from Supplier 1 only will always be less costly for the retailer.
- (c) $0 < Q_2 < C_2$: In this case, model M2 reduces to the following model.

$$(M3): \text{Maximize} \quad \frac{\lambda}{Q_1+Q_2} Q_1 p_1 \quad (16)$$

$$\text{subject to} \quad G_2^* \geq \frac{K_3 \lambda}{Q_1+Q_2} + \frac{\lambda}{Q_1+Q_2} (Q_1 p_1 + Q_2 p_2) + \frac{Q_1 p_1 + Q_2 p_2}{2} I \quad (17)$$

$$Q_1 = C_1 \quad (18)$$

$$Q_2 = \sqrt{\frac{2\lambda(K_3 + p_1C_1 - p_2C_1)}{p_2I}} - C_1 \quad (19)$$

$$0 \leq Q_2 \leq C_2 \quad (20)$$

$$0 \leq p_1 \leq p_2 \quad (21)$$

Replacing $Q_2 + C_1$ by x in the above model and using the KKT conditions, we have the following conditions for the local optima:

- (i) $\mu_1, \mu_2, \mu_3, \mu_4 \geq 0$
- (ii) $\mu_1(-x + e_1) = 0, \mu_2(x - e_2) = 0, \mu_3(a_3x^2 + b_3) = 0, \mu_4(a_2x^2 + b_2x + d_2) = 0$
- (iii) $-x + e_1 \leq 0, x - e_2 \leq 0, a_3x^2 + b_3 \leq 0, a_2x^2 + b_2x + d_2 \leq 0$
- (iv) $-a_1 + \frac{b_1}{x^2} - \mu_1 + \mu_2 + 2a_3x\mu_3 + \mu_4(2a_2x + b_2) = 0$

where $a_1 = \frac{p_2 I}{2}, b_1 = \lambda C_1 p_2 - \lambda K_3, a_2 = \frac{p_2 I^2}{4\lambda}, b_2 = p_2 I, d_2 = \lambda p_2 - \frac{K_3 I}{2} - G_2^*, a_3 = \frac{p_2 I}{2\lambda C_1}, b_3 = -\frac{K_3}{C_1}, e_1 = C_1, e_2 = C_1 + C_2$.

We find all the local optima by considering the following five different scenarios and checking the feasibility of the KKT conditions: (i) $\mu_1 \neq 0$, (ii) $\mu_1 = 0, \mu_2 \neq 0$, (iii) $\mu_1 = 0, \mu_2 = 0, \mu_3 \neq 0$, (iv) $\mu_1 = 0, \mu_2 = 0, \mu_3 = 0, \mu_4 \neq 0$, (v) $\mu_1 = 0, \mu_2 = 0, \mu_3 = 0, \mu_4 = 0$.

(ii) $p_2 \leq p_1$: In this case, we have $Q_2 = C_2$, and Q_1 is found as follows:

$$Q_1 = \min\{C_1, \max\{0, \sqrt{\frac{2\lambda(K_3 + p_2C_2 - p_1C_2)}{p_1I}} - C_2\}\} \quad (22)$$

Similar to the above scenario, we analyze the value of Q_1 in three cases:

- (a) $Q_1 = C_1$: Similar to case (a) above, using (11), (14) and (22) we have the following conditions on p_1 : $p_2 \leq p_1 \leq \min\{A_2, A_3\}$ where $A_3 = \frac{2\lambda K_3 + 2\lambda p_2 C_2}{2\lambda C_2 + I(C_1 + C_2)^2}$. Hence, if $\min\{A_2, A_3\} < p_2$, then no solution exists. Otherwise, we have $p_1 = \min\{A_2, A_3\}$.
- (b) $Q_1 = 0$: In this case, regardless of the unit price the revenue will be zero.
- (c) $0 < Q_1 < C_1$: Similar to the case (c) above, using $Q_1 = \min\{C_1, \max\{0, \sqrt{\frac{2\lambda(K_3 + p_2 C_2 - p_1 C_2)}{p_1 I}} - C_2\}\}$ and defining x as $Q_1 + C_2$, we obtain the following KKT conditions for the local optima:
 - (i) $\mu_1, \mu_2, \mu_3, \mu_4 \geq 0$
 - (ii) $\mu_1(-x + a_3) = 0, \mu_2(x - b_3) = 0, \mu_3(x^2 - a_4) = 0, \mu_4(a_5 x^2 + b_5 x + d_5) = 0$
 - (iii) $-x + a_3 \leq 0, x - b_3 \leq 0, x^2 - a_4 \leq 0, a_5 x^2 + b_5 x + d_5 \leq 0$
 - (iv) $-\frac{a_1(a_2 x^3 + b_2 x) - (a_1 x - b_1)(3a_2 x^2 + b_2)}{(a_2 x^3 + b_2 x)^2} - \mu_1 + \mu_2 + 2x\mu_3 + \mu_4(2a_5 x + b_5) = 0$
 where $a_1 = 2\lambda^2 K_3 + 2\lambda^2 p_2 C_2, b_1 = 2\lambda^2 K_3 C_2 + 2\lambda^2 p_2 C_2^2, a_2 = I, b_2 = 2\lambda C_2, a_3 = C_2, b_3 = C_1 + C_2, a_4 = \frac{2\lambda K_3}{p_2 I}, a_5 = C_2 p_2 I^2 - 2IG_2^*, b_5 = 4\lambda p_2 C_2 I + 4\lambda K_3 I, d_5 = 4\lambda^2 K_3 + 4\lambda^2 C_2 p_2 - 2\lambda K_3 C_2 I - 4\lambda C_2 G_2^*$.

We find all the local optima in a similar way.

Finally, we find the optimal joint-order pricing strategy by comparing the objective function value (revenue of Supplier 1) of the solutions found above.

2.3. Computational Study for Asymmetric Information Case

To provide managerial insights for the asymmetric information case, we perform a computational study. In addition to the base scenario defined below, we conduct a sensitivity analysis with respect to the problem parameters such as fixed ordering costs, holding cost and order size limits to deepen our understanding of this market dynamics. We also consider extreme scenarios where suppliers have significantly different order size limits or fixed costs of ordering. Finally, we extend the computational study by considering high and low fixed costs of ordering ².

We start with a base scenario where the values of the problem parameters are as follows: $K_1 = K_2 = 400$, $K_3 = 600$, $C_1 = 300$, $C_2 = 300$, $\lambda = 300$, $I = 0.2$, and $p_2 = 1$. In the base scenario, the retailer prefers giving a joint order since the order size constraints of the suppliers is a limiting factor while determining the order size. Starting from the base scenario, we investigate the effect of changes in each parameter on the pricing strategy of Supplier 1. While analyzing the effect of a parameter, we keep the other parameters constant while changing the value of the parameter under consideration. To present more generalizable results, we

²Note that high (low) fixed costs of ordering result in large (small) order sizes from the retailer's perspective. This has a similar effect as decreasing (increasing) the order size limits of the suppliers. Furthermore, increasing (decreasing) inventory holding cost rate and decreasing (increasing) retailer's demand have a similar effect as decreasing (increasing) the fixed cost of ordering on the order size of the retailer. Hence, to keep the computational study to the point, we consider high and low (compared to the base scenario) fixed costs of ordering.

consider different settings for the other parameters as well while changing the value of a certain parameter. We consider the following values of the parameters presented in Table 1.

Table 1: Problem parameter values tested (values in the base scenario are highlighted).

