On the Number and Membership Size of Consumer Managed Firms*

I. Introduction

Anderson, Porter, and Maurice [1] recently examined consumer-managed firms in a utility-maximization framework. They derived conditions for optimum production, pricing, and membership size of the consumer cooperative from the individual member's viewpoint, and related their results to the perfectly competitive firm. In this paper, their results are extended in a number of ways. First for the entire economy, the welfare-maximizing properties of the Anderson, Porter, and Maurice (APM) solution are shown to depend on the number of consumer cooperatives *and* the number of members in the cooperatives relative to the number of nonmembers. When only one cooperative is formed, the APM solution may or may not yield an *economy-wide* welfare maximum in spite of the fact that each member is enjoying maximum benefits. Therefore, an alternative formulation is provided that ensures an economy-wide welfare maximum. Second, when multiple cooperatives form, the APM solution is likely to yield a welfare maximum, although the cooperatives may or may not be stable in the sense that members may continually have incentives to transfer among the cooperatives.

II. The One-Cooperative Case

Following APM, the consumer-managed firm is organized and managed costlessly by identical members who can exclude nonmembers at no cost.¹ Each member is characterized by the strictly quasi-concave and twice-differentiable utility function, $U(q_x, q_y)$, where q_x is the cooperative good and q_y is a composite of all other goods. The cost of producing the cooperative good is $C(Q_x)$, where $Q_x \equiv mq_x$ is the total output of the cooperative, since m is the number of cooperative members.² Each member faces a budget constraint of the form:

$$Z = q_y + C(Q_x)/m. \quad (1)$$

Here, Z is income, the price of the composite good (P_y) is one, and since all members are identical, the production costs for the cooperative are equally shared.

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1. The ability to exclude nonmembers while sharing in the production of a good makes the problem very much like that found in the theory of clubs. For a survey of club theory and a discussion of conditions somewhat similar to those derived herein, see Sandler and Tschirhart [3].

2. We use the same notation as APM except that we set P_y equal to one. Our normalization of P_y simplifies the presentation.

APM maximize $U(q_x, q_y)$ subject to (1) and, in so doing, maximize the average utility of cooperative members. Substituting (1) into the utility function and taking derivatives with respect to q_x and m yields the following necessary conditions for a maximum:

$$U_x/U_y = C' \quad (2)$$

$$C' = C(Q_x)/Q_x \quad (3)$$

where

$$C' \equiv dC/dQ_x, U_x \equiv \partial U/\partial q_x, \text{ and } U_y \equiv \partial U/\partial q_y.$$

The optimal output condition (2) for the cooperative states that the marginal rate of substitution for each member must equal the ratio of marginal costs between the cooperative and composite goods. The marginal cost of the composite good does not explicitly appear in our representation, since P_y and, consequently, the composite good's marginal cost are equal to one.³ Condition (3) is the membership requirement. As APM state, "... the number of members in the cooperative is chosen so that average cost equals marginal cost" [1, 122]. Since APM [1, 121, 126] demonstrate that the effective price of the cooperative good equals average production costs (i.e., $P_x = C(Q_x)/Q_x$), the optimal effective price implied by (3) is *minimum average cost*, because nowhere else is average and marginal cost equal.

Conditions (2) and (3) ensure that each member receives the greatest possible benefit from participation in the cooperative. However, there is a possibility that the welfare of the economy as a whole is not at a maximum, and a larger membership would be Pareto-superior. By way of example, consider Table I where the economy consists of 4 individuals. The total utility values are the maximum utilities available to a cooperative consisting of m members. The average utility values can be thought of as solutions to the APM problem where the membership is fixed in each problem at $m = 1, \dots, 4$. When m is allowed to vary, the APM solution given by conditions (2) and (3) is a cooperative of three members where the average utility is maximized. Since the population consists of four consumers, a three-member cooperative implies one nonmember. This nonmember can be thought of as forming a one-member cooperative; a cooperative where the individual is self-sufficient in supplying the good.

In the APM solution, total utility for the economy is 9.5: 9 for the three-member cooperative and .5 for the nonmember. *However, if APM's requirement that all members of a cooperative receive equal shares is relaxed, a Pareto-superior solution can be obtained.* By admitting the nonmember to the three-member cooperative, total utility for the economy is increased to 10. The gain in utility of .5 can be apportioned to the four members so that none are made worse off and at least one is made better off as a result of the admittance.

This example suggests that maximizing *total* welfare for the cooperative is Pareto-superior to maximizing average welfare. This new problem is to maximize

$$mU[q_x, Z - C(Q_x)/m], \quad (4)$$

and the necessary conditions for optimum production and membership are given by (2) and (5) respectively.

$$U(\cdot)/U_y = [C'q_x - C(Q_x)/m]. \quad (5)$$

3. If P_y is not one, then (2) becomes (2') $U_x/U_y = C'/P_y$. For a comparative static analysis, P_y should not be set equal to one.

Table I.

m	1	2	3	4
Total Utility	.5	4	9	10
Average Utility	.5	2	3	2.5

Since the term in parenthesis is positive, optimum membership calls for marginal cost to exceed average cost, or a larger cooperative than that characterized by (2) and (3).⁴ Consequently, the optimal effective price of q_x will not be equal to minimum average cost when total welfare is maximized for the cooperative. Returning to Table I, the satisfaction of (2) and (5) corresponds to a cooperative of four members, or the entire population.

Unfortunately, this procedure does not always guarantee a welfare maximum either. Manipulating the numbers in Table I can make the admittance of individual 4 a Pareto-inferior move. For example, in Table II the numbers for a one-member cooperative have been changed so that admitting a fourth member to a three-member cooperative is undesirable. Maximizing average utility in this case yields a welfare maximum, while maximizing total welfare of the cooperative does not.

To avoid this ambiguity, the problem must be stated in terms of both members and nonmembers. An economy-wide welfare maximum is then attained for the single-cooperative case, where no other source of supply for the cooperative good exists.⁵ Let the total number of identical individuals in the economy be denoted by $\bar{m}$, and the utility of a nonmember be denoted by $\tilde{U}(0, \bar{q}_y)$ where $\bar{q}_y$ is the nonmember's consumption of the composite good. The production of the two goods is given by the transformation function, $F[mq_x, mq_y + (\bar{m} - m)\bar{q}_y]$. The problem is to maximize the Lagrangian expression⁶

$$L = mU(q_x, q_y) + (\bar{m} - m) \tilde{U}(0, \bar{q}_y) + F[mq_x, mq_y + (\bar{m} - m)\bar{q}_y]. \quad (6)$$

Taking the derivatives of this expression with respect to q_x , q_y , $\bar{q}_y$, and m yields the following conditions for a maximum.

$$U_x/U_y = F_x/F_y \quad (7)$$

where $F_x \equiv \partial F/\partial Q_x$, $F_y \equiv \partial F/\partial Q_y$, and $Q_y = mq_y + (\bar{m} - m)\bar{q}_y$.

$$U_y = \tilde{U}_y \quad (8)$$

$$U(\cdot) - \tilde{U}(\cdot) = U_x q_x + U_y(q_y - \bar{q}_y) \quad (9)$$

4. If P_y is not one, then (2') and (5') represent the necessary conditions, where (5') is $U(\cdot)/U_y = P_y^{-1}[C'q_x - C(Q_x)/m]$. Equation (5') implies that marginal cost exceeds average cost at the optimal membership; hence, the normalization of P_y does not affect our results.

5. This assumption is used to simplify the presentation and to highlight the membership benefits of the cooperative. If, however, nonmembers can purchase the good from firms, the right-hand side of the membership condition (9) becomes: $U_x(q_x - \bar{q}_x) + U_y(q_y - \bar{q}_y)$. A similar change would occur in (9'). Otherwise, our analysis remains unchanged when this alternative supply source is allowed.

6. This formulation presumes a Benthamite social welfare function. The problem could also be formulated in a Pareto-optimum framework with the same results; however, some additional notation would be required.

Table II.

m	1	2	3	4
Total Utility	1.5	4	9	10
Average Utility	1.5	2	3	2.5

Output condition (7) is the familiar result that the marginal rate of substitution in consumption is equal to the marginal rate of product transformation. Condition (8) requires that the marginal utility of the noncooperative good be the same for both members and nonmembers. Finally, (9) is the membership condition which illustrates the discreteness of the membership decision. In going from nonmember to member status, an individual experiences a discrete change in the consumption of both goods: from zero to q_x and from $\bar{q}_y$ to q_y . Thus, the right-hand side of (9) is the change in utility experienced by the marginal entrant. The left-hand side is the gain in utility over nonmembership experienced by each member of the cooperative. This gain is the average change in utility, and so the cooperative is expanded until these average and marginal utility changes are equal.

III. Discrimination, Stability, and Multiple Cooperatives

Given the various formulations of the cooperative problem, what membership size is most likely to be observed? In a world of equals, a strong case can be made for maximizing average utility as done by APM. The cooperative is a voluntary venture whose members seek the greatest possible payoffs from participation. This is accomplished where the member's average payoff is at a maximum. Whether or not this is an "optimum" solution depends on the viewpoint: from the member's viewpoint it is; however, from the entire economy's viewpoint, it may not be. This suggests that maximizing the average will be the observed outcome, provided that the members can thwart any attempts from the nonmembers at upsetting this outcome. If equal cost sharing is used, nonmembers will be unsuccessful in any attempt to join the cooperative, since their entry is at the expense of the existing members who enjoy maximum payoffs.

However, if unequal cost sharing is allowed via some form of price discrimination, membership size will grow beyond where the average is maximized until the welfare for the entire economy is maximized. Unequal cost shares will encourage members who have reached their maximum payoff to accept additional members, since the acceptance does not have to be at the expense of the existing members. Referring to Table I, a three-member cooperative would be willing to admit a fourth member, provided the entrant accepts a payoff (in terms of utility) of 1 or less. In that way, the three members maintain or exceed their payoffs of 3. And the entrant would be willing to join the cooperative, provided the payoff equals or exceeds .5.