Parameter	Tested Values
C_1	100, 200, 300 , 400, 500
C_2	100, 200, 300 , 400, 500
λ	100, 200, 300 , 400, 500
p_2	0.5, 0.75, 1.0 , 1.25, 1.5
K_1	300, 400 , 500
K_2	300, 400 , 500
K_3	500, 600 , 700
I	0.1, 0.2 , 0.3

As we increase the order size limit of Supplier 1, its annual revenue increases until a certain limit value (see Figure 1 for base scenario). We also point out the jump on the price curve. Although the revenue curve (for Supplier 1) is smooth, when Supplier 1 decides to force the retailer to switch to single-order policy at some point (just because it is more profitable for Supplier 1), Supplier 1 has to make a drastic decrease in the price in order to force retailer to order from itself only. In Figure 1, the sharp decrease in the price represents the point where Supplier 1 forces the retailer to switch from joint-order policy to single-order policy. Increase in order size limit beyond a certain limit value does not provide a positive return since order size restriction becomes non-binding for the retailer. Supplier 1 cannot increase the unit price by using its high order size limit advantage anymore beyond this point since retailer may switch to joint-order policy or single-order policy from Supplier 2 which decreases Supplier 1's revenue.

Figure 1: Effect of the order size limit of Supplier 1 on the pricing strategy and revenue of Supplier 1.

We also analyze the effect of retailer's demand, benefit of order consolidation, fixed cost of ordering from each supplier and inventory holding cost on this limit value. In Figure 2, we show how the revenue of Supplier 1 changes when the order size limit of Supplier 1 increases under different parameter values. In Figures 2(a)-(b), we illustrate how the order size limit affects the revenue of Supplier 1 under different values

of retailer's demand ($\lambda \in \{100, 200, 300, 400, 500\}$) and inventory holding cost rate ($I \in \{0.1, 0.2, 0.3\}$). We observe that the order size limit value of Supplier 1 at which its revenue stabilizes decreases as the retailer's demand decreases or the inventory holding cost rate increases. This is quite intuitive since retailer's order size decreases as the demand decreases, which makes the order size limits of the suppliers less of a concern for the retailer. Similarly, higher inventory holding cost rates decrease the order size of the retailer, and in this case the order size limits of the suppliers do not affect the ordering policy that much. Figures 2(c-e) show the effect of order size limit of Supplier 1 on its revenue under different scenarios for fixed cost of ordering. We observe that as the fixed cost of ordering from Supplier 1 (see Figure 2(c)) or the benefit of order consolidation (see Figure 2(e)) decreases, the revenue of Supplier 1 stabilizes at lower values of order size limit. Finally, the order size limit value of Supplier 1 at which its revenue stabilizes increases as the fixed cost of ordering from Supplier 2 decreases (see Figure 2(d)).

Next, we discuss how the pricing policy of Supplier 1 changes depending on its order size limit for different values of order size limit of Supplier 2. For smaller order size limit values, Supplier 1 prefers sharing the market with Supplier 2 whereas for higher order size limit values Supplier 1 prefers capturing the entire market (see the results for $C_2 \in \{200, 300, 400, 500\}$ in Figure 3). There is an exception to this for an extreme setting where Supplier 2 has a very low order size limit (see the curve for $C_2 = 100$ in Figure 3). In such a setting, instead of capturing the entire demand at a relatively low unit price, Supplier 1 increases its revenue by charging a higher unit price and allowing Supplier 2 capture a small portion of the market.

We also investigate the effect of order size limit of Supplier 2 on Supplier 1's revenue for different values of order size limit of Supplier 1. If Supplier 2 has a high order size limit, Supplier 1 either exits the market due to extremely low or even zero profit margins when it has low order size limit (see the results for $C_1 \in \{100, 200\}$ in Figure 4) or offers a lower unit price to capture the entire market when it has high enough order size limit (see the results for $C_1 \in \{300, 400, 500\}$ in Figure 4). In this case, high order size limit of Supplier 2 makes joint order less preferable for the retailer.

The unit price charged by Supplier 2 has almost a linear effect on the price offered by Supplier 1 and consequently on the revenue of Supplier 1. That is, as long as the ordering strategy of the retailer (joint- or single-order) does not change, the unit price offered by Supplier 1 increases linearly as the unit price of Supplier 2 increases. Beyond a certain value of the unit price of Supplier 2, Supplier 1 prefers threshold-pricing strategy in order to maximize its revenue due to reduced order size of the retailer.

We also analyze the effect of the retailer's cost structure. For low inventory holding cost settings, increased order size of the retailer makes sharing the market with Supplier 2 more preferable for Supplier 1 (see Figure 5(a)). Beyond a certain inventory holding cost rate value, order size limits of the suppliers become less of a concern for the retailer due to reduced order sizes. In that case, Supplier 1 prefers capturing the entire market.

Finally, for lower (higher) fixed cost of ordering from Supplier 1 (Supplier 2), Supplier 1 prefers capturing the entire market by using its cost advantage. Higher benefit of order consolidation makes joint order more

(a)

(b)

(c)

(d)

(e)

Figure 2: Effect of the order size limit of Supplier 1 on its revenue under different values of (a) retailer's demand, (b) inventory holding cost rate, (c) fixed cost of ordering from Supplier 1, (d) fixed cost of ordering from Supplier 2, and (e) fixed cost of joint order.

Figure 3: Pricing policy of Supplier 1 under different scenarios for order size limit of Supplier 2.

(a)

(b)

Figure 4: Effect of the order size limit of Supplier 2 on the (a) revenue, and (b) pricing strategy of Supplier 1 for different order size limit values of Supplier 1.

Figure 5: Effect of (a) inventory holding cost rate of the retailer, and (b) the benefit of order consolidation on the pricing strategy and revenue of Supplier 1.

attractive (in the presence of restrictive order size limits) for the retailer, and hence, Supplier 1 asks for a higher unit price than Supplier 2 using the limiting effect of order size limit and the retailer's desire to place joint orders (see Figure 5(b)). On the other hand, lower benefit of order consolidation makes joint order less preferable for the retailer, and in this case Supplier 1 prefers capturing the entire market.

In addition to the parameter setting provided in Table 1, we consider two extreme scenarios where (i) the order size limit of Supplier 1 (or Supplier 2) is much smaller than that of its competitor, and (ii) the fixed cost of ordering from Supplier 1 (or Supplier 2) is significantly lower compared to that of its competitor. More specifically, in the first extreme scenario, we consider two cases: (i) $C_1 = 100, C_2 = 1000$, and (ii) $C_1 = 1000, C_2 = 100$. All the other parameters are kept same as provided in Table 1. In the first case, Supplier 1 has zero revenue in the base scenario ($K_1 = K_2 = 400, K_3 = 600, \lambda = 300, I = 0.2, p_2 = 1$) due to its low order size limit. Supplier 1 can capture a certain portion of the market only if there is a high benefit of order consolidation (see Figure 6(a)), and it has higher revenue for high inventory holding cost rates (see Figure 6(b)). In the second case ($C_1 = 1000, C_2 = 100$), we observe that lower inventory holding cost rates and higher benefits of order consolidation force Supplier 1 to share the market with Supplier 2 (see Figures 7(a)-(b)). Similar to the observation for the parameter setting in Table 1, as the retailer's demand increases Supplier 1 prefers sharing the market with Supplier 2 since it can increase its revenue by charging a higher unit price using the capacity disadvantage of Supplier 2. Moreover, as the inventory holding cost rate of the retailer decreases, Supplier 1 prefers sharing the market with Supplier 2 as long as there is enough benefit of order consolidation for the retailer. If order consolidation does not bring much benefit to the retailer, capturing the entire market (at a relatively lower unit price) is more preferable for Supplier 1 (similar to Figure 5(b) for the base scenario).

(a)

(b)

Figure 6: Effect of (a) the benefit of order consolidation, and (b) the inventory holding cost rate of the retailer (for different values of K_3) on the pricing strategy of Supplier 1 when $C_1 = 100, C_2 = 1000$.

(a)

(b)

Figure 7: Effect of (a) the benefit of order consolidation, and (b) the inventory holding cost rate of the retailer (for different values of K_3) on the pricing strategy of Supplier 1 when $C_1 = 1000, C_2 = 100$.

In the second extreme scenario, we consider two cases: (i) $K_1 = 200, K_2 = 1000$, and (ii) $K_1 = 1000, K_2 = 100$. In this case, the fixed cost of joint order ranges from 1000 to 1200 (with the base value being 1100), and the other parameters are same as provided in Table 1. In the first case ($K_1 = 200, K_2 = 1000$), Supplier 1 captures the entire market using its low fixed cost of ordering advantage (see Figure 8(a) for different values of C_2). Moreover, Supplier 1 can charge a unit price significantly higher than that of Supplier 2 using its low fixed cost of ordering advantage. Even for higher values of C_2 , Supplier 1 still dominates the market but at a lower per unit price (see Figure 8(b) for different values of C_1). We also observe that revenue of Supplier 1 stabilizes at some order size limit value of Supplier 1. The order size limit value at which Supplier 1's revenue stabilizes increases as retailer's demand, order size limit of Supplier 2 increases or inventory holding cost rate of the retailer decreases. Finally, we observe that as the order size limit of Supplier 2 increases, the revenue of Supplier 1 decreases and then stabilizes after some point. In the second case ($K_1 = 1000, K_2 = 200$), order size limit of Supplier 1 has a significant impact on its revenue. Supplier 1 can only capture (the entire or a portion of) market if it has high order size limit and Supplier 2 has low order size limit (see Figures 9(a)-(b)). Moreover, due to its high fixed cost of ordering disadvantage Supplier 1 cannot charge a unit price as high as Supplier 2 unless Supplier 2 has significantly low order size limit. Similar to the observations above, Supplier 1's revenue stabilizes at some value of its order size limit. This order size limit value at which the revenue stabilizes increases as the retailer's demand, order size limit of Supplier 2 increases or inventory holding cost rate of the retailer decreases. If, in addition to low fixed cost of ordering, Supplier 2 has high enough order size limit, Supplier 1 cannot compete with Supplier 2.

Figure 8: Effect of (a) the order size limit of Supplier 1, and (b) the order size limit of Supplier 2 on the pricing strategy of Supplier 1 when $K_1 = 200, K_2 = 1000$.

Finally, we extend the computational study by considering significantly lower and higher fixed costs of ordering (for both suppliers) compared to the setting provided in Table 1. More specifically, keeping the order

Figure 9: Effect of (a) the order size limit of Supplier 1, and (b) the order size limit of Supplier 2 on the pricing strategy of Supplier 1 when $K_1 = 1000, K_2 = 200$.

size limit, demand and unit price charged by Supplier 2 values same as provided in Table 1, we consider fixed cost of ordering and holding cost values provided in Tables 2 and 3 for low and high fixed cost of ordering settings, respectively.

Table 2: Parameter values tested (values in the base scenario are highlighted) for the low fixed cost of ordering setting.

Parameter	Tested Values
K_1	150, 200 , 250
K_2	150, 200 , 250
K_3	250, 300 , 350
I	0.1, 0.2, 0.3 , 0.4, 0.5

Table 3: Parameter values tested (values in the base scenario are highlighted) for the high fixed cost of ordering setting.

Parameter	Tested Values
K_1	800, 900, 1000 , 1100, 1200
K_2	800, 900, 1000 , 1100, 1200
K_3	1100, 1300, 1500 , 1700, 1900
I	0.1, 0.2, 0.3 , 0.4, 0.5

When fixed costs of ordering are low, the retailer decreases the size of the orders to decrease the inventory holding cost. In that case, we observe that Supplier 1 shares the market with Supplier 2 if its order size limit is too low (see Figure 10(a)). However, compared to the high fixed cost of ordering setting, Supplier 1 switches to threshold-pricing policy at lower values of its order size limit, and the order size limit at which Supplier 1's revenue stabilizes decreases as the fixed costs of ordering decrease (see Figures 10(a)-(b)). We

also observe that when fixed cost of ordering is high for both suppliers, Supplier 1 can charge a higher unit price (compared to the low fixed cost of ordering setting). In the high fixed cost of ordering setting, the retailer prefers large order sizes and in this case order size limits of the suppliers become more of a concern. Because of this, Supplier 1 can charge a higher unit price at the same order size limit value.

For both low and high fixed cost of ordering settings, we make observations similar to the ones in Figures 2(a-e). As the retailer's demand, order size limit of Supplier 2 decreases or the inventory holding cost rate of the retailer increases, the order size limit value of Supplier 1 at which its revenue stabilizes decreases. We also observe that for higher order size limit values of Supplier 2, Supplier 1 prefers capturing the entire market at a relatively low unit price whereas for lower order size limit values Supplier 1 prefers sharing the market with Supplier 2. For the low fixed cost of ordering values, Supplier 1 is better off by capturing the entire market when it has high order size limit. However, similar to the observation for the setting in Table 1, Supplier 1 prefers sharing the market even at higher values of its order size limit if Supplier 2 has low order size limit and the fixed ordering costs are high. When Supplier 2 has a high order size limit, Supplier 1 either exits the market when it has low order size limit or offers a lower unit price to capture the entire market when it has high enough order size limit. When fixed ordering costs are low, Supplier 1 prefers capturing the entire market regardless of the inventory holding cost rate of the retailer (assuming other parameters take the highlighted values in the corresponding tables) whereas when fixed ordering costs are high, retailer's high order size preference forces Supplier 1 to share the market with Supplier 2. Finally, in Figures 11(a)-(b) we observe that as the fixed cost of ordering increases Supplier 1 prefers capturing the entire market when benefit of order consolidation is low.

Figure 10: Effect of the order size limit of Supplier 1 on the pricing strategy of Supplier 1 for (a) low fixed cost of ordering, and (b) high fixed cost of ordering.

Figure 11: Effect of the benefit of order consolidation on the pricing strategy of Supplier 1 for (a) low fixed cost of ordering, and (b) high fixed cost of ordering.

3. Symmetric Information Case

3.1. Proof of Proposition 1

Proof of Proposition 1. At $(0, \bar{p})$ there exists threshold-pricing equilibrium for Supplier 2 only if we have $G_2(Q_2^s) < \min\{\frac{\lambda K_1}{C_1}, G_3(C_1, Q_2^j)\}$. Supplier 2 has an incentive to slightly increase its price. Hence, such an equilibrium does not exist for Supplier 2. $\square$

3.2. Proof of Proposition 2

We first prove the following lemma which is auxiliary for Proposition 2.

Lemma 1. Suppose $p_2 = 0$. When $\frac{K_3}{K_1} \leq \frac{(C_1+C_2)^2 I + 2\lambda C_2}{C_1^2 I}$, $G_1(Q_1^s)$ increases (decreases) faster than $G_3(Q_1^j, C_2)$ as p_1 increases (decreases) for all p_1 .