A problem arises, however, whenever the average is maximized at a membership size

that is smaller than the entire population of potential cooperative members. The membership will not reach a stable size or composition, because there always exists incentives to disrupt the status quo. That is, any nonmember or group of nonmembers can seek to replace a member or group of members by their willingness to accept smaller payoffs than the existing members are receiving. For example, in Table I, nonmember four can replace any member by accepting a payoff less than 3 (but greater than .5). The other two members would not object, provided they receive a share of the ejected member's payoff. But the ejected member is then on the outside, and in a position to disrupt the cooperative by bidding back into it. The process is never ending; instability characterizes the solution.⁷

Allowing now for multiple cooperatives, the instability problem may be solved. If the total population is an integer multiple of the optimal membership size when average utility is maximized, a stable welfare maximum is attained where the entire population is distributed between the cooperatives. All cooperatives are providing members with their maximum possible payoffs, and total welfare, therefore, is maximized. In addition, there are no incentives for members to transfer among cooperatives, when all cooperatives provide the same payoff. Any cooperative that practices price discrimination will find either its discriminating members being replaced by members of other cooperatives, or those members discriminated against leaving for other cooperatives. Consequently, the existence of multiple cooperatives curtails the degree of discrimination. This would seem to be the equilibrium solution of APM.

The instability problem with multiple cooperatives resurfaces, if the population is *not an integer multiple*. For then, any nonmembers have the ability and incentive to disrupt the cooperatives. And again, an optimum-sized cooperative is elusive, because of the differing viewpoints.

IV. Heterogeneous Individuals

When individuals are not identical, the outcomes of cooperative formation are considerably more complicated. Maximizing average utility is not useful, since members' contributions to the cooperative vary. The average does not mean much to a member who contributes significantly more or less than others. Nevertheless, as APM indicate, the output where the average cost of the cooperative good is minimized defines the optimum provision level. But the number of cooperative members is immaterial; the optimum can result from several "large" members or numerous "small" members.

To maximize welfare for an entire economy of heterogeneous consumers with one cooperative, (6) must be modified as follows:

$$L = \sum_{i=1}^m U^i(q_x^i, q_y^i) + \sum_{i=m+1}^{\bar{m}} \bar{U}^i(0, \bar{q}_y^i) + \lambda F \left(\sum_{i=1}^m q_x^i, \sum_{i=1}^m q_y^i + \sum_{i=m+1}^{\bar{m}} \bar{q}_y^i \right) \quad (10)$$

where superscripts refer to individuals. The membership condition, previously (9), now becomes (9')

7. That there exist incentives to disrupt the cooperative is equivalent to stating that the distribution of payoffs is not in the core of the game of cooperative formation. This is closely related to Pauly's [2] work on cores and club formation. Pauly illustrates the limits of price discrimination by using the definition of the core.

$$U^m - \tilde{U}^m = U_x^m q_x^m + U_y^m (q_y^m - \tilde{q}_y^m) \quad (9')$$

which is in terms of the marginal entrant. The size of the cooperative is determinant, given the ordering implied in numbering the individuals from 1 to m . Different orderings will yield different cooperative sizes.

Perhaps the most important aspect of moving from a homogeneous to a heterogeneous population is the introduction of significantly greater transaction costs, since heterogeneous fees would have to be charged to each member. Moreover, heterogeneous tastes would have to be analyzed when making allocative decisions. The introduction of transaction costs may expand or contract optimal membership for the cooperative. Scale economies of administration tend to expand optimal membership, while bargaining costs tend to reduce optimal membership.

V. Conclusions

If the total utility of the collective is maximized, the consumer-managed firm will not reach the same long-run equilibrium as the perfectly competitive firm, because the optimal effective price of q_x is greater than minimum average cost. When multiple cooperatives form, average utility is maximized, and the number of cooperatives divides evenly into the population (i.e., no integer problem), the consumer-managed firm then reaches the perfectly competitive long-run equilibrium as characterized by price equals minimum average cost.

Although APM analysis is correct for the problem that they examined, the analysis here has extended their study. In particular, this paper has shown that the pricing and membership decisions for the consumer cooperative are responsive to the following: the maximizing viewpoint, whether equal cost sharing is required, the number of cooperatives, and the ratio of population to the number of cooperatives. By introducing the number of cooperatives, this paper has analyzed whether memberships of the cooperatives will be stable or unstable.

Todd Sandler
John Tschirhart
University of Wyoming
Laramie, Wyoming

References

1. Anderson, Richard K., Philip K. Porter, and S. Charles Maurice, "The Economics of Consumer-Managed Firms." *Southern Economic Journal*, July 1979, 119-30.
2. Pauly, Mark V., "Clubs, Commonality, and the Core: An Integration of Game Theory and the Theory of Public Goods." *Economica*, August 1967, 314-24.
3. Sandler, Todd and John Tschirhart, "The Economic Theory of Clubs: An Evaluative Survey," *Journal of Economic Literature*, December 1980, 1481-1521.