Proof of Lemma 1. We have $G_1'(Q_1^s) = \begin{cases} \frac{C_1 I}{2} + \lambda & \text{if } p_1 \leq \frac{2\lambda K_1}{C_1^2 I} \\ \frac{\sqrt{\lambda K_1 I}}{\sqrt{2} p_1} + \lambda & \text{otherwise} \end{cases}$.

$$G_3'(Q_1^j, C_2) = \begin{cases} \frac{C_1 I}{2} + \lambda \frac{C_1}{C_1 + C_2} & \text{if } p_1 \leq \frac{2\lambda K_3}{(C_1 + C_2)^2 I + 2\lambda C_2} \\ \lambda - \frac{C_2 I}{2} + \frac{\sqrt{\lambda I}(K_3 - 2p_1 C_2)}{\sqrt{2}(K_3 - p_1 C_2)p_1} & \text{if } \frac{2\lambda K_3}{(C_1 + C_2)^2 I + 2\lambda C_2} < p_1 < \frac{2\lambda K_3}{C_2^2 I + 2\lambda C_2} \\ 0 & \text{if } p_1 \geq \frac{2\lambda K_3}{C_2^2 I + 2\lambda C_2} \end{cases}$$

When $\frac{2\lambda K_1}{C_1^2 I} \geq \frac{2\lambda K_3}{(C_1 + C_2)^2 I + 2\lambda C_2}$, $G_1'(Q_1^s) > G_3'(Q_1^j, C_2)$ holds for all $p_1 \leq \frac{2\lambda K_1}{C_1^2 I}$. When $p_1 > \frac{2\lambda K_1}{C_1^2 I}$, $G_1'(Q_1^s) > G_3'(Q_1^j, C_2)$ if $\sqrt{K_1} > -C_2 \sqrt{\frac{p_1 I}{2\lambda}} + \sqrt{K_3 - p_1 C_2} - \frac{p_1 C_2}{\sqrt{K_3 - p_1 C_2}}$. Note that at $p_1 = \frac{2\lambda K_1}{C_1^2 I}$ this inequality holds. Moreover, as p_1 increases, the right hand side of the inequality decreases. Hence, $G_1'(Q_1^s) > G_3'(Q_1^j, C_2)$

for all $p_1 > \frac{2\lambda K_1}{C_1^2 I}$ as well. Therefore, we conclude that when $\frac{K_3}{K_1} \leq \frac{(C_1+C_2)^2 I + 2\lambda C_2}{C_1^2}$, $G_1(Q_1^s)$ increases (decreases) faster than $G_3(Q_1^j, C_2)$ does as p_1 increases (decreases) for all p_1 . $\square$

Proof of Proposition 2. If $\frac{K_3}{C_1+C_2} = \frac{K_1}{C_1}$, then Supplier 1 cannot capture the entire market by offering a positive price.

If $\frac{K_3}{C_1+C_2} < \frac{K_1}{C_1}$, we analyze two cases: (i) $\frac{K_2}{C_2} < \frac{K_3}{C_1+C_2}$, and (ii) $\frac{K_3}{C_1+C_2} \leq \frac{K_2}{C_2}$. In the first case, Supplier 2 can always offer a unit price low enough to capture the entire market. Hence, threshold-pricing equilibrium does not exist for Supplier 1. In the second case, due to Lemma 1, $G_1(Q_1^s)$ increases faster than $G_3(Q_1^j, C_2)$ as p_1 increases for all p_1 . Moreover, we have $\lim_{p_1 \rightarrow 0^+} G_1(Q_1^s) = \frac{\lambda K_1}{C_1}$ and $\lim_{p_1 \rightarrow 0^+} G_3(Q_1^j, C_2) = \frac{\lambda K_3}{C_1+C_2}$. Hence, threshold-pricing equilibrium does not exist for Supplier 1 as $G_3(Q_1^j, C_2) < G_1(Q_1^s)$ for all $p_1 > 0$. $\square$

3.3. Proof of Theorem 1

Proof of Theorem 1. In order to have a threshold-pricing equilibrium for Supplier 1, ordering from Supplier 1 only should not be more costly than ordering from Supplier 2 only even if Supplier 2 offers a zero unit price. Hence, there may exist a unique threshold-pricing equilibrium at $(\bar{p}, 0)$ where $\bar{p}$ solves $G_1(Q_1^s) = \frac{\lambda K_2}{C_2}$.

If $G_3(Q_1^j, C_2) \geq \frac{\lambda}{C_2} K_2$, then $(\bar{p}, 0)$ is the unique threshold-price equilibrium. Otherwise, i.e., if $G_3(Q_1^j, C_2) < \frac{\lambda}{C_2} K_2$ at $(\bar{p}, 0)$, Supplier 1 has to decrease the price further to make sure that single-order policy is more preferable than joint-order policy from the retailer's perspective. Hence, $\tilde{p}$ where $\tilde{p} = \max\{p_1 | G_1(Q_1^s) = G_3(Q_1^j, C_2) \text{ and } p_1 < \bar{p}\}$ is the candidate for threshold-pricing equilibrium. Furthermore, to have an equilibrium at this point, the threshold-pricing strategy should be more preferable than joint-order pricing strategy from Supplier 1's perspective. $\square$

3.4. Proof of Proposition 3

We first prove the following lemma which is auxiliary for Proposition 3.

Lemma 2. Suppose at (p_1, p_2) we have $G_3(Q_1^j, Q_2^j) = G_2(Q_2^s) < G_1(Q_1^s)$. Then, Supplier 2 can increase its revenue by decreasing the price slightly to $p_2 - \epsilon$ for some small enough positive ϵ .

Proof of Lemma 2. First, we observe that joint-order can be preferable only if (i) $Q_1' > C_1, Q_2' \leq C_2$ and $p_1 \leq p_2$, or (ii) $Q_1' \leq C_1, Q_2' > C_2$ and $p_2 \leq p_1$, or (iii) $Q_1' > C_1, Q_2' > C_2$ (see Section 1 of the Online Supplement).

Let $\hat{Q}_2^s$ be the optimal order quantity at price $p_2 - \epsilon$ under single-order policy from Supplier 2 only for some small positive ϵ . Similarly, let $\hat{Q}_1^j$ and $\hat{Q}_2^j$ be the optimal order quantities under joint-order policy at prices $(p_1, p_2 - \epsilon)$.

It is obvious that we have $G_2(\hat{Q}_2^s) \geq G_2(Q_2^s)$ at price p_2 and $G_2(\hat{Q}_2^s) \leq G_2(Q_2^s)$ at price $p_2 - \epsilon$. Hence, the decrease in G_2 is not less than $\frac{\epsilon Q_2^s I}{2} + \lambda \epsilon$ when Supplier 2 decreases its price by ϵ . Similarly, we have $G_3(\hat{Q}_1^j, \hat{Q}_2^j) \geq G_3(Q_1^j, Q_2^j)$ at prices (p_1, p_2) and $G_3(\hat{Q}_1^j, \hat{Q}_2^j) \leq G_3(Q_1^j, Q_2^j)$ at prices $(p_1, p_2 - \epsilon)$. Hence, the

decrease in G_3 is not more than $\frac{\epsilon \hat{Q}_2^j I}{2} + \frac{\lambda \epsilon \hat{Q}_2^j}{\hat{Q}_1^j + \hat{Q}_2^j}$. If the following inequality is satisfied, this concludes the proof since Supplier 2 increases its revenue to $\lambda(p_2 - \epsilon)$ from $\frac{\lambda Q_2^j p_2}{Q_1^j + Q_2^j}$ for any ϵ such that $\epsilon < \frac{Q_1^j p_2}{Q_1^j + Q_2^j}$ in that case:

$$\frac{\epsilon Q_2^s I}{2} + \lambda \epsilon > \frac{\epsilon \hat{Q}_2^j I}{2} + \frac{\lambda \epsilon \hat{Q}_2^j}{\hat{Q}_1^j + \hat{Q}_2^j}.$$

The above inequality is equivalent to the following:

$$\frac{2\lambda \hat{Q}_1^j}{\hat{Q}_1^j + \hat{Q}_2^j} > \hat{Q}_2^j - Q_2^s.$$

Note that this inequality is satisfied for cases (ii) and (iii) discussed above since $Q_2^s = C_2$ in those cases.

Now, we want to show that our claim is true for the remaining case: $Q_1' > C_1, Q_2' \leq C_2$ and $p_1 \leq p_2$. If $Q_2' = C_2$, then the above discussion is still valid since $Q_2^s = C_2$. Therefore, we have $Q_1' > C_1, Q_2' < C_2$ and $p_1 \leq p_2$. Moreover, if $p_1 = p_2$, then after decreasing p_2 slightly to $\hat{p}_2 (= p_2 - \epsilon)$ we have the following conditions: $Q_1' > C_1, \hat{Q}_2' < C_2$ and $\hat{p}_2 < p_1$ where $\hat{Q}_2'$ is the EOQ value at price $\hat{p}_2$. We know that joint-order is not preferable in this case (see Section 1 of the Online Supplement). Hence, the retailer prefers ordering from Supplier 2 only, and Supplier 2 increases its revenue.

The only remaining case left is $Q_1' > C_1, Q_2' < C_2$ and $p_1 < p_2$. First, we show that $Q_2^j < C_2$. We prove it by contradiction. Assume $Q_2^j = C_2$. Then, we have $\sqrt{\frac{2\lambda(K_3 + p_1 C_1 - p_2 C_1)}{p_2 I}} - C_1 \geq C_2$. After taking the square of both sides of the following inequality

$$\sqrt{\frac{2\lambda(K_3 + p_1 C_1 - p_2 C_1)}{p_2 I}} - C_1 \geq C_2$$

and dividing both sides by $2(C_1 + C_2)$, we have

$$\frac{\lambda K_3}{C_1 + C_2} + \frac{\lambda p_1 C_1}{C_1 + C_2} \geq \frac{(C_1 + C_2)p_2 I}{2} + \frac{\lambda p_2 C_1}{C_1 + C_2}$$

Then, adding $\frac{\lambda p_2 C_2}{C_1 + C_2} + \frac{I(C_1 p_1 + C_2 p_2)}{2}$ to both sides, we obtain the following inequality:

$$G_3(C_1, C_2) = \frac{\lambda K_3}{C_1 + C_2} + \frac{\lambda(p_1 C_1 + p_2 C_2)}{C_1 + C_2} + \frac{I(C_1 p_1 + C_2 p_2)}{2} \geq \lambda p_2 + C_2 p_2 I + \frac{I C_1 (p_1 + p_2)}{2} > G_2(Q_2^s)$$

The last inequality follows from the fact that $Q_2' < C_2$ at price p_2 . Therefore, we have $Q_2^j = \frac{2\lambda(K_3 + p_1 C_1 - p_2 C_1)}{p_2 I} - C_1 < C_2$. At prices (p_1, p_2) , we know that $G_3(Q_1^j, Q_2^j) = G_2(Q_2^s)$. Hence, we have the following equality:

$$G_3(Q_1^j, Q_2^j) - G_2(Q_2^s) = \sqrt{2\lambda p_2 I(K_3 + p_1 C_1 - p_2 C_1)} + \frac{C_1(p_1 - p_2)I}{2} - \sqrt{2\lambda K_2 p_2 I} = 0.$$

After decreasing p_2 slightly by ϵ , at the new price levels we have:

$$f(\epsilon) := G_3(\hat{Q}_1^j, \hat{Q}_2^j) - G_2(\hat{Q}_2^s) = \sqrt{2\lambda(p_2 - \epsilon)I(K_3 + C_1(p_1 + \epsilon - p_2))} + \frac{C_1(p_1 + \epsilon - p_2)I}{2} - \sqrt{2\lambda K_2(p_2 - \epsilon)I} = 0.$$

Using $f(0) = 0$, one can show that $f'(\epsilon) > 0$ for positive ϵ values such that $\epsilon < \min\{2p_1, p_2 - p_1, \frac{p_2}{2}\}$. $\square$

Proof of Proposition 3. At (p_1, p_2) , the retailer prefers joint-order policy. Thus, $G_3(Q_1^j, Q_2^j) < G_1(Q_1^s)$ and $G_3(Q_1^j, Q_2^j) \leq G_2(Q_2^s)$. Suppose that $G_3(Q_1^j, Q_2^j) = G_2(Q_2^s)$. Supplier 2 will deviate by charging a slightly lower price, $p_2^d = p_2 - \epsilon$, and capturing the entire market. In this case, Supplier 2 generates a higher revenue and hence deviates (see Lemma 2). Suppose that $G_3(Q_1^j, Q_2^j) < G_2(Q_2^s)$. If $p_1 = p_2$, again Supplier 2 will deviate by charging a slightly lower price, $p_2^d = p_2 - \epsilon < p_1$ and by doing so increases its ordered amount to C_2 and increases its revenue. If $p_1 < p_2$, then Supplier 1 has two options: (i) charging a slightly higher price, $p_1^d = p_1 + \epsilon < p_2$, (ii) charging a slightly lower price, $p_1^d = p_1 - \epsilon < p_2$.

(i) $p_1^d = p_1 + \epsilon$: Let $\tilde{Q}_2^j$ be the order amount from Supplier 2 when Supplier 1 deviates to p_1^d where $\tilde{Q}_2^j > Q_2^j$.

At (p_1, p_2) , the revenue of Supplier 1 is equal to $\frac{\lambda p_1 C_1}{C_1 + Q_2^j}$. At (p_1^d, p_2) , the revenue of Supplier 1 is equal to $\frac{\lambda(p_1 + \epsilon)C_1}{C_1 + \tilde{Q}_2^j}$. Supplier 1 does not deviate if $\frac{(p_1 + \epsilon)C_1}{C_1 + \tilde{Q}_2^j} \leq \frac{p_1 C_1}{C_1 + Q_2^j}$ or, equivalently, $1 + \frac{\epsilon}{p_1} \leq \frac{C_1 + \tilde{Q}_2^j}{C_1 + Q_2^j}$.

(ii) $p_1^d = p_1 - \epsilon$: Let $\hat{Q}_2^j$ be the order amount from Supplier 2 when Supplier 1 deviates to p_1^d where $\hat{Q}_2^j < Q_2^j$. At (p_1^d, p_2) , the revenue of Supplier 1 is equal to $\frac{\lambda(p_1 - \epsilon)C_1}{C_1 + \hat{Q}_2^j}$. Supplier 1 does not deviate if $\frac{(p_1 - \epsilon)C_1}{C_1 + \hat{Q}_2^j} \leq \frac{p_1 C_1}{C_1 + Q_2^j}$ or, equivalently, $1 - \frac{\epsilon}{p_1} \leq \frac{C_1 + \hat{Q}_2^j}{C_1 + Q_2^j}$.

If conditions above both hold, then $1 + \frac{\epsilon}{p_1} + 1 - \frac{\epsilon}{p_1} \leq \frac{C_1 + \tilde{Q}_2^j}{C_1 + Q_2^j} + \frac{C_1 + \hat{Q}_2^j}{C_1 + Q_2^j}$ or, equivalently, $2 \leq \frac{2C_1 + \tilde{Q}_2^j + \hat{Q}_2^j}{C_1 + Q_2^j} \rightarrow 2Q_2^j \leq \tilde{Q}_2^j + \hat{Q}_2^j$. As $Q_2^j = \sqrt{\frac{2\lambda(K_3 + p_1 C_1 - p_2 C_1)}{p_2 I}} - C_1$ is a strictly concave function of p_1 , this is a contradiction. Thus, we conclude that Supplier 1 will deviate by charging a slightly higher or lower price. If $p_2 < p_1$, Supplier 2 will deviate by the same reasoning. $\square$

3.5. Proof of Proposition 4

Proof of Proposition 4. At (p_1, p_2) , the retailer prefers full-size orders from both suppliers. Thus, $G_3(C_1, C_2) < G_1(Q_1^s)$ and $G_3(C_1, C_2) \leq G_2(Q_2^s)$. Suppose that $G_3(C_1, C_2) < G_2(Q_2^s)$. If $p_1 < p_2$, then Supplier 1 will deviate by charging a slightly higher price, $p_1^d = p_1 + \epsilon < p_2$ as at (p_1^d, p_2) , the retailer still orders at the full-size order level from both suppliers. In this case, Supplier 1 generates a higher revenue and hence deviates. If $p_2 < p_1$, Supplier 2 will deviate by the same reasoning. If $G_3(C_1, C_2) = G_2(Q_2^s)$, then Supplier 2 will deviate by charging a slightly lower price, $p_2^d = p_2 - \epsilon$ and captures the entire market. In this case, Supplier 2 generates a higher revenue and hence deviates (see Lemma 2). $\square$

3.6. Proof of Theorem 2

Proof of Theorem 2. First, due to Theorem 1, at joint-order equilibrium prices (p_1, p_2) , we must have $Q_1^j = C_1$ and $Q_2^j = C_2$. That is, without loss of generality, if $p_1 \leq p_2$ then $p_2 \leq \frac{K_3 + p_1 C_1}{\frac{I(C_1 + C_2)^2}{2\lambda} + C_1}$ must hold. Second, due to Proposition 4, at a joint-order equilibrium, we must have $p_1 = p_2$. Therefore, joint-order equilibrium prices must satisfy $p_1 = p_2 = \tilde{p}$ where $\tilde{p} \leq \frac{2\lambda K_3}{I(C_1 + C_2)^2}$.

We divide the price space (p_1, p_2) into four regions and analyze the existence of a joint-order equilibrium in each region. In Region I, we have $Q_1' \leq C_1$ and $Q_2' \leq C_2$. In Region II, we have $Q_1' \leq C_1$ and $Q_2' > C_2$. In Region III, we have $Q_1' > C_1$ and $Q_2' \leq C_2$. In Region IV, we have $Q_1' > C_1$ and $Q_2' > C_2$.

From the proof of Lemma 2, we know that if $Q_i' \leq C_i$ for some $i \in \{1, 2\}$, then joint-order policy with $Q_1^j = C_1$ and $Q_2^j = C_2$ cannot be the optimal order policy. Therefore, a joint-order equilibrium does not exist in Regions I, II and III.

In Region IV, when $p_1 = p_2$, the retailer compares, $G_1(C_1)$, $G_2(C_2)$ and $G_3(C_1, C_2)$. At $p_1 = p_2 = \tilde{p}$ the retailer prefers joint order if either $G_3(C_1, C_2) < \min\{G_1(C_1), G_2(C_2)\}$ or $G_3(C_1, C_2) = G_2(C_2) < G_1(C_1)$ holds. First, we show that $\tilde{p} < \frac{2\lambda K_3}{I(C_1 + C_2)^2}$ cannot be a joint-order equilibrium. When $p_1 = p_2 = \tilde{p}$, we have $p_2 < \frac{K_3 + p_1 C_1}{\frac{I(C_1 + C_2)^2}{2\lambda} + C_1}$. Then, at $p_1 = \tilde{p}$ if Supplier 2 charges a price slightly higher than $\tilde{p}$, the inequality still holds, and hence, the retailer orders C_2 from Supplier 2 in joint-order policy. That is, at the prices $(\tilde{p}, \tilde{p} + \epsilon)$ the retailer still compares $G_1(C_1)$, $G_2(C_2)$ and $G_3(C_1, C_2)$, and chooses $G_3(C_1, C_2)$. Hence, Supplier 2 always deviates with a higher revenue. Therefore, we conclude that at a joint-order equilibrium, we must have $p_1 = p_2 = \tilde{p}$ with $\tilde{p} = \frac{2\lambda K_3}{I(C_1 + C_2)^2}$. Second, we show that if we have $G_3(C_1, C_2) = G_2(C_2) < G_1(C_1)$ at $p_1 = p_2 = \tilde{p}$, then (p_1, p_2) cannot be an equilibrium. When $p_1 = \tilde{p}$, if Supplier 2 deviates by charging a slightly lower price, then the retailer is still in Region IV but compares $G_1(C_1)$, $G_2(C_2)$ and $G_3(Q_1^j, C_2)$ where $Q_1^j = \sqrt{\frac{2\lambda(K_3 - \epsilon C_2)}{\tilde{p}I}} - C_2$. Since the decrease in the cost of ordering from Supplier 2 is greater than that of ordering from both suppliers, the retailer prefers single-order from Supplier 2 at $(\tilde{p}, \tilde{p} - \epsilon)$, and hence, Supplier 2 deviates (see Lemma 2).

Therefore, necessary conditions for the existence of a joint-order equilibrium are:

- (i) $p_1 = p_2 = \tilde{p} = \frac{2\lambda K_3}{I(C_1 + C_2)^2}$,
- (ii) $\frac{K_3}{K_i} < \frac{(C_1 + C_2)^2}{(C_1 + C_2)^2 - C_j^2}$ for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$.

Note that, when (ii) holds, we have $G_3(C_1, C_2) < G_i(C_i)$ for all i .

We now identify the conditions so that individual price deviations from $p_1 = p_2 = \frac{2\lambda K_3}{I(C_1 + C_2)^2}$ would not be profitable. First, we investigate slight price deviations which make the deviating supplier better off while the retailer still prefers joint-order policy. Then, we investigate considerable price cuts to make the retailer prefer ordering from one supplier only.

For slight price increases:

- Consider Supplier 1: If Supplier 1 increases its price slightly and charges $\tilde{p} + \epsilon$, we stay in the same region where the retailer compares $G_1(C_1)$, $G_2(C_2)$ and $G_3(\hat{Q}_1^j, C_2)$ and chooses $G_3(\hat{Q}_1^j, C_2)$ where $\hat{Q}_1^j$ denotes

the joint-order quantity from Supplier 1 after deviation. The deviation revenue of Supplier 1 becomes $\frac{\lambda \hat{Q}_1^j}{\hat{Q}_1^j + C_2}(\tilde{p} + \epsilon)$. Hence, Supplier 1 prefers deviation if $\frac{\hat{Q}_1^j}{\hat{Q}_1^j + C_2}(\tilde{p} + \epsilon) > \frac{C_1}{C_1 + C_2}\tilde{p}$ where $\hat{Q}_1^j = \sqrt{\frac{2\lambda(K_3 - \epsilon C_2)}{(\tilde{p} + \epsilon)I}} - C_2$. That is, deviation occurs if $\epsilon^2(IC_2^2(C_1 + C_2)^2 + 2\lambda C_2^3 + 2\lambda C_1 C_2(C_1 + 2C_2)) + \epsilon(2\tilde{p}IC_2^2(C_1 + C_2)^2 + 4\lambda\tilde{p}C_2^3 - 2\lambda K_3 C_1(C_1 + 2C_2) + 4\lambda\tilde{p}C_1 C_2^2) + \tilde{p}(\tilde{p}C_2^2(I(C_1 + C_2)^2 + 2\lambda C_2) - 4\lambda K_3 C_1 C_2) < 0$. Thus, Supplier 1 cannot deviate by charging a slightly higher price if $\tilde{p}C_2(I(C_1 + C_2)^2 + 2\lambda C_2) - 4\lambda K_3 C_1 \geq 0$. That is, $\frac{(2C_1 - C_2)(C_1 + C_2)^2}{2C_2^2} \leq \frac{\lambda}{I}$ must hold.

- Consider Supplier 2: Similar to the case above, Supplier 2 cannot deviate by charging a slightly higher price if $\frac{(2C_2 - C_1)(C_1 + C_2)^2}{2C_1^2} \leq \frac{\lambda}{I}$ holds.

For slight price decreases:

- Consider Supplier 1: If Supplier 1 decreases its price slightly and charges $\tilde{p} - \epsilon$, we stay in the same region where the retailer compares $G_1(C_1)$, $G_2(C_2)$ and $G_3(C_1, \hat{Q}_2^j)$ and chooses $G_3(C_1, \hat{Q}_2^j)$ where $\hat{Q}_2^j$ denotes the joint-order quantity of Supplier 2 after deviation. The deviation revenue of Supplier 1 becomes $\frac{\lambda C_1}{C_1 + \hat{Q}_2^j}(\tilde{p} - \epsilon)$. Hence, Supplier 1 prefers deviation if $\frac{C_1}{C_1 + \hat{Q}_2^j}(\tilde{p} - \epsilon) > \frac{C_1}{C_1 + C_2}\tilde{p}$ where $\hat{Q}_2^j = \sqrt{\frac{2\lambda(K_3 - \epsilon C_1)}{\tilde{p}I}} - C_1$. That is, deviation occurs if $\epsilon I(C_1 + C_2)^2 + 2\tilde{p}(\lambda C_1 - I(C_1 + C_2)^2) > 0$. Thus, as a necessary condition for Supplier 1 not to deviate by charging a slightly lower price we have $\lambda C_1 - I(C_1 + C_2)^2 < 0$. That is, $\frac{\lambda}{I} < \frac{(C_1 + C_2)^2}{C_1}$ must hold.
- Consider Supplier 2: Similar to Supplier 1, we have to have $\frac{\lambda}{I} < \frac{(C_1 + C_2)^2}{C_2}$ for Supplier 2 not to deviate by charging a slightly lower price.

Therefore, the third necessary condition for the existence of a joint-order equilibrium becomes:

$$(iii) \max\left\{\frac{2C_1 - C_2}{C_2^2}, \frac{2C_2 - C_1}{C_1^2}\right\} \frac{(C_1 + C_2)^2}{2} \leq \frac{\lambda}{I} < \frac{(C_1 + C_2)^2}{\max\{C_1, C_2\}}.$$

Note that condition (iii) implies that $\frac{\sqrt{17}+1}{4} > \frac{C_1}{C_2} > \frac{\sqrt{17}-1}{4}$ is a necessary condition for a joint-order equilibrium.

However, in the case of slight price deviations Supplier 1 (or similarly Supplier 2) may still deviate if $\epsilon > \frac{4\lambda K_3}{I(C_1 + C_2)^2} - \frac{4\lambda^2 K_3 C_1}{I^2(C_1 + C_2)^4} = \epsilon_0$ although $\frac{\lambda}{I} < \frac{(C_1 + C_2)^2}{C_1}$ holds. In that case, we have to make sure that the retailer prefers single-order policy (which is analyzed as considerable price deviations below) for such ϵ values. Otherwise, Supplier 1 can deviate by charging $\tilde{p} - \epsilon$ for some $\epsilon > \epsilon_0$. At $\tilde{p} - \epsilon$, we want to make sure that single-order policy is preferable:

$$\frac{\lambda K_1}{C_1} + \frac{C_1(\tilde{p} - \epsilon)I}{2} + \lambda(\tilde{p} - \epsilon) \leq \frac{\lambda K_3}{C_1 + \hat{Q}_2^j} + \frac{I}{2}(C_1(\tilde{p} - \epsilon) + \hat{Q}_2^j \tilde{p}) + \frac{\lambda}{C_1 + \hat{Q}_2^j}(C_1(\tilde{p} - \epsilon) + \hat{Q}_2^j \tilde{p})$$

where $\hat{Q}_2^j = \sqrt{\frac{2\lambda(K_3 - \epsilon C_1)}{\tilde{p}I}} - C_1$. After rearranging the terms and replacing $\tilde{p}$ by its equivalent, we obtain the following inequality:

$$\epsilon^2 + \epsilon\left(\frac{2K_3 C_1}{(C_1 + C_2)^2} - \frac{2K_1}{C_1}\right) + \left(\frac{K_1^2}{C_1^2} + \frac{K_3^2 C_1^2}{(C_1 + C_2)^4} + \frac{2K_1 K_3}{(C_1 + C_2)^2} - \frac{4K_3^2}{(C_1 + C_2)^2}\right) \leq 0.$$

The second order expression (in terms of ϵ) on the left hand side has two positive roots: ϵ_1 and ϵ_2 such that $\epsilon_1 = \frac{K_1}{C_1} - \frac{K_3 C_1}{(C_1 + C_2)^2} - \frac{2}{C_1 + C_2} \sqrt{K_3^2 - K_3 K_1}$ and $\epsilon_2 = \frac{K_1}{C_1} - \frac{K_3 C_1}{(C_1 + C_2)^2} + \frac{2}{C_1 + C_2} \sqrt{K_3^2 - K_3 K_1}$. For $\epsilon > \epsilon_2$, joint-order policy cannot be optimal since one can show that $\hat{Q}_2^j$ becomes negative in that case. When $\epsilon \in [\epsilon_1, \epsilon_2]$, single-order policy is preferable. Hence, we have to make sure that $\epsilon_0 \geq \epsilon_1$ in order to make deviation by slight price decreases non-profitable for Supplier 1. $\epsilon_0 \geq \epsilon_1$ is equivalent to following:

$$\frac{4\lambda K_3}{I(C_1 + C_2)^2} - \frac{4\lambda^2 K_3 C_1}{I^2(C_1 + C_2)^4} \geq \frac{K_1}{C_1} - \frac{K_3 C_1}{(C_1 + C_2)^2} - \frac{2}{C_1 + C_2} \sqrt{K_3^2 - K_3 K_1}.$$

After rearranging the terms, we obtain the following inequality:

$$\left(\frac{\lambda}{I}\right)^2 \frac{4K_3 C_1}{(C_1 + C_2)^4} - \frac{\lambda}{I} \frac{4K_3}{(C_1 + C_2)^2} + \left(\frac{K_1}{C_1} - \frac{K_3 C_1}{(C_1 + C_2)^2} - \frac{2}{C_1 + C_2} \sqrt{K_3^2 - K_3 K_1}\right) \leq 0.$$

If we solve the above inequality for $\frac{\lambda}{I}$, we have the following conditions on $\frac{\lambda}{I}$:

$$\frac{(C_1 + C_2)^2}{2C_1} \left(1 + \sqrt{1 - \frac{K_1}{K_3}}\right) + \frac{C_1 + C_2}{2} \geq \frac{\lambda}{I} \geq \frac{(C_1 + C_2)^2}{2C_1} \left(1 - \sqrt{1 - \frac{K_1}{K_3}}\right) - \frac{C_1 + C_2}{2}.$$

Note that the current lower bound on $\frac{\lambda}{I}$ ($\max\{\frac{2C_1 - C_2}{C_2^2}, \frac{2C_2 - C_1}{C_1^2}\} \frac{(C_1 + C_2)^2}{2}$) is tighter than the above. However, the new upper bound is tighter than the current one ($\frac{(C_1 + C_2)^2}{\max\{C_1, C_2\}}$). Hence, condition (iii) is updated as follows:

$$(iii) \max\left\{\frac{2C_1 - C_2}{C_2^2}, \frac{2C_2 - C_1}{C_1^2}\right\} \frac{(C_1 + C_2)^2}{2} \leq \frac{\lambda}{I} < \frac{C_1 + C_2}{2} + \frac{(C_1 + C_2)^2}{2} \min\left\{\frac{1 + \sqrt{1 - \frac{K_1}{K_3}}}{C_1}, \frac{1 + \sqrt{1 - \frac{K_2}{K_3}}}{C_2}\right\}.$$

For considerable price deviations:

- Consider Supplier 1 and let $\hat{p}_1$ be the deviation price. Deviation is possible at $(\hat{p}_1, \tilde{p})$ if $G_1(C_1) \leq G_3(C_1, \hat{Q}_2^j)$ where $\hat{Q}_2^j = \min\{C_2, \max\{0, \sqrt{\frac{2\lambda(K_3 + \hat{p}_1 C_1 - \tilde{p} C_1)}{\tilde{p} I}} - C_1\}\}$. This is equivalent to having $\frac{\hat{Q}_2^j}{C_1 + \hat{Q}_2^j} (\hat{p}_1 - \tilde{p}) \leq \frac{K_3}{C_1 + \hat{Q}_2^j} - \frac{K_1}{C_1} + \frac{K_3}{(C_1 + C_2)^2} \hat{Q}_2^j$. Deviation is profitable if $\frac{\hat{p}_1}{\tilde{p}} > \frac{C_1}{C_1 + C_2}$. Hence, if $G_1(C_1) \leq G_3(C_1, \hat{Q}_2^j)$ when $\hat{p}_1 > \frac{C_1}{C_1 + C_2} \tilde{p}$, Supplier 1 deviates. Moreover, using borderline condition $\hat{p}_1 = \frac{C_1}{C_1 + C_2} \tilde{p}$ and $\tilde{p} = \frac{2\lambda K_3}{I(C_1 + C_2)^2}$, we obtain $\hat{Q}_2^j = \max\{0, \sqrt{(C_1 + C_2)^2 - \frac{2\lambda C_1 C_2}{(C_1 + C_2)I}} - C_1\}$.
- Similar to the case above, Supplier 2 deviates by charging $\hat{p}_2$ such that $\frac{\hat{p}_2}{\tilde{p}} > \frac{C_2}{C_1 + C_2}$ if $G_2(C_2) < G_3(\hat{Q}_1^j, C_2)$ where $\hat{Q}_1^j = \max\{0, \sqrt{(C_1 + C_2)^2 - \frac{2\lambda C_1 C_2}{(C_1 + C_2)I}} - C_2\}$. $\square$

3.7. Proof of Proposition 5

Proof of Proposition 5. When $\min\{\frac{K_1}{C_1}, \frac{K_2}{C_2}\} < \frac{K_3}{C_1 + C_2}$ and $\frac{K_1}{C_1} \neq \frac{K_2}{C_2}$, a threshold price exists and hence the supplier with the lowest per unit fixed cost has a tendency to deviate from charging 0 when the other supplier charges 0. Thus, the necessary condition for the existence of an equilibrium at $(0, 0)$ is $\frac{K_3}{C_1 + C_2} \leq \min\{\frac{K_1}{C_1}, \frac{K_2}{C_2}\}$ or $\frac{K_1}{C_1} = \frac{K_2}{C_2}$. If $\frac{K_1}{C_1} = \frac{K_2}{C_2}$, then we have $\frac{K_3}{C_1 + C_2} \leq \frac{K_1 + K_2}{C_1 + C_2} = \frac{K_1}{C_1} = \frac{K_2}{C_2}$ since $K_3 \leq K_2 + K_1$. Hence, $\frac{K_3}{C_1 + C_2} \leq \min\{\frac{K_1}{C_1}, \frac{K_2}{C_2}\}$ is a necessary condition.

In order to establish the sufficiency, consider, for example, Supplier 1. Suppose $\frac{K_3}{C_1+C_2} < \frac{K_2}{C_2}$ holds. When $p_2 = 0$, the best response of Supplier 1 is charging a low enough price strictly greater than 0. Thus, $(0, 0)$ is an equilibrium if and only if $\frac{K_3}{C_1+C_2} = \frac{K_i}{C_i}$ for all i .

We prove the uniqueness as follows. When $\frac{K_1}{C_1} = \frac{K_2}{C_2} = \frac{K_3}{C_1+C_2}$, a necessary condition for the existence of a joint-order equilibrium at a price $p_1 = p_2 = \tilde{p} \neq 0$ is $\frac{\hat{Q}_2}{C_1+\hat{Q}_2} \frac{C_2}{C_1+C_2} \tilde{p} < \frac{K_1}{C_1} - \frac{K_3}{C_1+\hat{Q}_2} - \frac{K_3\hat{Q}_2}{(C_1+C_2)^2}$, where $\hat{Q}_2 = \max\{0, \sqrt{(C_1+C_2)^2 - \frac{2\lambda C_1 C_2}{(C_1+C_2)I}} - C_1\}$. However, since $\frac{K_1}{C_1} - \frac{K_3}{C_1+\hat{Q}_2} < 0$, this condition does not hold. Hence, a joint-order equilibrium with positive prices does not exist. It is clear that a threshold-pricing equilibrium does not exist either. Hence, $p_1 = p_2 = 0$ is the unique equilibrium. $\square$

3.8. Proof of Corollary 1

Proof of Corollary 1. Suppose $\frac{K_1}{C_1} = \frac{K_2}{C_2}$. Then, a threshold-pricing equilibrium does not exist. First, we identify the cost of the retailer when there is no cost synergy under joint-order procurement. When $K_3 = K_1 + K_2$, due to Proposition 5, there exists a joint-order equilibrium at $p_1 = p_2 = 0$. Then, the cost of the retailer becomes $\frac{\lambda}{C_1+C_2}(K_1 + K_2)$.

Second, we identify the cost of the retailer under the lowest joint-order cost environment. Without loss of generality, assume that $K_1 \leq K_2$. When $K_3 = K_2$, due to Theorem 2, if there exists a joint-order equilibrium, it occurs at $p_1 = p_2 = \frac{2\lambda K_2}{I(C_1+C_2)^2}$. Then, the cost of the retailer becomes $\frac{2\lambda}{C_1+C_2}K_2 + \lambda p_1$. Hence, we conclude that when suppliers have identical per unit fixed costs, the retailer prefers no cost synergy in joint-order procurement. $\square$